



SNS COLLEGE OF TECHNOLOGY
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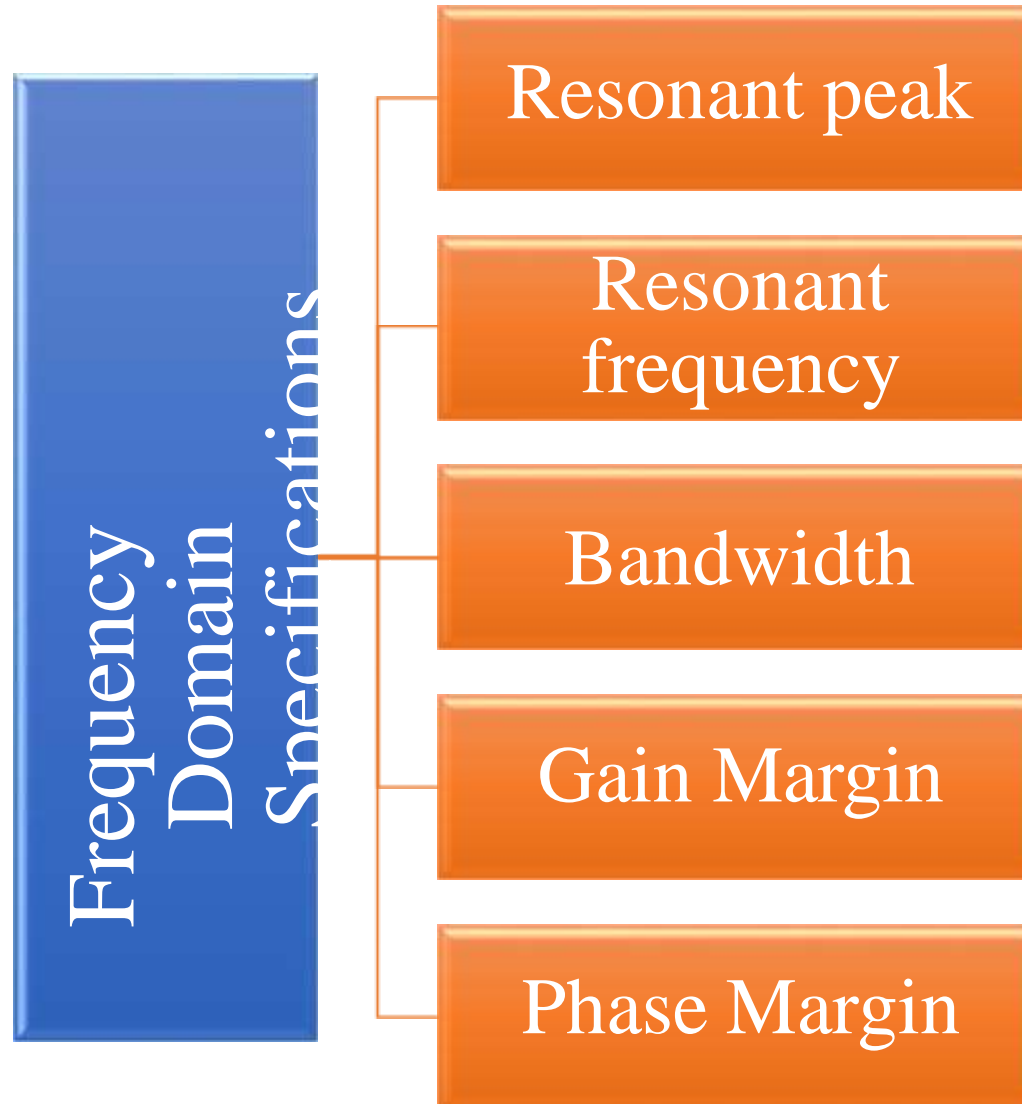


UNIT III

**FREQUENCY DOMAIN
SPECIFICATIONS**



INTRODUCTION





GAIN AND PHASE MARGIN



Gain Margin:

“ It is the reciprocal of the magnitude of the open loop transfer function at phase cross over frequency. Phase cross over frequency is the frequency at which the phase angle is 180° ”.

Phase Margin:

“ It is the additional phase lag at gain cross over frequency. Gain cross over frequency is the frequency at which the gain of $G(j\omega)$ is unity or 0db.”



GAIN AND PHASE MARGIN

- GM and PM tells you how much uncertainty one can tolerate in the open loop system before the closed loop system goes to instability.
- Phase margin is related to closed loop damping ratio and so to the overshoot. To show this, consider an open loop system,

$$G_{oL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$



GAIN AND PHASE MARGIN

$$G_{cL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\zeta\omega_n)} = \frac{-\omega_n^2}{\omega^2 - j2\zeta\omega_n\omega}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\sqrt{\omega^4 + 4\zeta^2\omega_n^2\omega^2}}$$

Find ω_c , the crossover frequency.

$$\begin{aligned} |G(j\omega_c)| &= 1 \\ \Rightarrow \omega_c^4 &= \omega_c^4 + 4\zeta^2\omega_n^2\omega_c^2 \\ \Rightarrow \omega_c &= \omega_n \left[\sqrt{1 + 4\zeta^4} - 2\zeta^2 \right]^{1/2} \end{aligned}$$



GAIN AND PHASE MARGIN

Now, find the phase margin as follows,

$$\begin{aligned}\text{PM} &= \tan^{-1} \left[\frac{-\text{Im}(G(j\omega))}{-\text{Re}(G(j\omega))} \right]_{\omega=\omega_c} \\ &= \tan^{-1} \left[\frac{\text{Im}(G(j\omega))}{\text{Re}(G(j\omega))} \right]_{\omega=\omega_c}\end{aligned}$$

Now,

$$G(j\omega) = \frac{-\omega_n^2}{\omega^2 - j2\zeta\omega_n\omega} = \frac{-\omega_n^2(\omega^2 + j2\zeta\omega_n\omega)}{\omega^4 - 2\zeta^2\omega_n^2\omega^2}$$

So,

$$\begin{aligned}\text{PM} &= \tan^{-1} \left(\frac{2\zeta\omega_n\omega_c}{\omega_c^2} \right) = \tan^{-1} \left(\frac{2\zeta\omega_n}{\omega_c} \right) \\ &= \tan^{-1} \left[\frac{2\zeta}{[\sqrt{1 + 4\zeta^4} - 2\zeta^2]^{1/2}} \right]\end{aligned}$$