Relationship between Fourier transform and Laplace transform

Fourier transform

$$F[x(t)] = X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt \quad (or)$$

$$= X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad \text{— ①}$$

Laplace transform

$$L[x(t)] = X(S) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$S = \sigma + j\omega$$

$$X(S) = \int_{-\infty}^{\infty} x(t)\, e^{-(\sigma+j\omega)t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt \quad \text{— ②}$$

comparing ① & ② equations

$$L[x(t)] = X(S) = F[x(t) \cdot e^{-\sigma t}]$$

If $\sigma = 0$,

$$L[x(t)] = F[x(t)]$$

$$X(S) = X(j\omega) \quad \text{where } s = j\omega$$

convergence of Laplace transform [condition for existance]

If F.T of $x(t)\, e^{-\sigma t}$ exist, then $L[x(t)]$ also exist.

F.T of $x(t)\, e^{-\sigma t}$ must be absolutely integrable

$$\int_{-\infty}^{\infty} |x(t)\, e^{-\sigma t}|\, dt < \infty \Rightarrow \text{condition}$$

ROC [Region of Convergence]

The range of value of σ for which the laplace transform convergence is called Region of convergence [ROC]