Differential Equations :-

Differential equations are used to represe[nt] continuous time LTI system. The Differential equations relates the I/p and o/p of the systems.

Solving Differential equations using Fourier Method :-

$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f) \cdot H(f)$$

$$\boxed{\therefore H(f) = \frac{Y(f)}{X(f)}} \quad (or) \quad H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$H(f)$ (or) $H(\omega) \rightarrow$ System transfer function (or) Frequency response.

① The system produces the o/p $y(t) = e^{-t} u(t)$ for an input $x(t) = e^{-2t} u(t)$. Determine the Impulse response and frequency response of the system.

$$y(t) = e^{-t} u(t)$$

$$x(t) = e^{-2t} u(t)$$

$$X(f) = \frac{1}{2+j2\pi f}, \quad Y(f) = \frac{1}{1+j2\pi f}$$

$$\therefore H(f) = \frac{Y(f)}{X(f)} = \frac{2+j2\pi f}{1+j2\pi f}$$

$$h(t) = F^{-1}[H(f)]$$

$$= F^{-1}\left[\frac{2+j2\pi f}{1+j2\pi f}\right]$$

$$= F^{-1}\left[\frac{1+1+j2\pi f}{1+j2\pi f}\right]$$

$$= F^{-1}[1] + F^{-1}\left[\frac{1}{1+j2\pi f}\right]$$

$$h(t) = \delta(t) + e^{-t}u(t).$$

(2) The Diff equation of the system is given as

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = \frac{-d}{dt}x(t).$$

Determine Freq response and Impulse response.

By taking Fourier transform

$$(j\omega)^2 Y(\omega) + 5j\omega Y(\omega) + 6Y(\omega) = (-j\omega)X(\omega)$$

$$Y(\omega)\left[(j\omega)^2 + 5j\omega + 6\right] = -j\omega X(\omega)$$

$$\therefore H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$H(\omega) = \frac{-j\omega}{(j\omega+2)(j\omega+3)}$$

$$= \frac{2}{j\omega+2} - \frac{3}{j\omega+3}$$

$$= 2\cdot\frac{1}{j\omega+2} - 3\cdot\frac{1}{j\omega+3}$$