SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

COIMBATORE-35

Accredited by NBA-AICTE and Accredited by NAAC – UGC with A+ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF BIOMEDICAL ENGINEERING

COURSE NAME: 19BMT301 Bio Control Systems

III YEAR / V SEMESTER

Unit 1–Introduction To Physiological Modelling

Topic 1: Mathematical Modelling of Systems

What We'll Discuss

TOPIC OUTLINE

Transfer Function

Mechanical System

Electrical System

Introduction

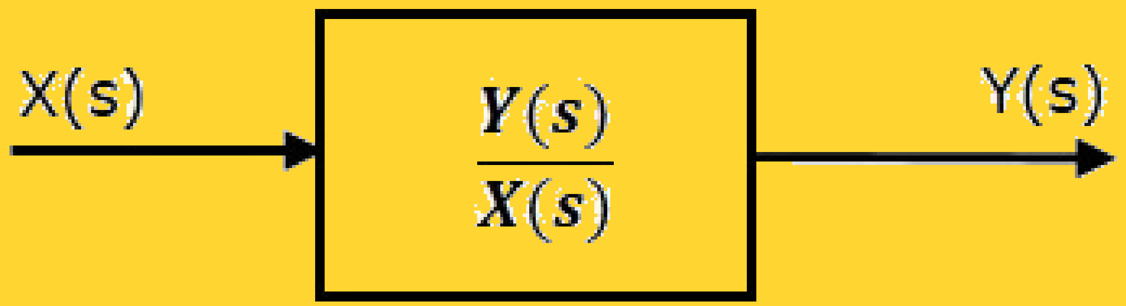
- The control systems can be represented with a set of mathematical equations known as mathematical model. These models are useful for analysis and design of control systems.
- The following mathematical models are mostly used.
  - ✓ Differential equation model
  - ✓ Transfer function model
  - ✓ State space model

Mathematical Model

- A mathematical model is a set of equations (usually differential equations) that represents the dynamics of systems.
- In practice, the complexity of the system requires some assumptions in the determination model.
- How do we obtain the equations?
  - Physical law of the process
  - Examples:
    - ✓ Mechanical system (Newton's laws)
    - ✓ Electrical system (Kirchhoff's laws)

Transfer Function

- Transfer function model is an s-domain mathematical model of control systems.
- The Transfer function of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.



Mechanical System

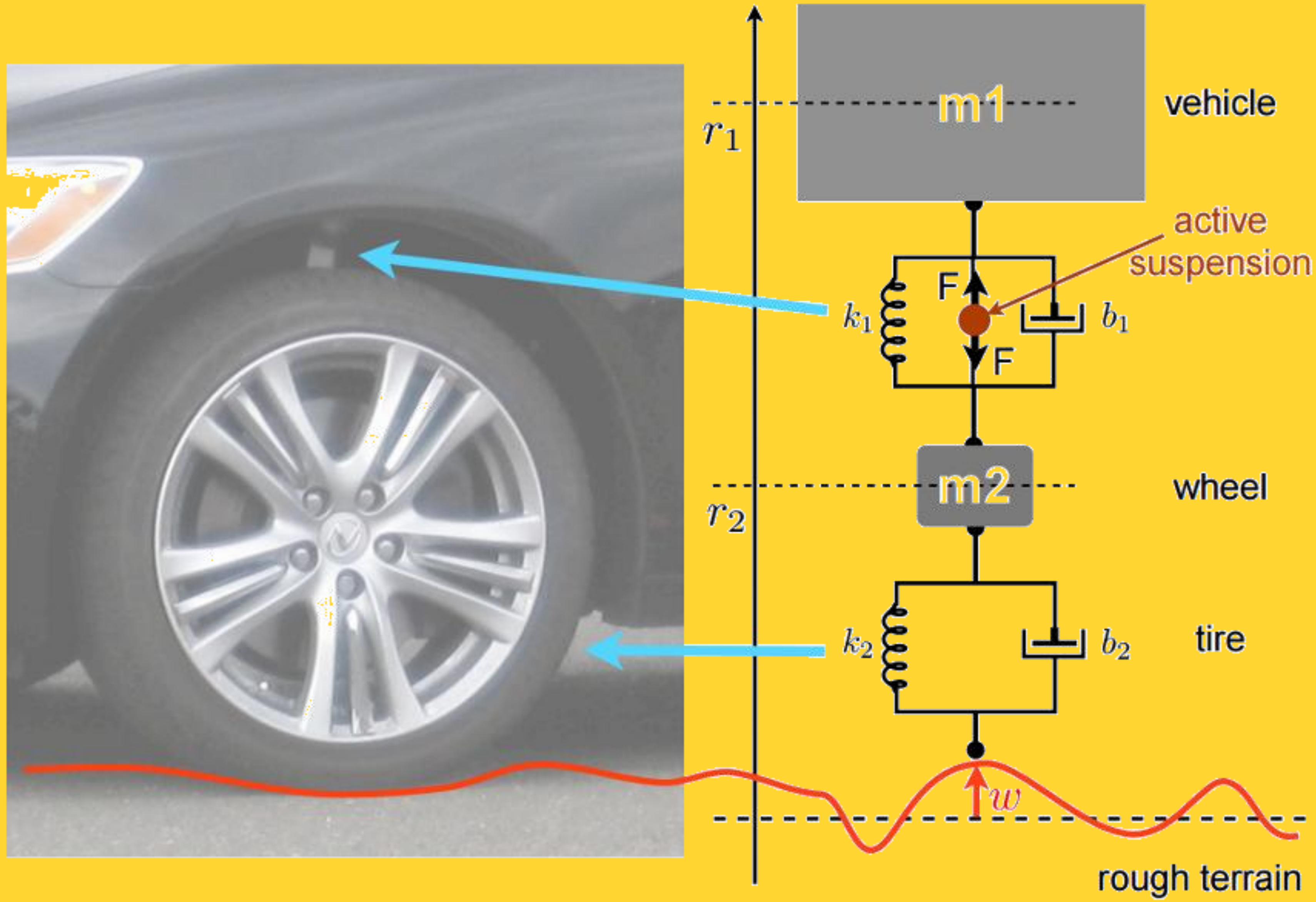
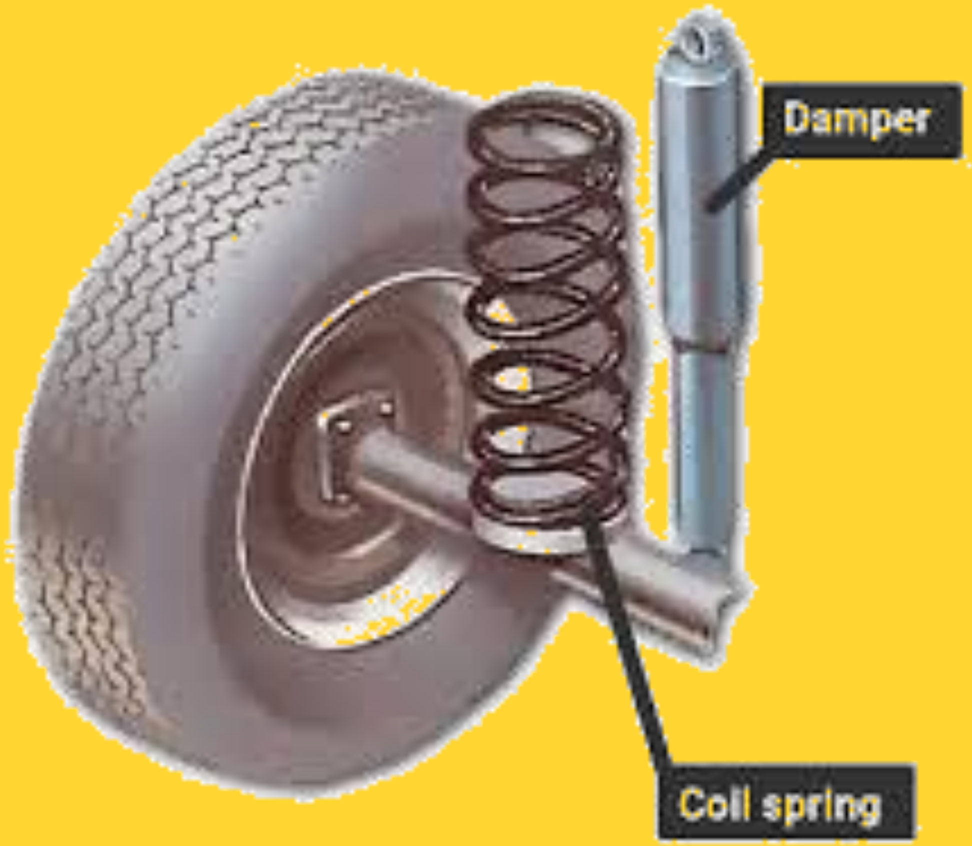


Figure 1: Scheme of an active vehicle suspension system

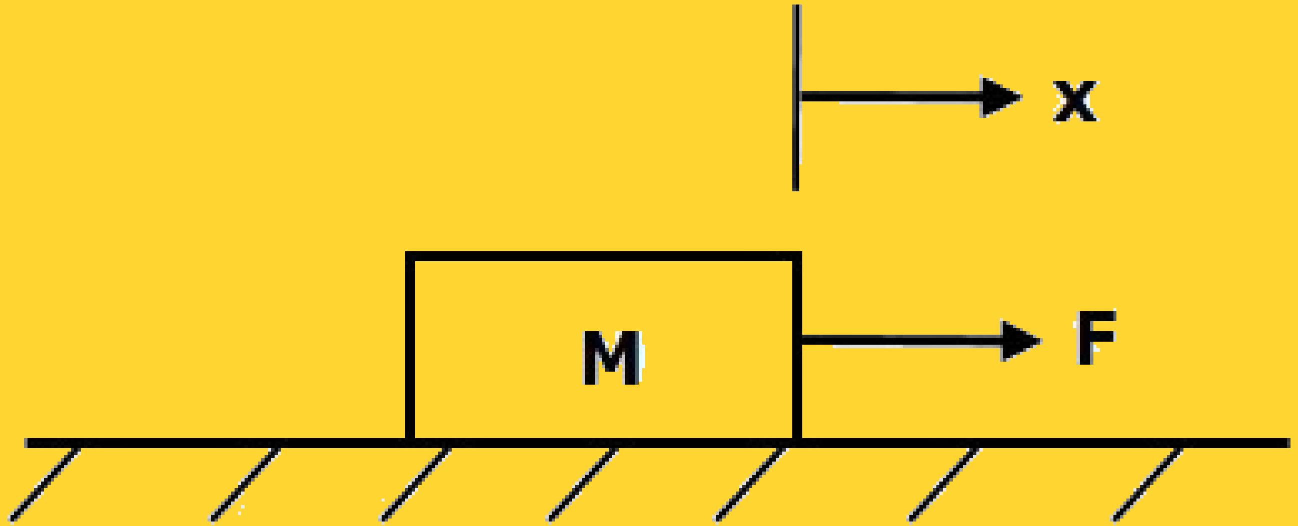


Mechanical System

- Mechanical systems mainly consists of three main elements namely mass, dashpot and spring.
- If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system.
- Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero.

Mechanical System

- Mass:



$$F_m \propto a$$

$$F_m = M_a = M\frac{d^2x}{dt^2}$$

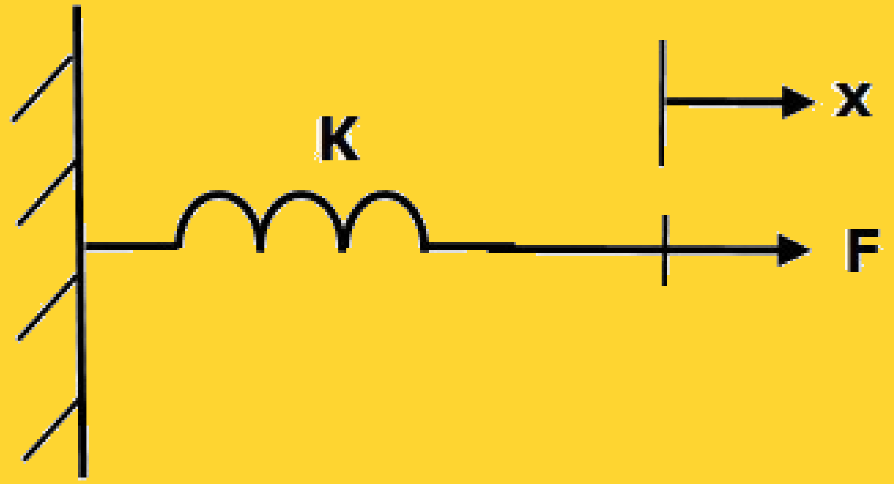
$$F = F_m = M\frac{d^2x}{dt^2}$$

Where,

- **F** is the applied force
- **F_m** is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement

Mechanical System

- Spring:



$$F \propto x$$

$$F_k = Kx$$

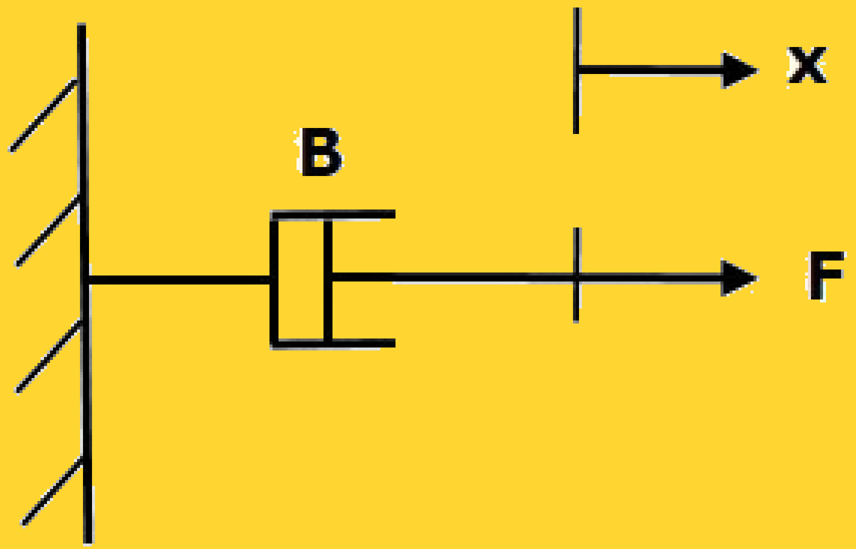
$$F = F_k = Kx$$

Where,

- **F** is the applied force
- **F_k** is the opposing force due to elasticity of spring
- **K** is spring constant
- **x** is displacement

Mechanical System

- Dashpot:



$$F_b \propto v$$

$$F_b = Bv = B\frac{dx}{dt}$$

$$F = F_b = B\frac{dx}{dt}$$

Where,

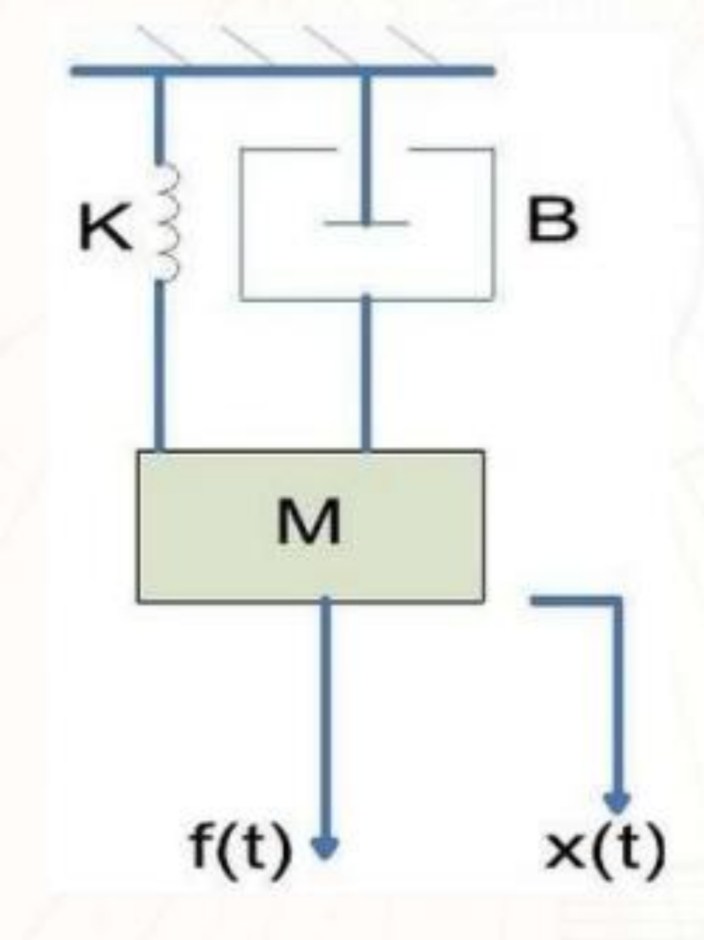
- **F** is the applied force
- **F_k** is the opposing force due to friction of dashpot
- **B** is spring constant frictional coefficient
- v is velocity
- x is displacement

Mechanical System

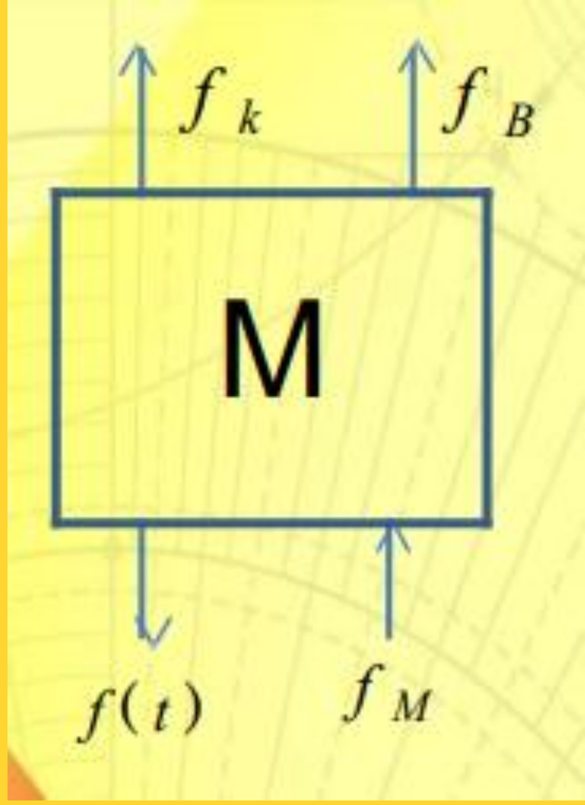
- First, draw a free-body diagram, placing on the body all forces that act on the body either in the direction of motion or opposite to it.
- Second, use Newton’s law to form a differential equation of motion by summing the forces and setting the sum equal to zero.
- Finally, assuming zero initial conditions, we take the Laplace transform of the differential equation, separate the variables, and arrive at the transfer function.

Transfer Function of Mechanical System

- Find the transfer function of the mechanical translational system given in figure.



$$f(t) = f_k + f_M + f_B$$



$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

Electrical System

Resistance

V-I in time domain

$$v_R(t) = i_R(t)R$$

V-I in s domain

$$V_R(s) = I_R(s)R$$

Inductance

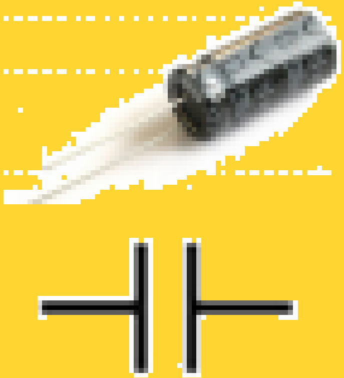
V-I in time domain

$$v_L(t) = L\frac{di_L(t)}{dt}$$

V-I in s domain

$$V_L(s) = sLI_L(s)$$

Capacitance



V-I in time domain

$$v_c(t) = \frac{1}{C}\int i_c(t)dt$$

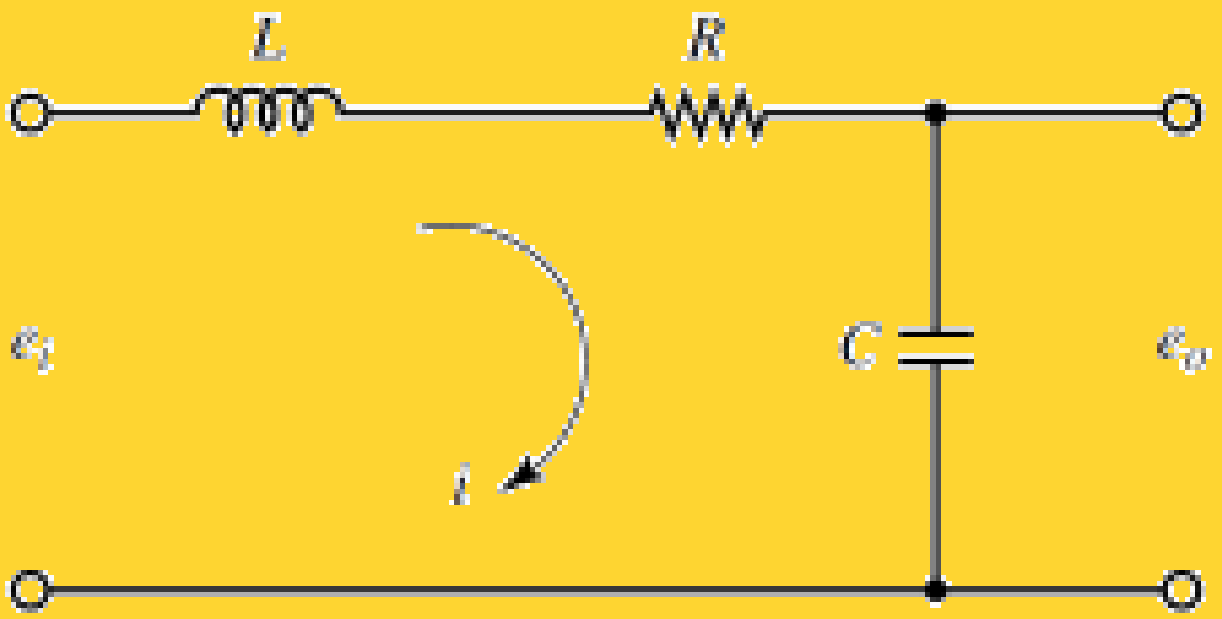
V-I in s domain

$$V_c(s) = \frac{1}{Cs}I_c(s)$$

Electrical System

Find the transfer function $G(s) = E_o(s) / E_i(s)$ of the RLC network

Applying the Kirchhoff's voltage law:



RLC circuit

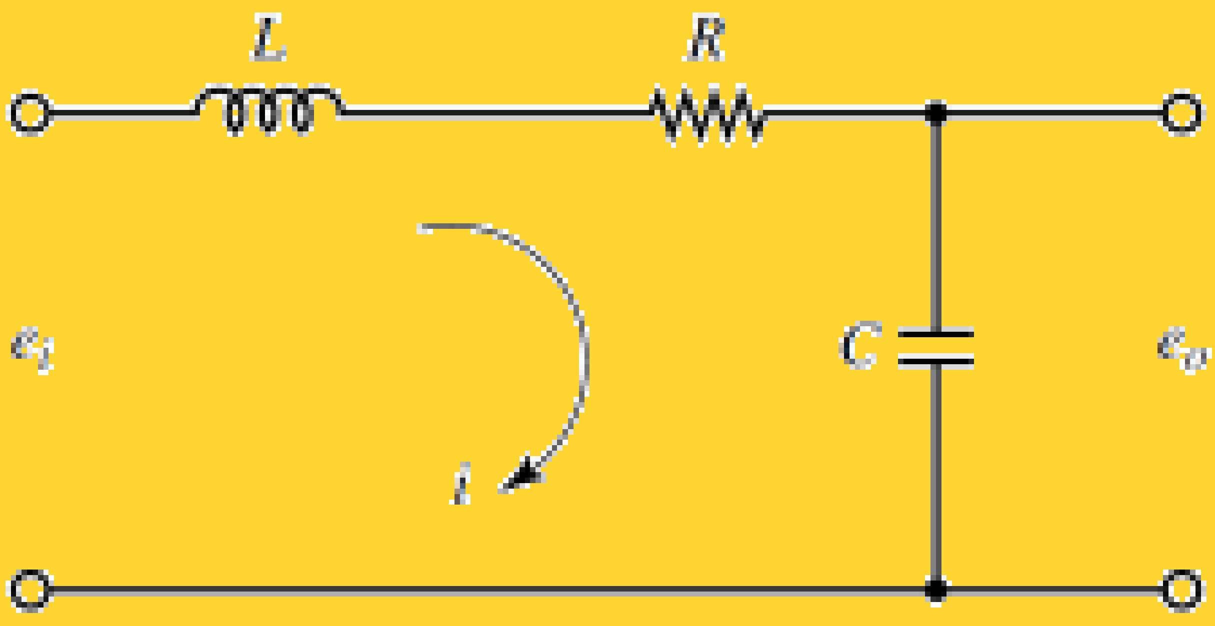
$$\boxed{\sum V = 0}$$

$$e_i(t) - L\frac{di}{dt} - R.i - \frac{1}{C}\int i\,dt = 0$$

$$\frac{1}{C}\int i\,dt = e_o$$

Electrical System

Find the transfer function $G(s) = E_o(s) / E_i(s)$ of the RLC network



RLC circuit

Taking Laplace transform with zero initial conditions:

$$L.s.I(s) + RI(s) + \frac{1}{C}\frac{1}{s}I(s) = E_i(s)$$

$$\frac{1}{C}\frac{1}{s}I(s) = E_o(s)$$

So,

$$\boxed{G(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1}}$$

RECALL TIME

ASSESSMENT TIME

THANK YOU