Method of Pattern Multiplication

The total field pattern of an array of non-isotropic but similar point sources is the multiplication of the individual source patterns and the pattern of an array of isotropic point sources each located at the phase centre of individual source and having the same relative amplitude and phase, while the total phase pattern is the addition of phase pattern of the individual sources and the array of isotropic point sources.

The total field pattern of an array of non-isotropic but similar sources may be written as,

$$E = \{ E_i(\theta,\phi) \times E_a(\theta,\phi) \} \times \{ E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi) \}$$

E = (Multiplication of field pattern) × (Addition of phase pattern)

This principle can be applied to any number of sources provided only that they are similar.

$E_i(\theta,\phi)$ → Field pattern of individual source

$E_a(\theta,\phi)$ → Field pattern of array of isotropic point sources

$E_{pi}(\theta,\phi)$ → phase pattern of individual source

$E_{pa}(\theta,\phi)$ → phase pattern of array of isotropic point sources.

$\theta \rightarrow$ polar angle

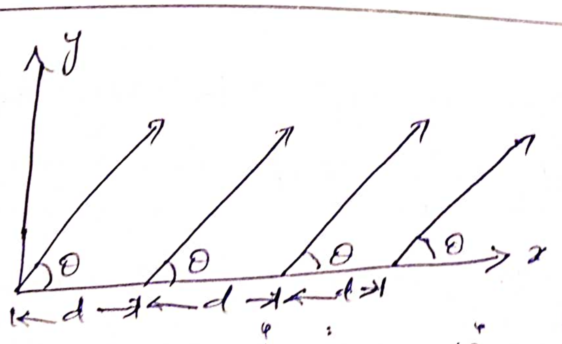
$\phi \rightarrow$ azimuth angle.

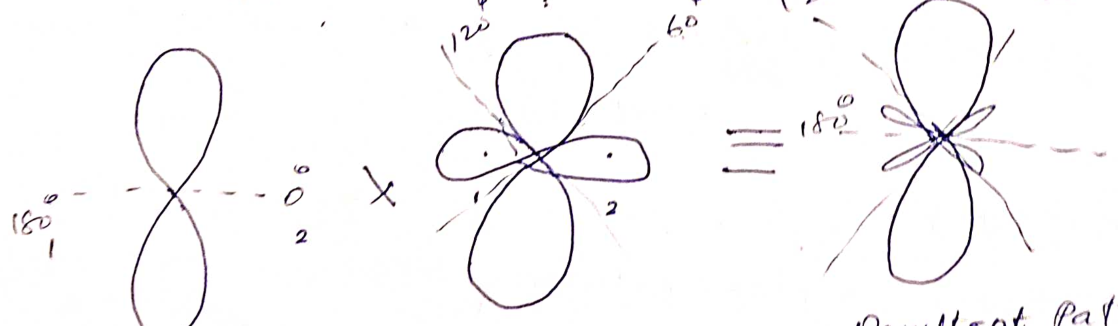
Features

* Speedy method for sketching the pattern of complicated arrays just by inspection.
* Useful tool in design of antenna arrays.
* Width of principle lobe & the corresponding width of array pattern are same.
* Secondary lobes are determined from the number of nulls in the resultant pattern.
* No. of nulls in the resultant pattern are equal to sum of nulls of individual pattern & array pattern.

Ex: 1

Radiation pattern of 4 isotropic ~~#~~ elements fed in phase ~~[illegible]~~ & spaced $\lambda/2$ apart.





Individual (or) Unit pattern

Group pattern

Resultant pattern of four isotropic elements

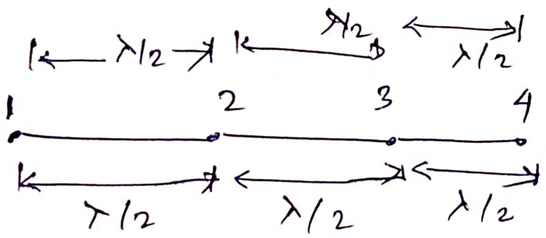
Unit pattern → Pattern due to individual source.

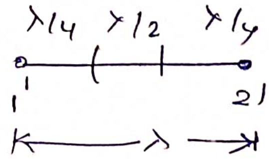
Group pattern → due to an array of two isotropic elements.

Let the 4 isotropic elements fed in phase, spaced $\lambda/2$ apart. Two isotropic points sources spaced $\lambda/2$ apart & fed in phase provides a bidirectional pattern as shown in figure.

Elements 1 & 2 are considered as one unit and placed between the midway of elements. Similarly elements 3 & 4 are considered as another unit & placed between the two elements.

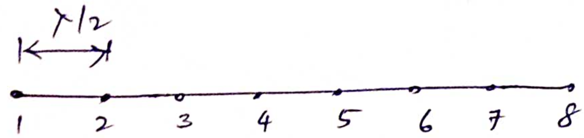
The two units (1' & 2') have the same radiation pattern as in figure & the radiation pattern of two isotropic antennas spaced λ as shown in figure.

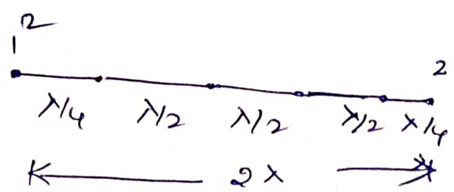


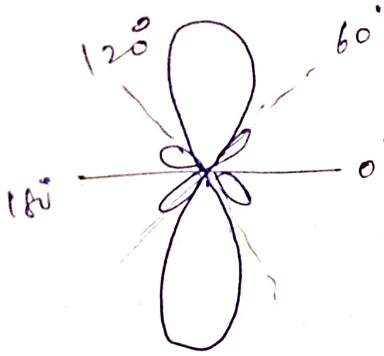


Ex. 2

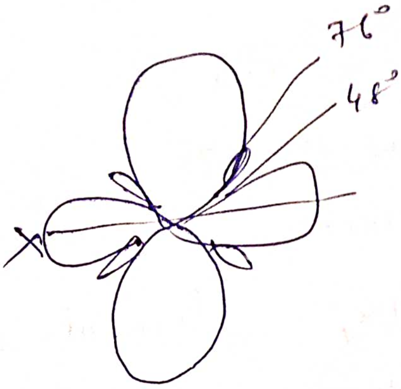
Radiation pattern of 8 isotropic elements fed in ~~phase~~. and spaced $\lambda/2$ apart





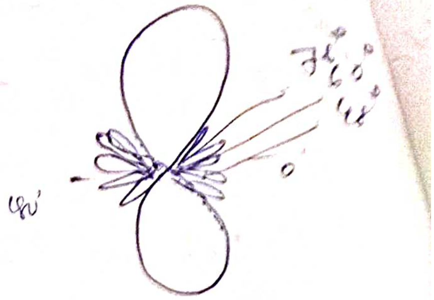


Unit pattern due to 4 individual elements



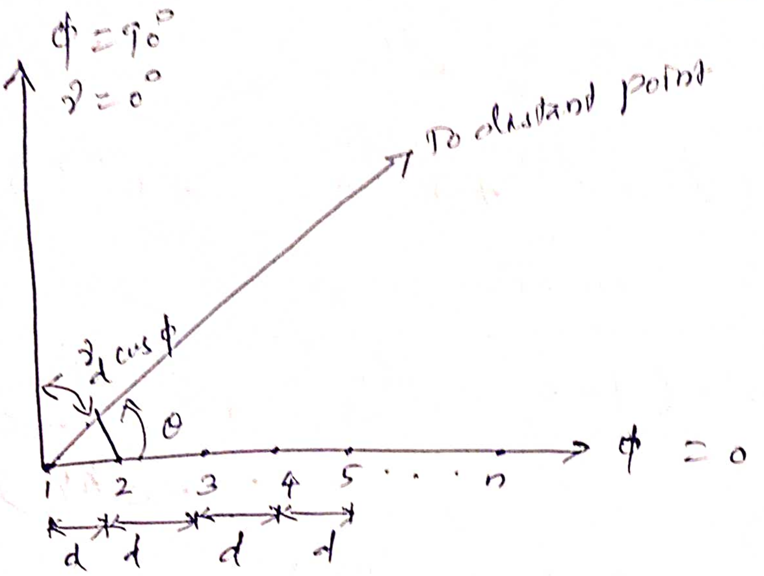
Group pattern due to 2 isotropic elements spaced 2λ apart

$\equiv$



Resultant pattern of 8 isotropic elements.

N Element Uniform Linear Arrays



(Fig) Linear array of n isotropic point sources

Consider an array of 'n' isotropic point sources of equal amplitude & spacing arranged as a linear array. 'n' is +ve integer.

The total field E at a large distance in the direction of ϕ is given by,

$$E_t = E_0\left(1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \dots + e^{j(n-1)\psi}\right) \rightarrow (1)$$

$\psi \rightarrow$ total phase difference of the fields from adjacent sources is given by

$$\psi = \frac{2\pi}{\lambda} d\cos\theta + \alpha$$

$$= \beta d\cos\phi + \alpha \rightarrow (2)$$

Amplitude of the fields are equal and taken as unity source '1' is the reference.

Thus at a distant point in the direction 'ϕ' the field from source 2 is advance in phase with respect to source 1 by ψ, the field from source 3 is advanced w.r.t source 1 by 2ϕ etc.

Multiplying eq ① by $e^{j\psi}$

$$E_t e^{j\psi} = E_0 (e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\phi}) \rightarrow ③$$

Subtract ③ from ①

$$E_t - E_t e^{j\psi} = E_0 (1 + \cancel{e^{j\psi}} + \cancel{e^{j2\psi}} + \dots + e^{j(n-1)\psi} - \cancel{e^{j\psi}} - \cancel{e^{-j2\psi}} \dots - e^{jn\psi})$$

$$E_t (1 - e^{j\psi}) = E_0 (1 - e^{jn\psi})$$

$$E_t = E_0 \frac{(1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

$$= \frac{E_0 (e^{jn\psi 2} \cdot e^{-jn\psi/2} - e^{jn\psi/2} e^{jn\psi/2})}{(e^{j\psi/2} \times e^{-j\psi/2} - e^{j\psi/2} \cdot e^{j\psi/2})}$$

$$= \cancel{E_0 e^{j(n-1}}$$

$$= \frac{E_0 e^{jn\psi/2}}{e^{j\psi/2}} \times \frac{(e^{-jn\psi/2} - e^{jn\psi/2})}{(e^{-j\psi/2} - e^{j\psi/2})}$$

$$= E_0 e^{j(n-1)\psi/2} \times \frac{\sin n\psi/2}{\sin \psi/2}$$

$$E_t = E_0 e^{j\phi} \frac{\sin n\psi/2}{\sin \psi/2} = E_0 \frac{\sin (n\psi/2)}{\sin \phi/2} \angle \phi \rightarrow ④$$

where $\boxed{\phi = \frac{n-1}{2} \psi}$

If phase is referred to centre point of the array ϕ is eliminated.

$$\frac{E_t}{E_0} = \frac{\sin(n\psi/2)}{\sin \psi/2} \rightarrow (5)$$

when $\phi = 0$, Eq (4) & (5) indeterminants

$$E_{norm} = \frac{E_t}{E_{t\,max}}$$

$$= \frac{\cancel{E_0}\,\frac{\sin n\psi/2}{\sin \psi/2}}{\cancel{E_0}\, n}$$

$$E_{norm} = \frac{1}{n}\,\frac{\sin n\psi/2}{\sin \psi/2} = (AF)_n \rightarrow (6)$$

Applying L'Hospital rule

$$\lim_{\psi \to 0} E_t = E_0 \frac{\lim_{\psi \to 0} \frac{d}{d\psi} \sin n\psi/2}{\lim_{\psi \to 0} \frac{d}{d\psi} \sin \psi/2}$$

$$= \lim_{\psi \to 0} \frac{n/2 \cos n\psi/2}{1/2 \cos \psi/2}$$

$$\boxed{E_{t\,max} = E_0\, n}$$

Eq (6) is the array factor.