



SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai



DEPARTMENT OF MECHATRONICS ENGINEERING

19MCT201 - DESIGN OF DIGITAL CIRCUITS

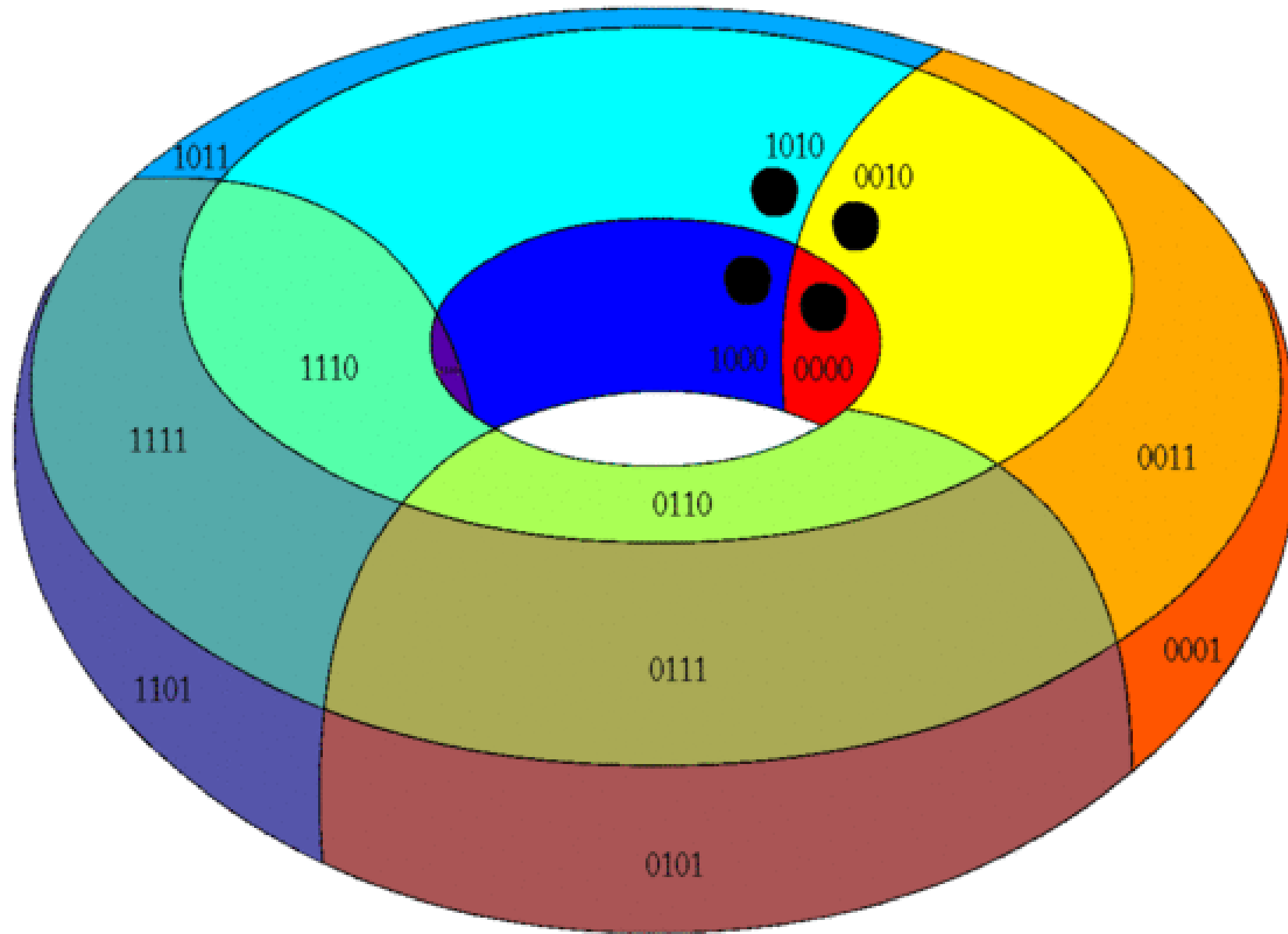
II YEAR - III SEM

UNIT 1 – MINIMIZATION TECHNIQUES AND LOGIC GATES

TOPIC 3 –K-MAP



Meaning of this Symbol ?



●	0000	0100	1100	●	1000	
	0001	0101	1101		1001	
	0011	0111	1111		1011	
●	0010	0110	1110		●	1010

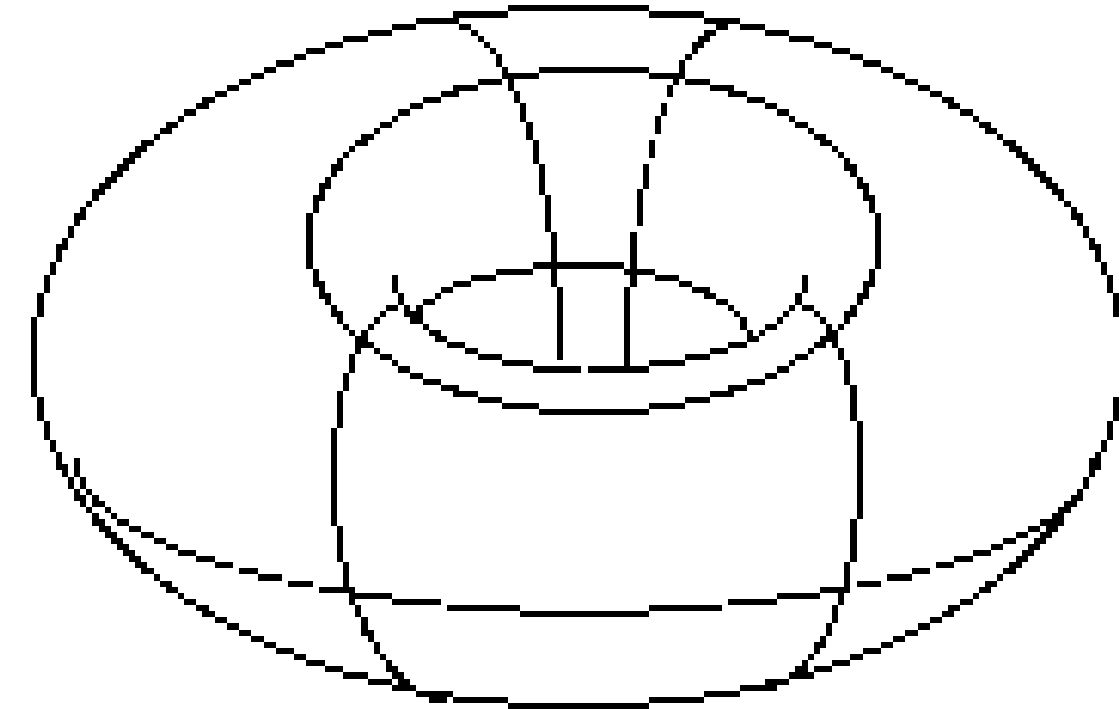
How Boolean Expression can easily be minimized by KMap?



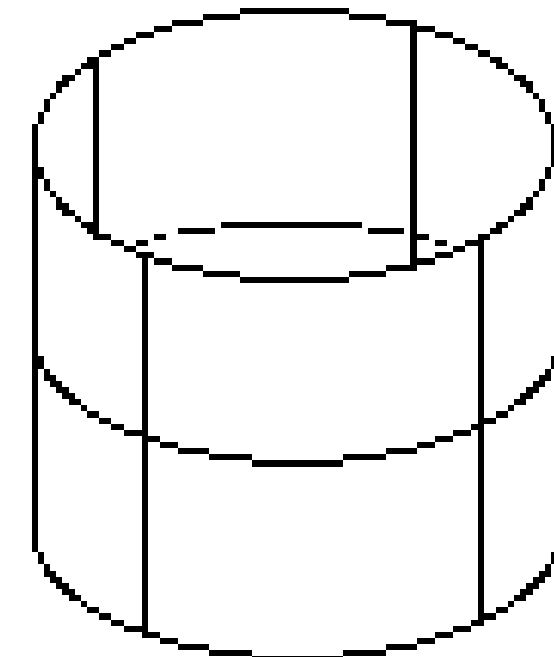
Sum of Product (SOP) - Minterm



- ✓ A Karnaugh map (K-map) is a pictorial method used to minimize Boolean expressions without having to use Boolean algebra theorems and equation manipulations.
- ✓ A K-map can be thought of as a special version of a truth table .
- ✓ Using a K-map, expressions with two to four variables are easily minimized.



4 VARIABLE MAP



3 VARIABLE MAP



Karnaugh map Minimization



The K-map method is graphical technique of simplifying Boolean expressions

The number of cells in the K-map = 2^n , where the number of input variables is n .

To simplify a logical expression with two inputs, we require a K-map with 4 ($=2^2$) cells.

A four-input logical expression would lead to a 16 ($=2^4$) celled-K-map, and so on.

	\bar{B}	B
\bar{A}	$\bar{A}\bar{B}$ 0	$\bar{A}B$ 1
A	$A\bar{B}$ 2	AB 3

2-Variable map

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	$\bar{A}\bar{B}\bar{C}$ 0	$\bar{A}\bar{B}C$ 1	$\bar{A}BC$ 3	$\bar{A}B\bar{C}$ 2
A	$A\bar{B}\bar{C}$ 4	$A\bar{B}C$ 5	ABC 7	$AB\bar{C}$ 6

3-Variable map

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}C\bar{D}$ 3	$\bar{A}\bar{B}CD$ 2
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BC\bar{D}$ 7	$\bar{A}BCD$ 6
AB	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABC\bar{D}$ 15	$ABCD$ 14
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}C\bar{D}$ 11	$A\bar{B}CD$ 10

4-Variable map



Karnaugh map Minimization



Steps to solve expression using K-map-

- ✓ Select K-map according to the number of variables.
- ✓ Identify minterms or maxterms as given in problem.
- ✓ For SOP put 1's in blocks of K-map respective to the minterms (0's elsewhere).
- ✓ For POS put 0's in blocks of K-map respective to the maxterms(1's elsewhere).
- ✓ Make rectangular groups containing total terms in power of two like 2,4,8 ..(except 1) and try to cover as many elements as you can in one group.
- ✓ From the groups made in step 5 find the product terms and sum them up for SOP form.



Karnaugh map Minimization



$$\begin{aligned} Y &= \bar{A}\bar{B}C + \bar{A}BC \\ &= \bar{A}C(\bar{B} + B) \\ &= \bar{A}C \end{aligned}$$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
\bar{A} 0	0	1	1	0
A 1	0	0	0	0

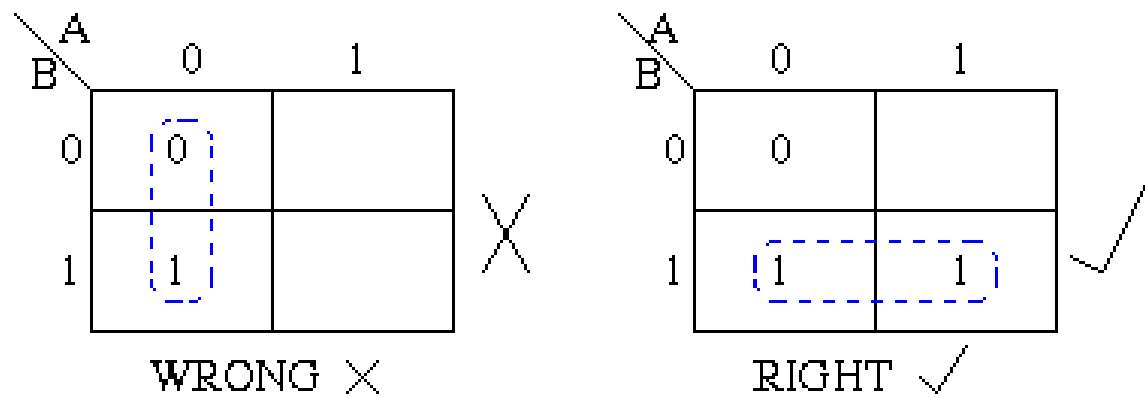
A dashed box highlights the two '1's in the \bar{A} 0 row, and a line points to this box with the label $\bar{A}C$.



Karnaugh map Minimization

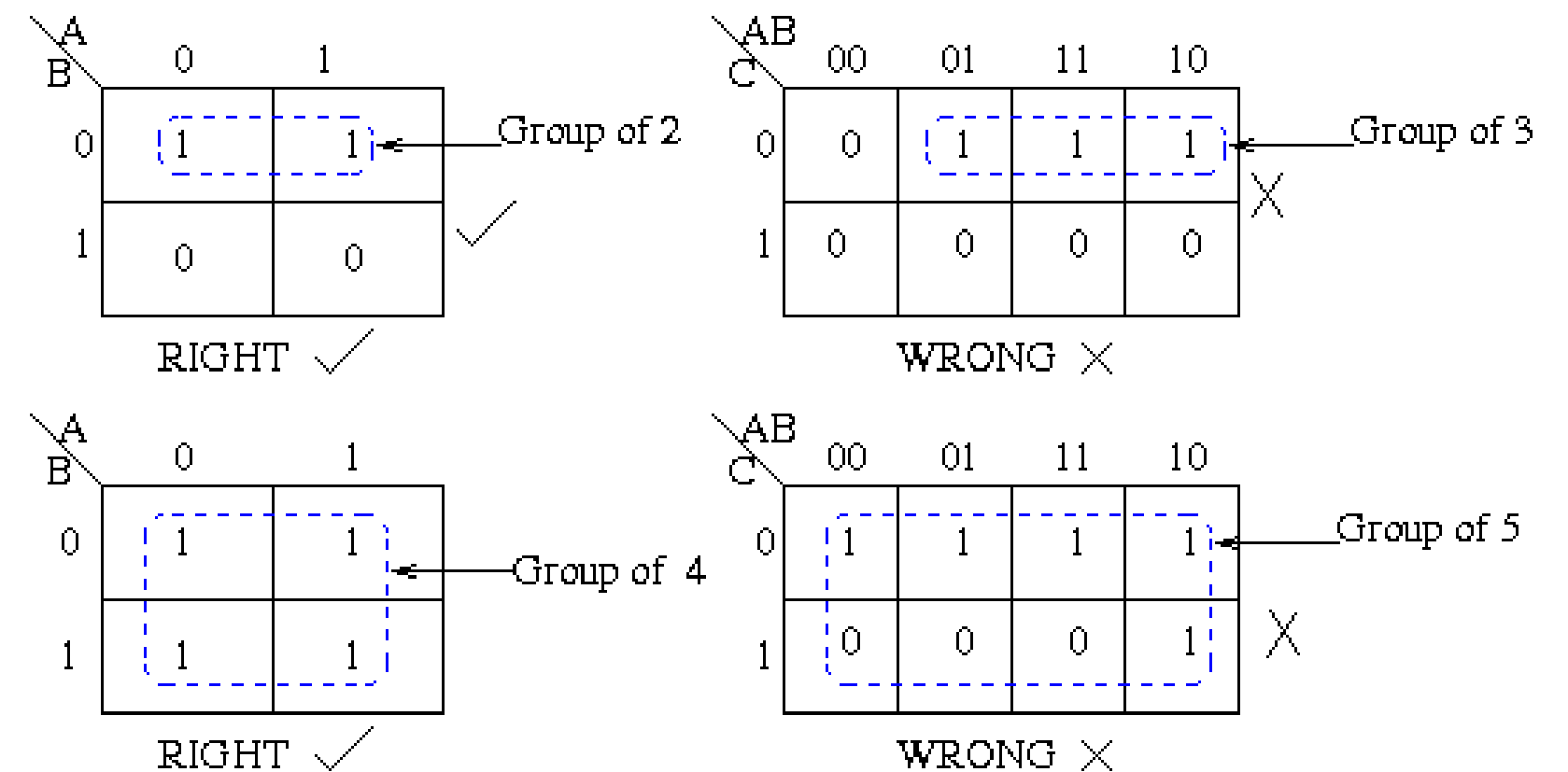
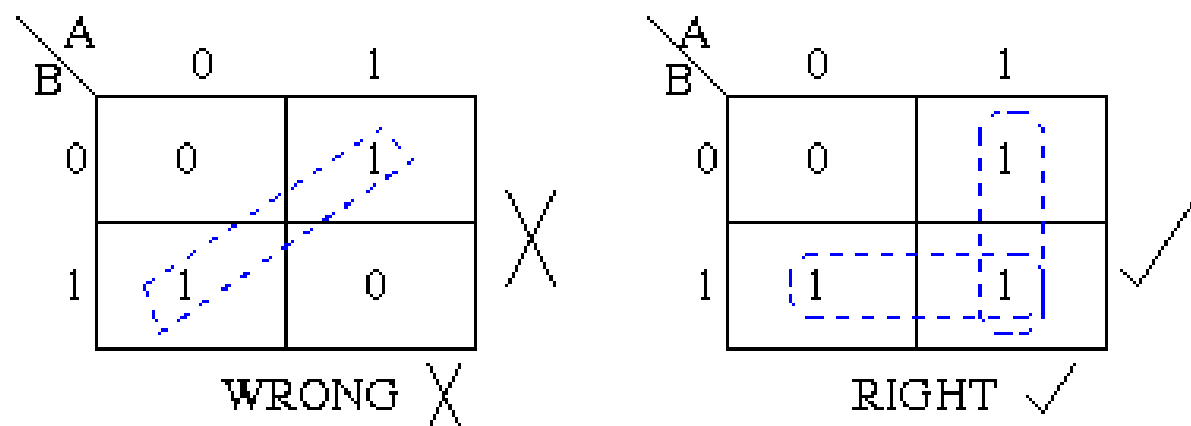


- Groups may not include any cell containing a **zero**



Groups must contain 1, 2, 4, 8, or in general 2^n cells.
That is if $n = 1$, a group will contain two 1's since $2^1 = 2$.
If $n = 2$, a group will contain four 1's since $2^2 = 4$.

- Groups may be horizontal or vertical, but not diagonal.

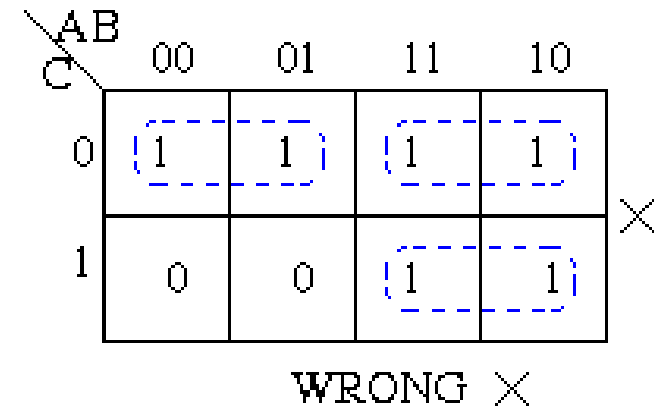
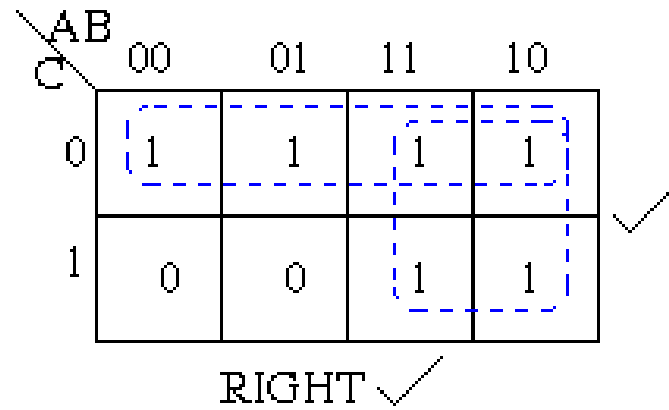




Karnaugh map Minimization

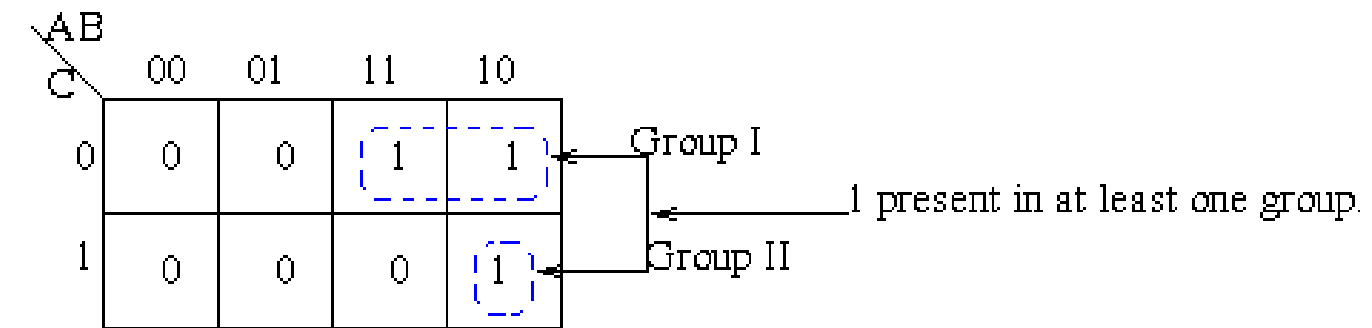


Each group should be as large as possible

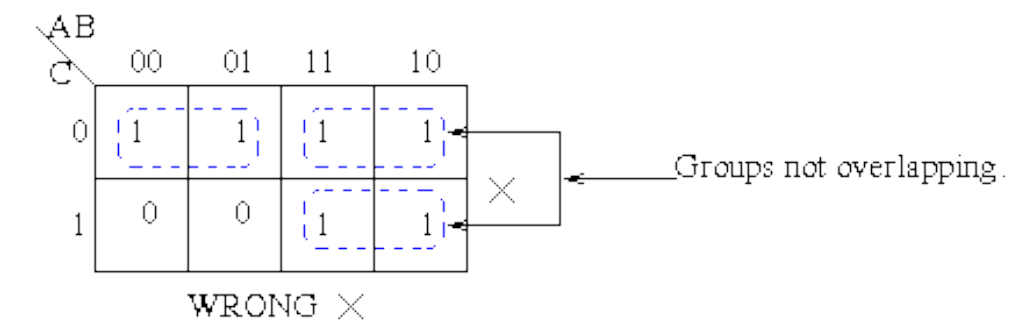
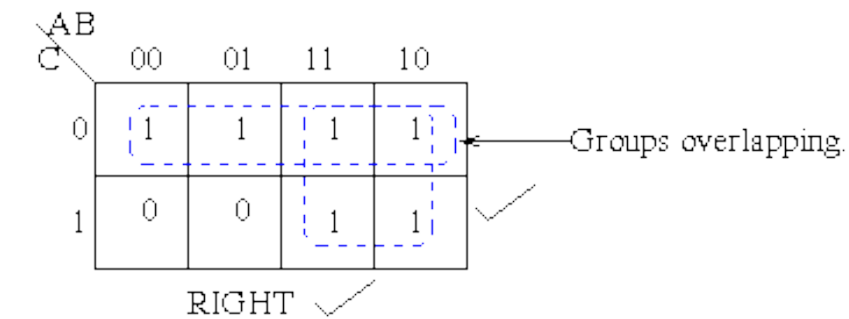


(Note that no Boolean laws broken, but not sufficiently minimal)

• Each cell containing a *one* must be in at least one group



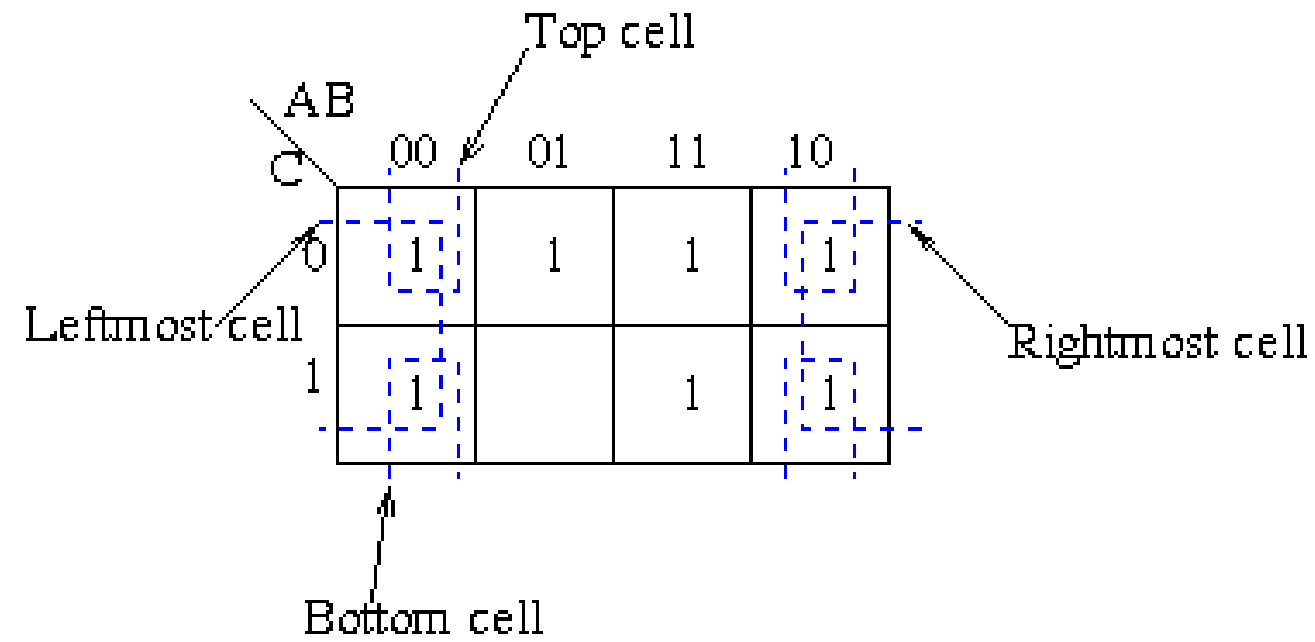
• Groups may overlap.



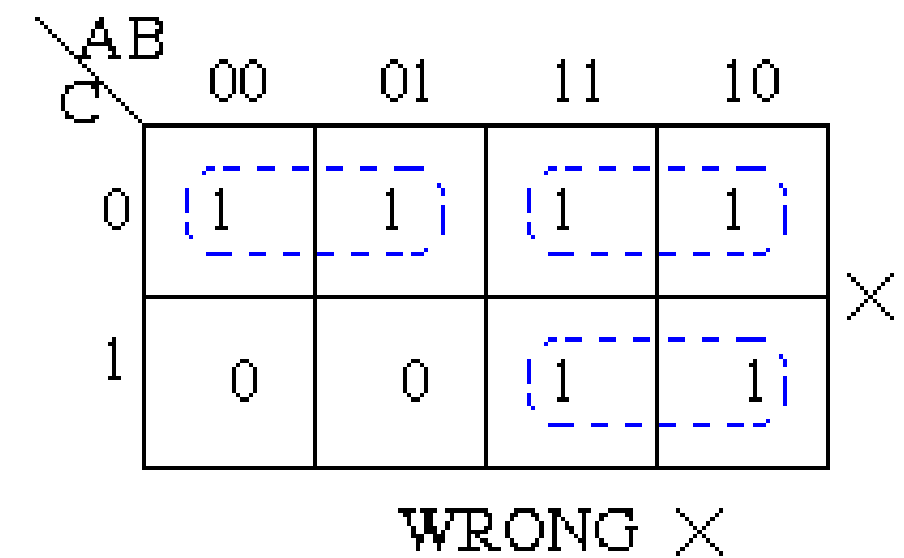
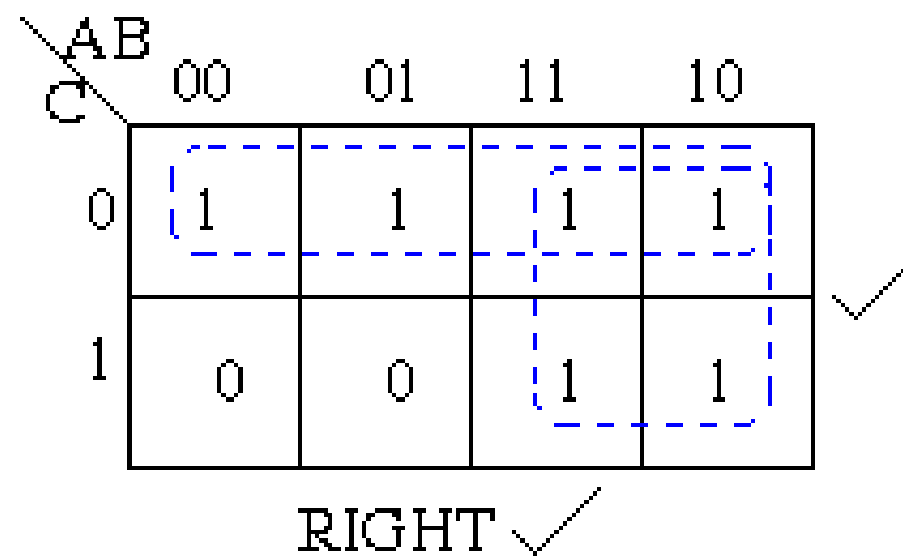


Karnaugh map Minimization

Groups may wrap around the table. The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell.



There should be as few groups as possible, as long as this does not contradict any of the previous rules.





Karnaugh map Minimization



Summary:

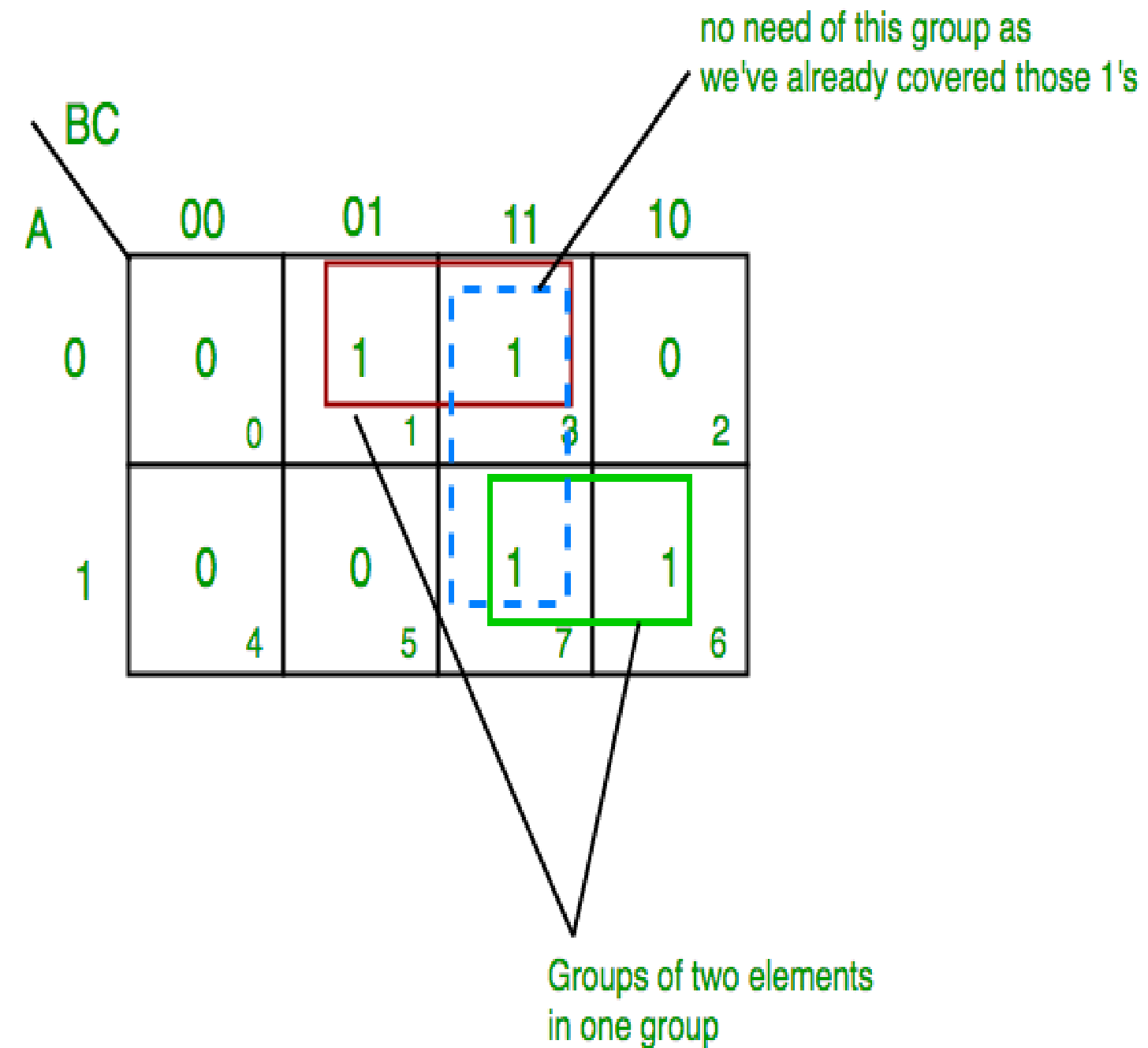
- 1.No zeros allowed.
- 2.No diagonals.
- 3.Only power of 2 number of cells in each group.
- 4.Groups should be as large as possible.
- 5.Every one must be in at least one group.
- 6.Overlapping allowed.
- 7.Wrap around allowed.
- 8.Fewest number of groups possible.



Karnaugh map Minimization



1. K-map of 3 variables-
 $Z = \sum A, B, C(1, 3, 6, 7)$



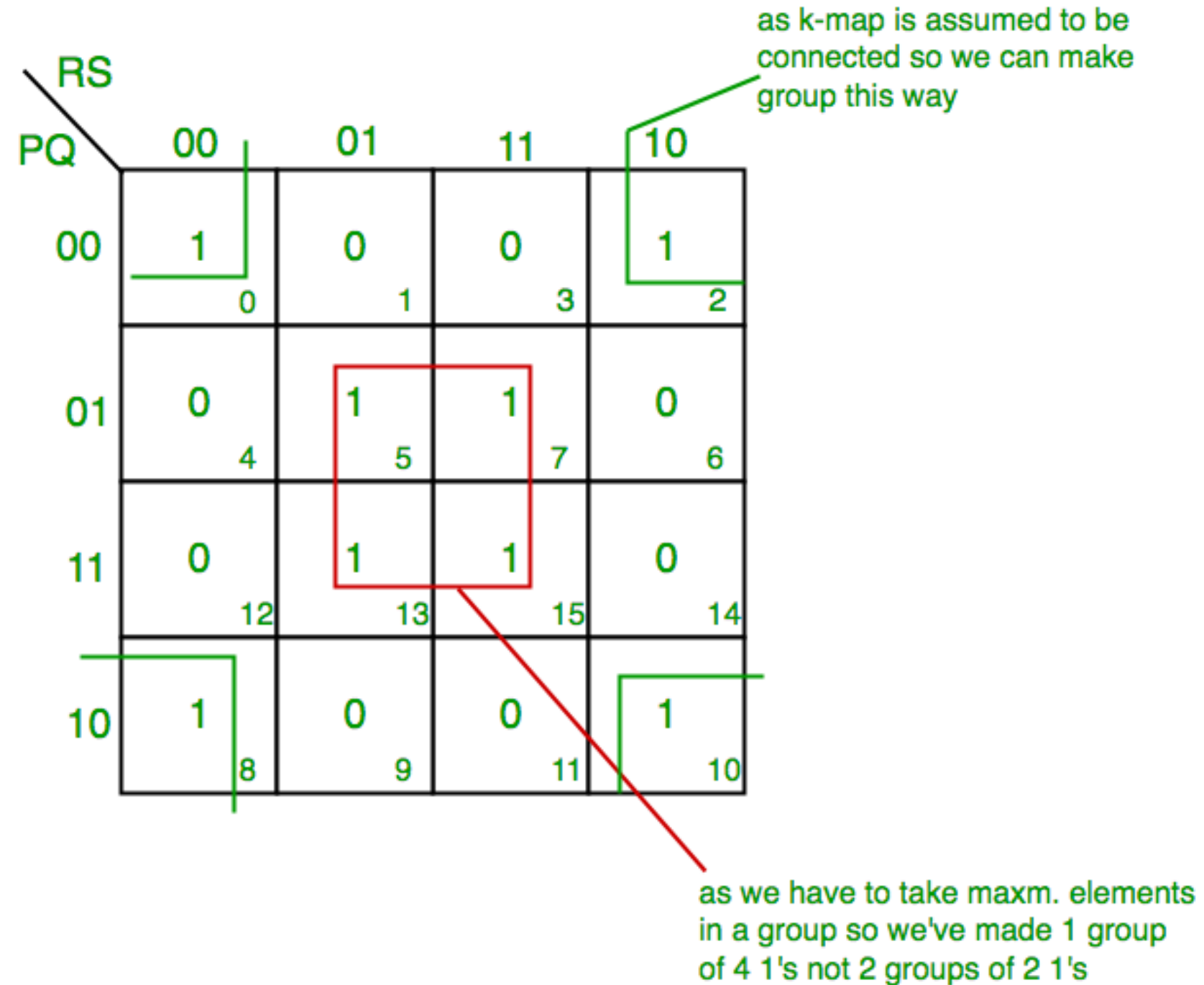


Karnaugh map Minimization



K-map for 4 variables

$$F(P,Q,R,S) = \sum(0,2,5,7,8,10,13,15)$$





Karnaugh map Minimization





K Map- Problems for Practise

1. Simplify following logical expressions using Karnaugh maps

i) $Y = A \bar{B} + A B + \bar{A} B$

Ans. : $Y = A + B$

ii) $Y = \bar{A} \bar{B} \bar{C} + \bar{A} B \bar{C} + A \bar{B} \bar{C} + \bar{A} \bar{B} C + A B \bar{C}$

Ans. : $Y = \bar{A} \bar{B} + \bar{C}$

iii) $Y = \bar{A} \bar{B} C D + A \bar{B} \bar{C} \bar{D} + \bar{A} \bar{B} \bar{C} \bar{D} + A B \bar{C} D + \bar{A} \bar{B} \bar{C} D + A \bar{B} \bar{C} D + A B C D$

Ans. : $Y = A B D + \bar{A} \bar{B} D + \bar{B} \bar{C}$



ASSESSMENT - 1

How Laws relates with us....

Question 1

A Karnaugh map is a systematic way of reducing which type of expression?

- a) product-of-sums
- a) exclusive NOR
- a) sum-of-products
- a) those with overbars

Question 2

Occasionally, a particular logic expression will be of no consequence in the operation of a circuit, such as a BCD-to-decimal converter. These result in _____ terms in the K-map and can be treated as either _____ or _____, in order to _____ the resulting term

- A.don't care, 1s, 0s, simplify
- B.spurious, ANDs, ORs, eliminate
- C.duplicate, 1s, 0s, verify
- D.spurious, 1s, 0s, simplify



References



- <https://brilliant.org/wiki/de-morgans-laws/>
- <https://circuitglobe.com/demorgans-theorem.html>
- <https://www.electrical4u.com/>