Human Visual System & Digital Image

Digital Image Fundamentals

- Human visual system
- A simple image model
- Sampling and quantization
- Color models and Color imaging

Why to study Human Visual System

- **In many image processing applications, the objective is to help a human observer perceive the visual information in an image. Therefore, it is important to understand the human visual system.**

- **The human visual system consists mainly of**

  the eye (image sensor or camera)

  optic nerve (transmission path)

  brain (image information processing unit or computer).

- **It is one of the most sophisticated image processing and analysis systems.**

Why to study Human Visual System(Cont..)

- There is a large difference between the image we display and the image we actually perceive, i.e. the luminance and the perceived brightness of a pixel.
- Brightness also depends on other factors, such as contrast around the pixel and various other cognitive processes
- Human visual system can perform a number of image processing tasks in a manner vastly superior to any computer vision system.
- If we want to mimic such processing, the way our eyes and brain work needs to be carefully studied and understood.

Human Visual System

- Brightness adaptation
- Brightness discrimination
- Weber ratio
- Mach band pattern
- Simultaneous contrast

Structure of Human Eye

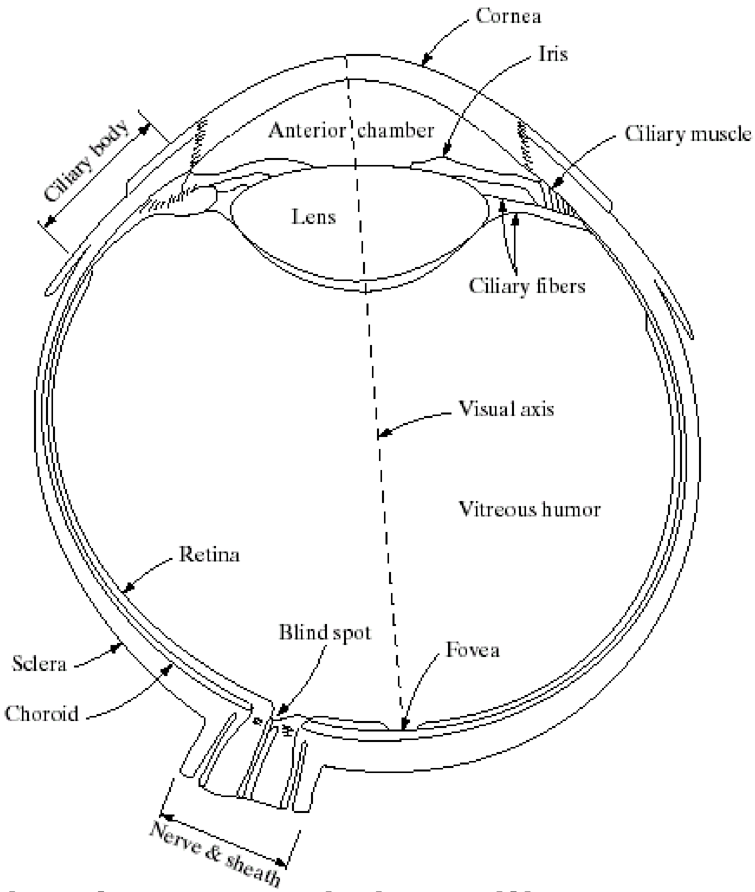
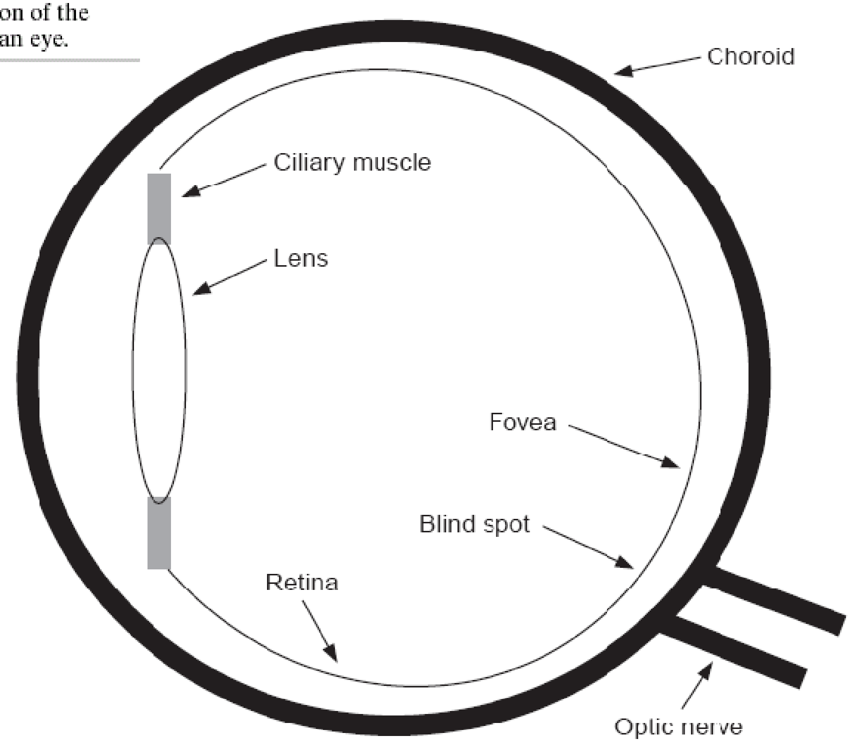
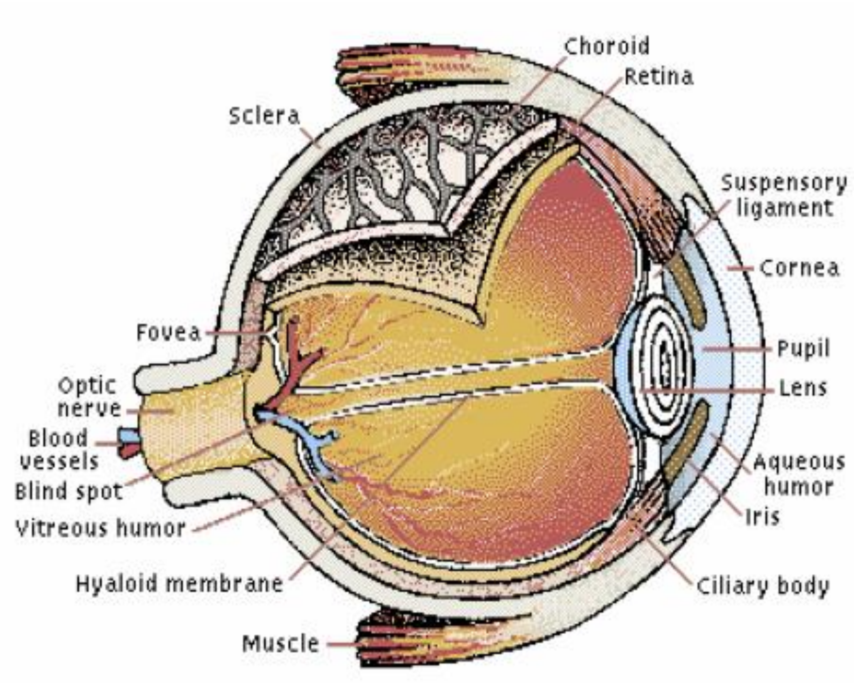


FIGURE 2.1
Simplified diagram of a cross section of the human eye.



The lens and the ciliary muscle focus the reflected lights from objects into the retina to form an image of the objects.

Elements of visual perception



The amount of light entering the eye is controlled by the pupil, which dilates and contracts accordingly.

The cornea and lens, whose shape is adjusted by the ciliary body, focus the light on the retina, where receptors convert it into nerve signals that pass to the brain.

Human Eye Structure

- The eye is nearly a sphere
- Average Diameter of approximately 20 mm
- Three membranes:-

  - The outer cover

    + Cornea (tough, transparent tissue)

    + Sclera (opeque membrane)

  - The Choroid (below the sclera)

    +ciliary body

    +iris diaphragm (pupil) contracts or expands to control the amount of light

  - The Retina (the inner most membrane of the eye)

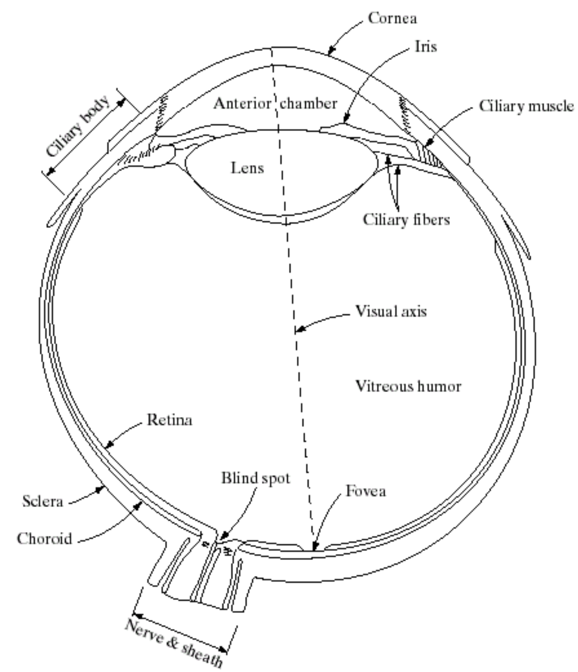


FIGURE 2.1 Simplified diagram of a cross section of the human eye.

Lens & Retina

- Lens: Infrared & ultraviolet light are absorbed appreciably by proteins within the lens structure &, in excessive amounts, can cause damage to the eye

- Innermost membrane of the eye which lines inside of the wall's entire posterior portion. When the eye is properly focused, light from an object outside the eye is imaged on the retina

Receptors

- Pattern vision is afforded by the distribution of discrete light receptors over the surface of the retina
- Receptors are divided into 2 classes
  - Cones
  - Rods

Cones

- 6-7 million, located primarily in the central position of the retina(the fovea, muscles controlling the eye rotate the eyeball until the image falls on the fovea
- Highly sensitive to color
- Each is connected to its own nerve end thus luman can resolve fine details
- Cone vision is called photopic or bright light vision

Rods

- 75-150 million, distributed over the retina surface
- Several rods are connected to a single nerve end reduce the amount of detail discernible
- Serve to give a general, overall picture of the field of view
- Sensitive to low level of illumination
- Rod vision os called scotopic or dim-light vision

Components Human Eye

When the eye is properly focused, light from an object outside the eye is imaged on the retina

1. The ***lens*** contains 60-70% water, 6% of fat.
2. The ***iris*** diaphragm controls amount of light that enters the eye.
3. ***Light receptors*** in the ***retina***

- - About 6-7 millions ***cones*** for bright light vision called ***photopic***

  - Density of cones is about 150,000 elements/mm^2.

  - In each eye located in the fovea, one cone-one nerve end

  - **Cones involve in color vision**.

  - Cones are concentrated in ***fovea*** about 1.5x1.5 mm^2.

  - About 75-150 millions ***rods*** **for dim light vision** called ***scotopic***

  - Rods are sensitive to low level of light and are not involved color vision.

- 4. ***Blind spot*** is the region of emergence of the optic nerve from the eye.

- Lens: Infrared & ultraviolet light are absorbed appreciably by protiens within the lens structure & in amounts, can cause damage to the eye

- Retina: Innermost membrane of the eye which lines inside of the wall’s entire posterion portion. When the eye is properly focused, light from an object outside the eye is imaged on the retina

Question

- An object appears brightly colored in daylight will be seen colorless in moonlight (why)

- (Only the rods are stimulated at night or at low light).

Distribution of Visible units in Human Eye

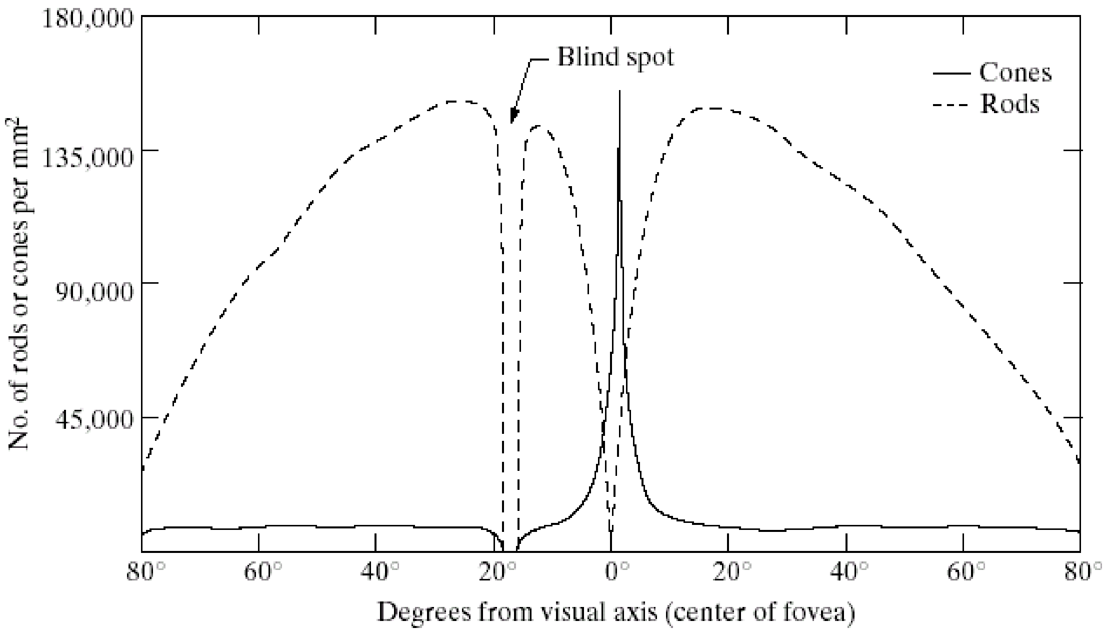


FIGURE 2.2 Distribution of rods and cones in the retina.

- Blind spot (Receptors are absent in this area)
- Fovea: a circular indentation in the retina of about 1.5 mm in diameter
- The density of cones in the fovea is approx 150000 elements per square mm

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Human Visual System

- Image formation in the eye
- The focal length varies form aprox 17 mm to about 14 mm, as the refractive power of the lens increses from the min to its max.
  - Object farther away than 3 meter — lowest refractive power and focal length 17 mm
  - Nearby object — Max refractive power and focal length 14 mm
  - It is easy to calculate the size of the retinal image of any object
  - Image length h = 17(mm) x (15/100) = 2.55 mm

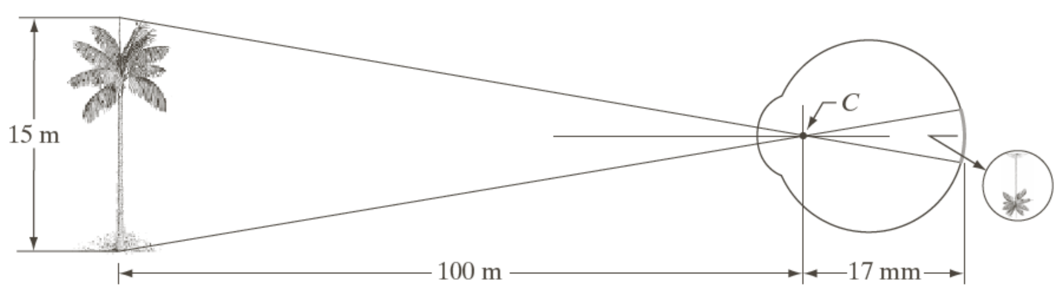
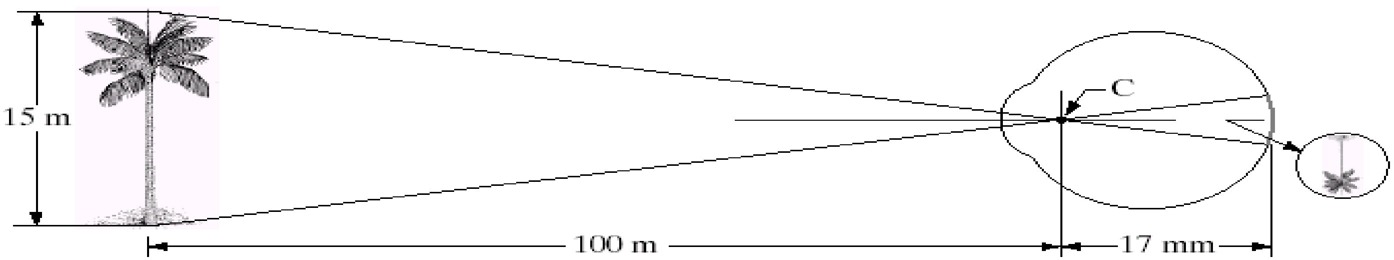


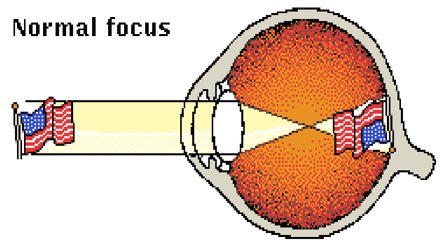
FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point *C* is the optical center of the lens.

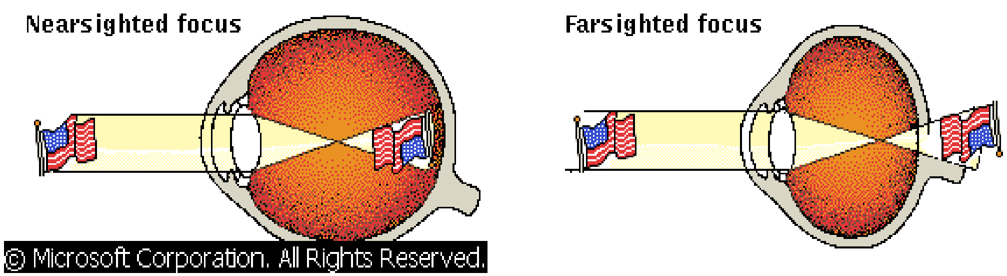
- Principal difference b/w the lens of the eye & an ordinary optical lens is that the former is flexible
- Radius of the curvature → the anterion surface → its postirior surface
- Shape→ is controlled by tension in the fibles of the ciliary body
- Focus on distinct objects, the controlling muscles cause the lens to be relatively flattened
- These muscles allow the lens to become thicker in order to focus on objects near the eye

Image Formation in the Human Eye



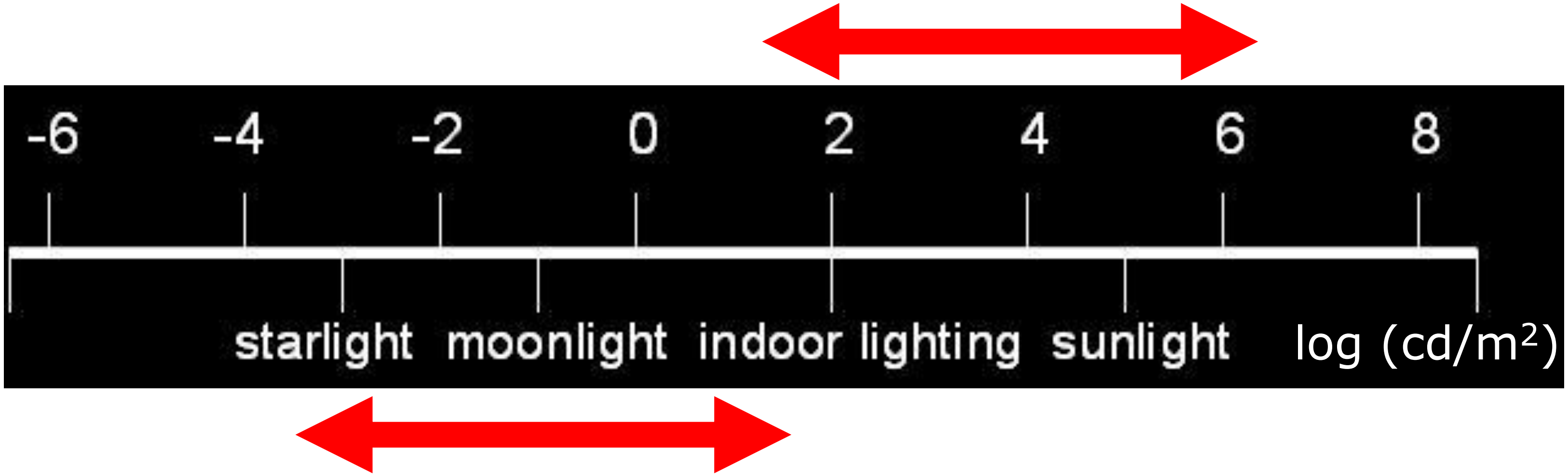
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.





Human Visual System

Human simultaneous luminance vision range
(5 orders of magnitude)



Brightness Adaptation & Discrimination

- The human visual system can perceive (the ability to see, hear, or become aware of something through the senses) approximately 10^{10} different light intensity levels
- However, at any one time we can only discriminate between a much smaller number – *brightness adaptation*
- The ability of the eye to discriminate between *changes* in light intensity at any specific brightness adaptation level is governed by *Weber's law* which states that **the just noticeable difference ΔI is proportional to the background luminance I**
- Similarly, the *perceived intensity* of a region is related to the light intensities of the regions surrounding it

Human Visual System

- Brightness adaptation

  – HVS can adapt to light intensity range on the order of 10^{10} from the scotopic to the glare limits

  – Subjective brightness(Brightness as perceived by the human visual systems) is a logarithmic function of the light intensity incident on the eye

Brightness adaptation

- **Simutaneous range is smaller than Total adaptation range**

The **lambert** (symbol L) is a unit of luminance named for Johann Heinrich Lambert (1728 - 1777), a German mathematician, physicist and astronomer. A related unit of luminance, the foot-lambert, is used in the lighting, cinema and flight simulation industries. The SI unit is the candela per square metre (cd/m²).

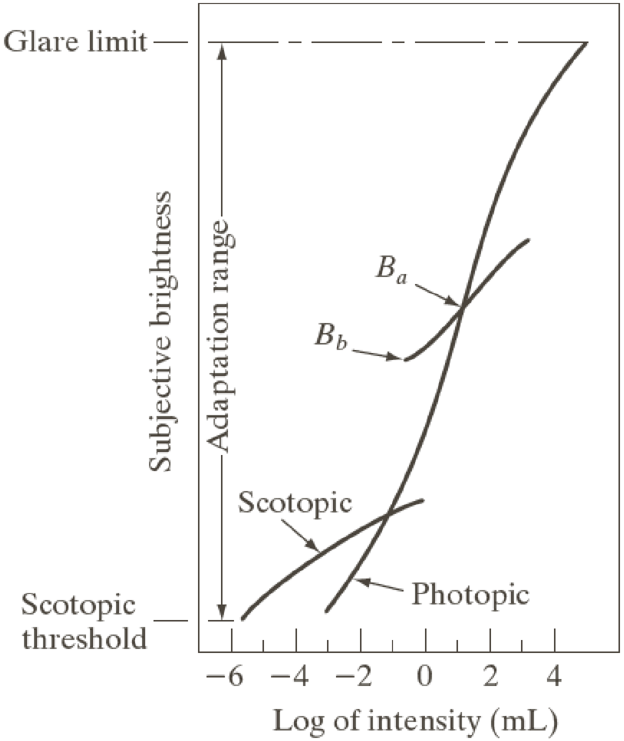


FIGURE 2.4 Range of subjective brightness sensations showing a particular adaptation level.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

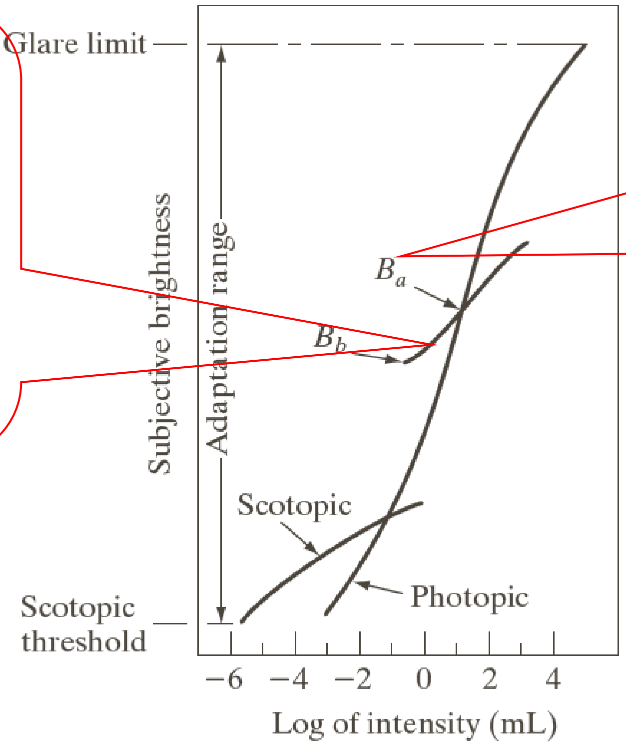
Human Visual System

- Brightness adaptation
  - The HVS cannot operate on such range (10^6 orders of magnitude) simultaneously
  - It accomplishes this through (brightness) adaptation
  - The total intensity level the HVS can discriminate simultaneously is rather small in comparison (about 4 orders of magnitude)

Brightness adaptation

Sensitivity of the HVS for the given adaptation level

Anything below Bb will be perceived as indistinguishable blacks



For a given observation condition, the current sensitivity level is call the brightness adaptation level

- The range of brightness that the eye can adapt to is enormous, roughly around 10^{10} to 1.
- Photopic vision alone has a range of around 10^6 to 1.
- Brightness adaptation: example " B_a"
- mL: millilambert

Brightness Adaptation and Discrimination

- Subjective brightness (Intensity as perceived by the human visual system) is a logarithmic function of the light intensity incident on the eye.
- Photopic vision: range 10^6
- Transition from scotopic to photopic vision : range from 0.001 to 0.1 milliambert -3 to -1 mL in the log scale
- Visual system cannot operate over such a total range simultaneously.
  Brightness adaptation

- Brightness adaptation level (the current sensitivity level of the visual system) at level Ba is (Bb to Ba)

Brightness Adaptation and Discrimination

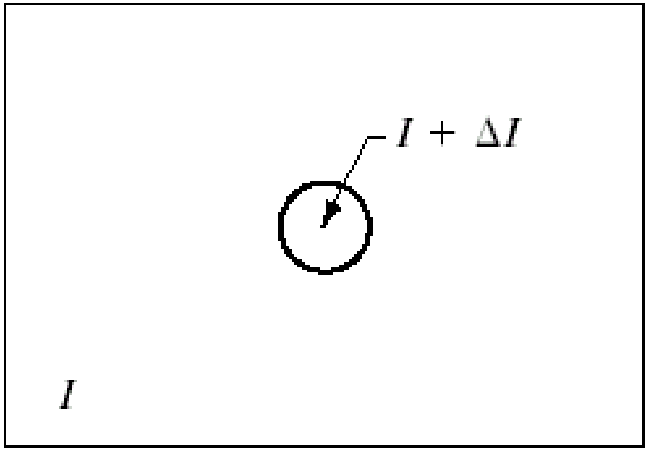


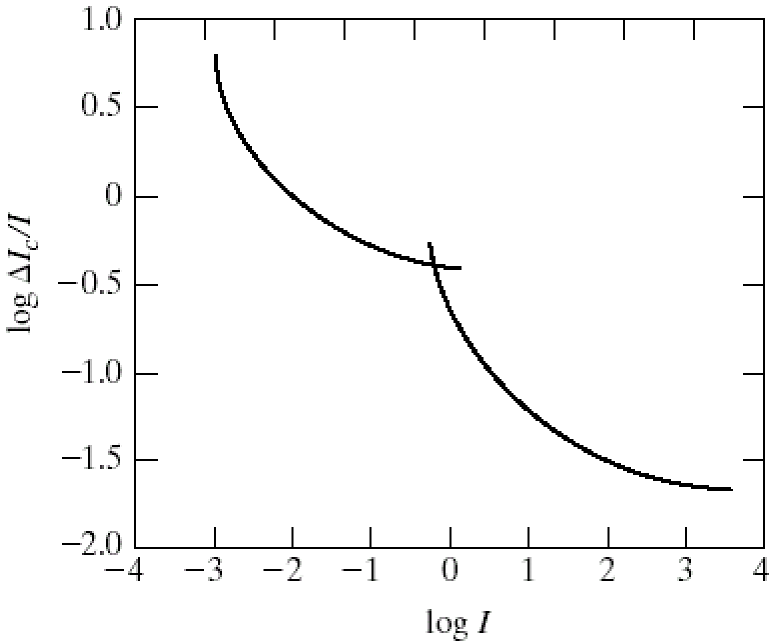
FIGURE 2.5 Basic experimental setup used to characterize brightness discrimination.

- Brightness discrimination is the ability of the eye to discriminate between changes in light intensity at any specific adaptation level.
- The quantity $\Delta I_c/I$, where ΔI_c is the increment of illumination discriminable 50% of the time with background illumination I, is called the Weber ratio. A small value of Weber ratio, means good brightness discrimination.

Brightness Adaptation and Discrimination

- Brightness discrimination is poor at low levels of illumination.The two branches in the curve indicate that at low levels of illumination vision is carried out by the rods, whereas at high level by the cones.

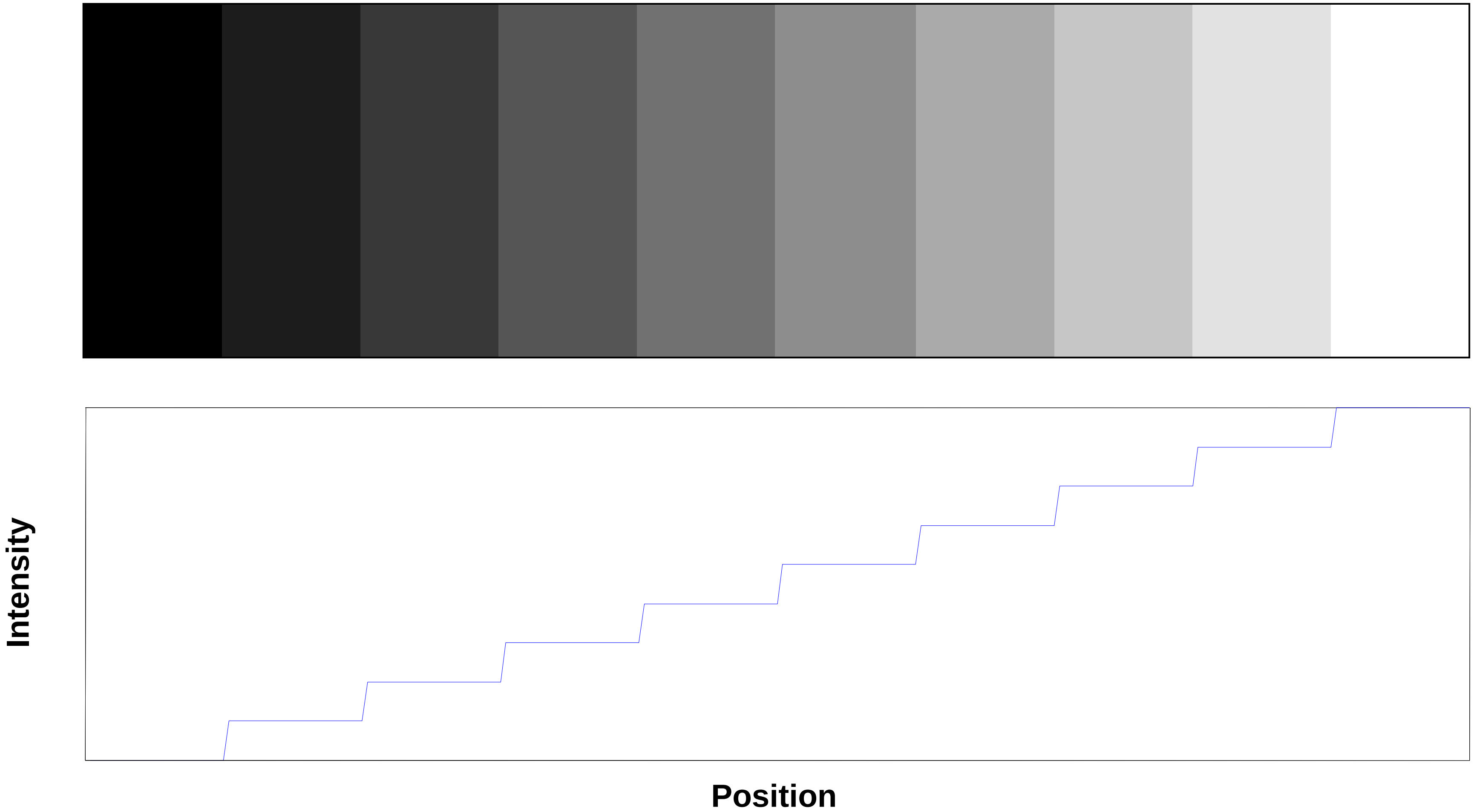
FIGURE 2.6
Typical Weber ratio as a function of intensity.



Perceived Brightness

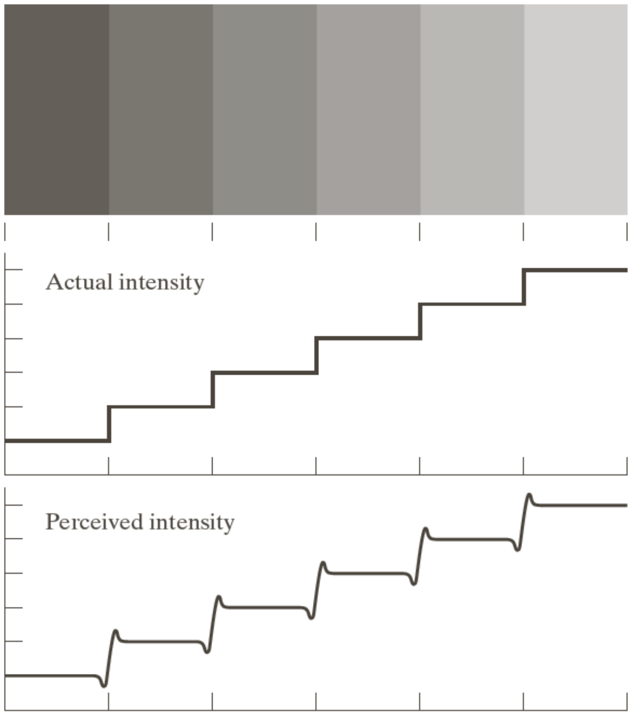
- **Two phenomena clearly demonstrate that perceived brightness is not a simple function of intensity.**
  - **Mach bands**
  - **Simultaneous contrast**

Brightness Adaptation of Human Eye : Mach Band Effect



Perceived Brightness

- Perceived brightness is not a simple function of intensity – Mach band pattern



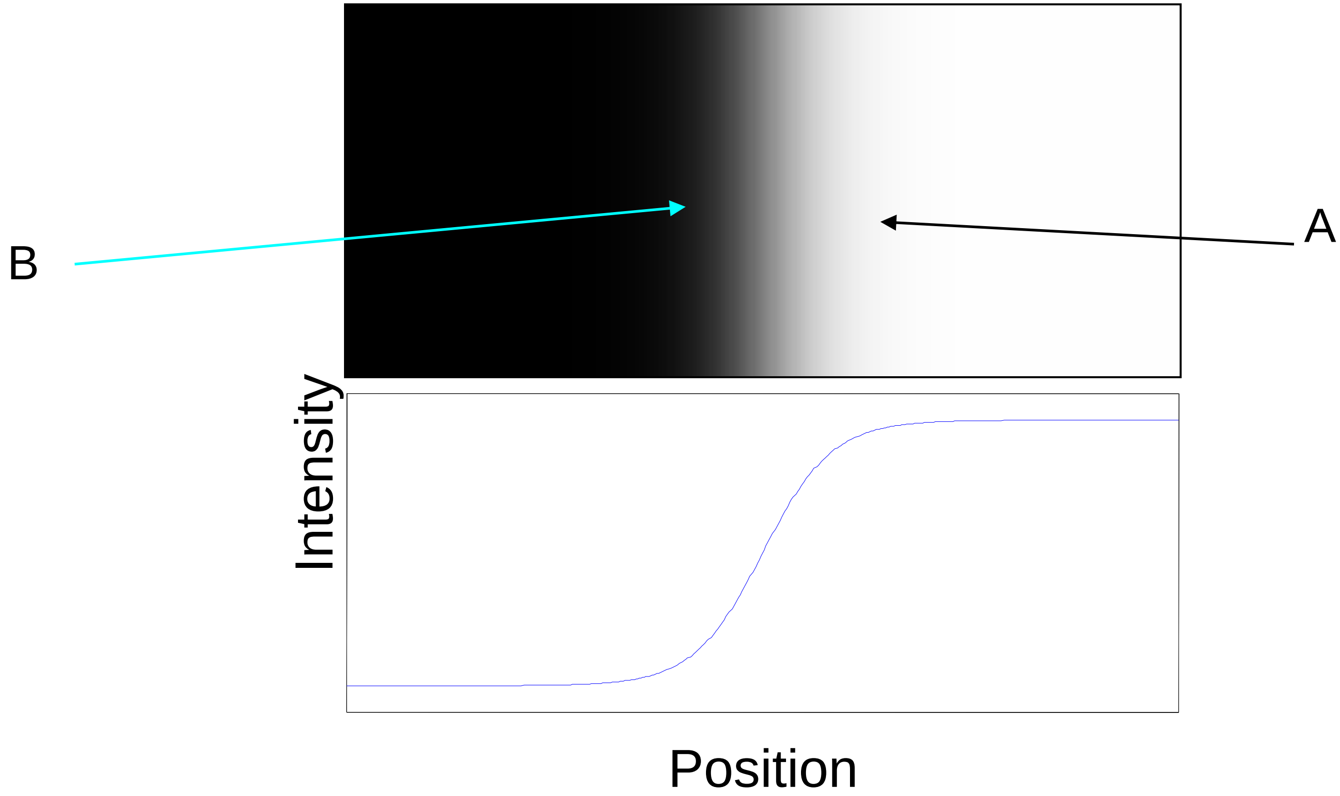
a
b
c

FIGURE 2.7 Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.

Intensities of surrounding points effect perceived brightness at each point.

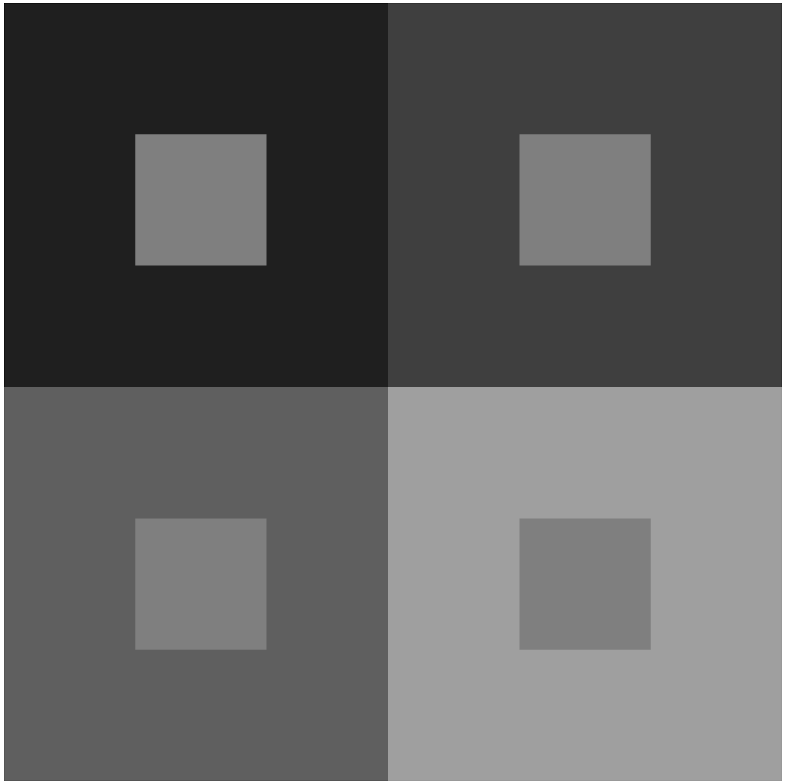
In this image, edges between bars appear brighter on the right side and darker on the left side.

Mach Band Effect (Cont)



In area A, brightness perceived is darker while in area B is brighter. This phenomenon is called ***Mach Band Effect***.

Brightness Adaptation of Human Eye :
Simultaneous Contrast



Simultaneous contrast. All small squares have exactly the same Intensity but they appear progressively darker as background becomes lighter.

Perceived Brightness

- Perceived brightness is not a simple function of intensity – Simultaneous contrast

a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

In this phenomena, called simultaneous contrast, a spot may appears to the eye to become darker as the background gets lighter.

Optical Illusions

Examples for Human Perception Phenomena

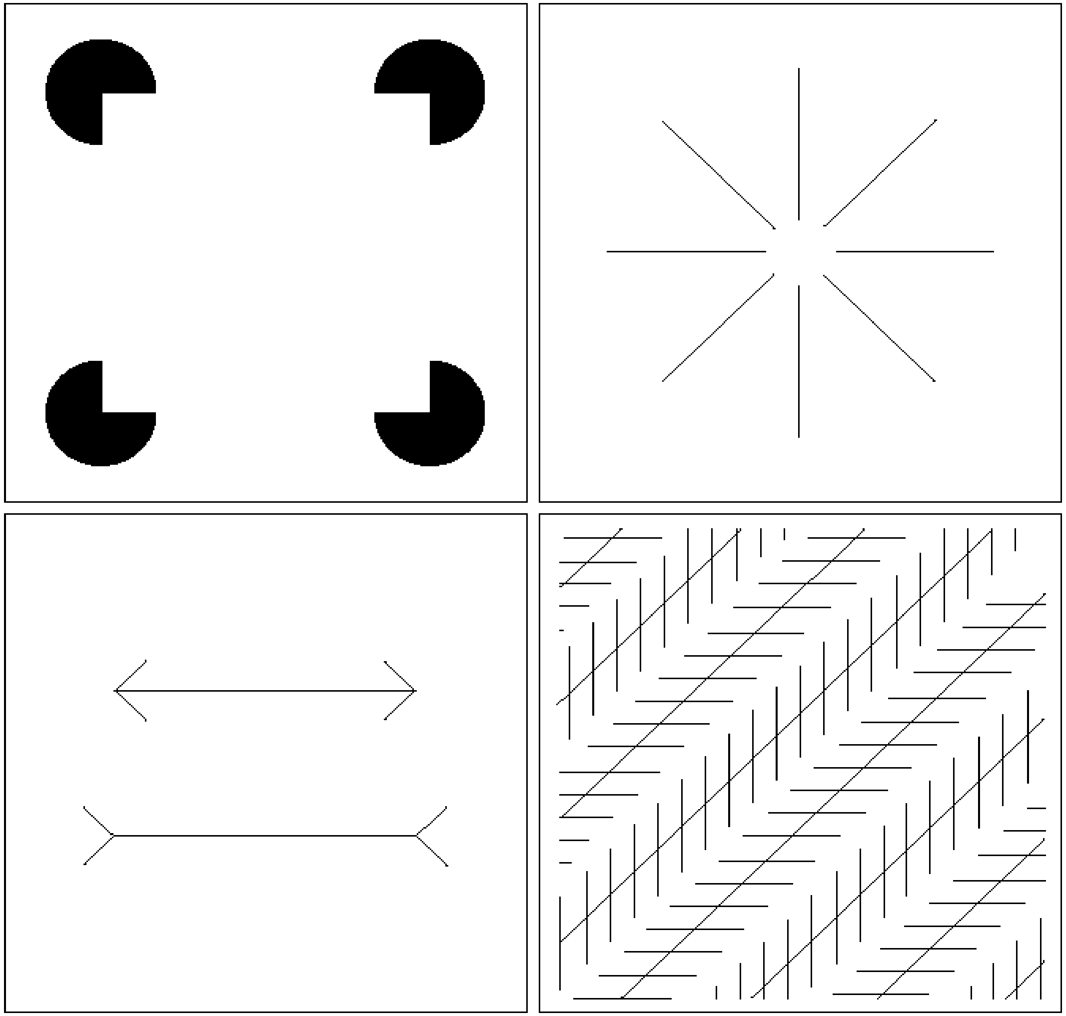


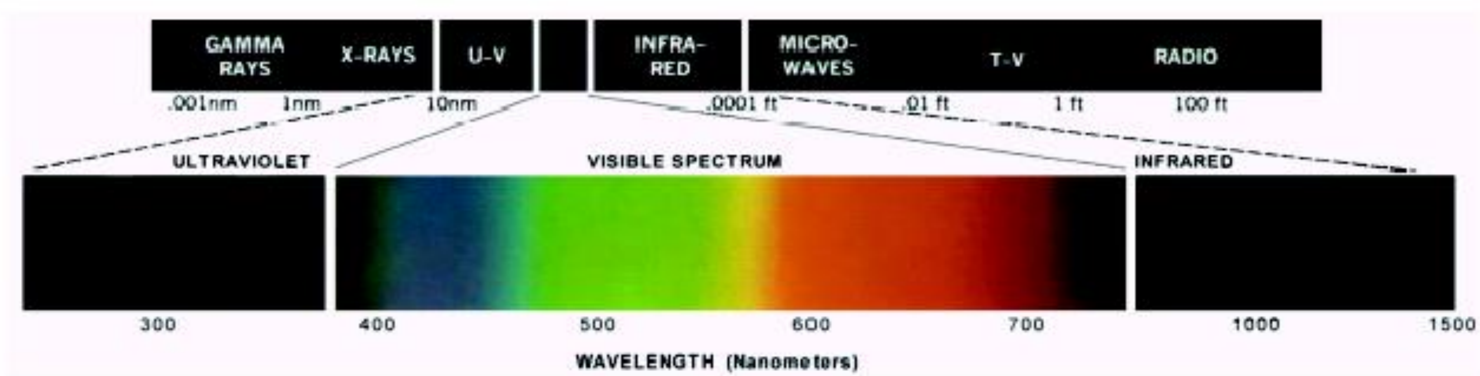
FIGURE 2.9 Some well-known optical illusions.

Examples for Human Perception Phenomena

Light And The Electromagnetic Spectrum

Light is just a particular part of the electromagnetic spectrum that can be sensed by the human eye

The electromagnetic spectrum is split up according to the wavelengths of different forms of energy



Light and the Electromagnetic Spectrum

- **The colors that humans perceive in an object are determined by the nature of the light reflected from the object.**
- **Wavelength = velocity (c)/frequency(v)**
- **Energy of eletromagnetic spectrum E = hv**
- Achromatic or monochromatic light is void of color, and is described by its intensity (gray level).
- Chromatic light spans the electromagnetic energy spectrum from 0.43 to 0.79 μm, and is described by
  - **Radiance:** Total amount of energy that flows from the light source, and measured in watts (W)
  - **Luminance:** Measured in lumens (lm), gives a measure of the amount of energy an observer perceives from a light source
  - **Brightness:** Subjective descriptor of light perception that is practically impossible to measure.

A simple image model

- Two-dimensional light-intensity function

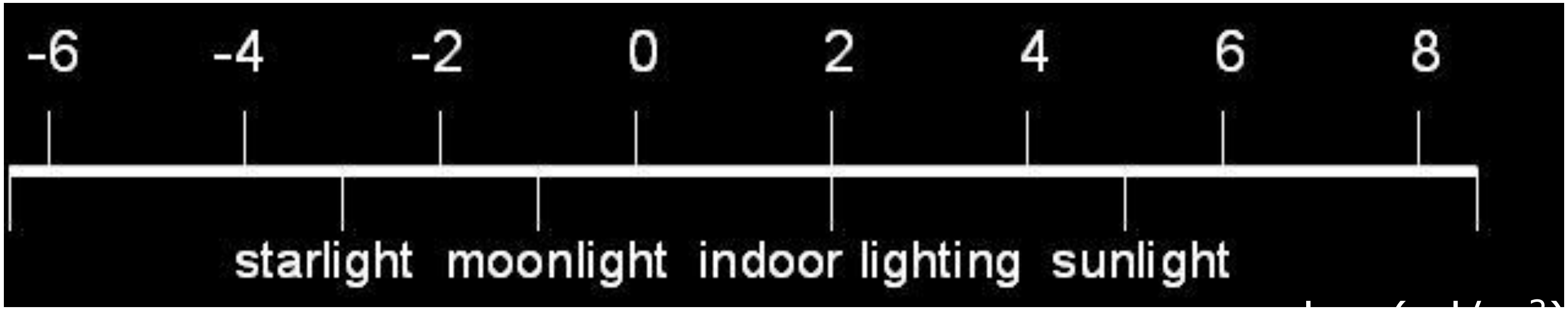
$$f(x,y) = l(x,y)\, r(x,y)$$

$l(x,y)$ - **illumination component**

$r(x,y)$ – **reflectance component**

A simple image model

- ***l*(*x*,*y*) - illumination range**

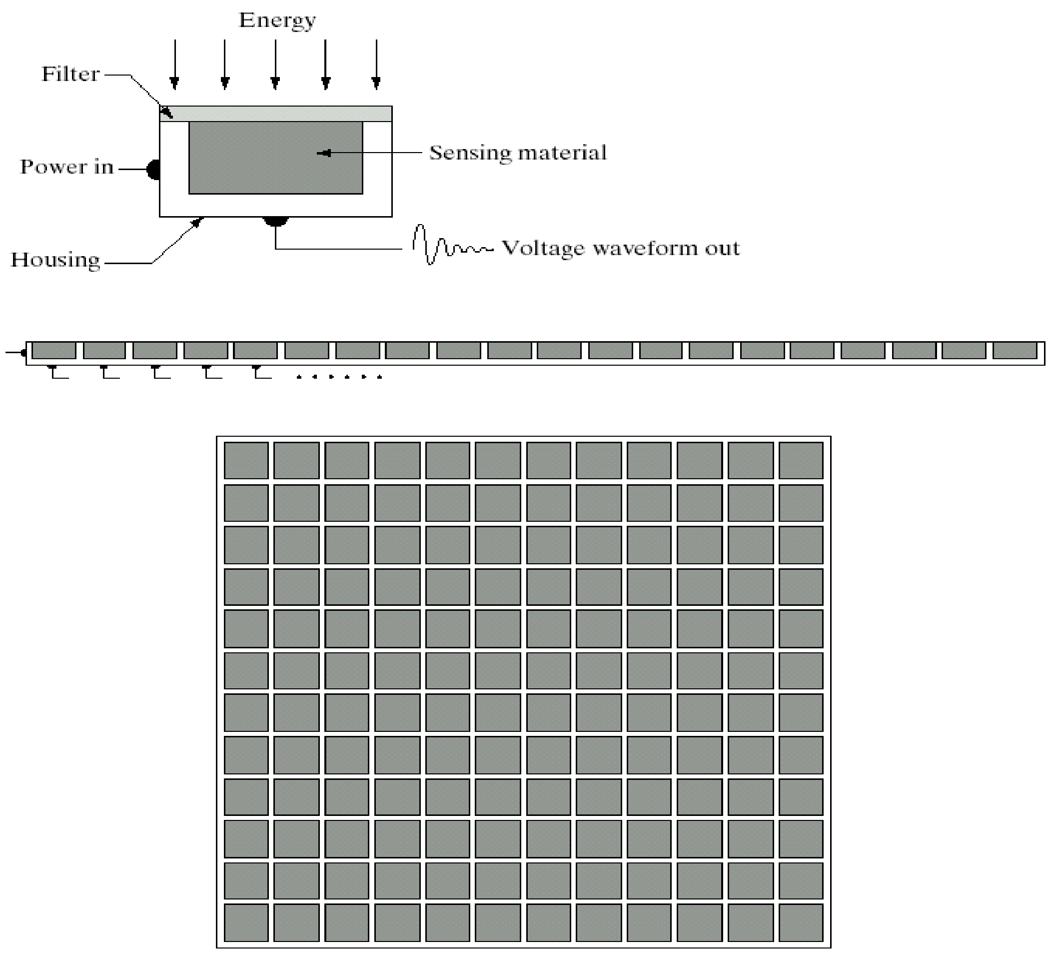


- ***r*(*x*,*y*) – typical reflectance indixes**
  - **black velvet (0.01)**
  - **stainless steel (0.65)**
  - **white paint (0.80)**
  - **silver plate (0.90)**
  - **snow (0.93)**

Image Sensing and Acquisition

- **Electromagnetic energy source and sensor that can detect the energy of the electromagnetic source are needed to generate an image.**
- EM source will illuminate the objects that need to be imaged and then a sensor will detect the reflected energy from the objects.
- **Different objects will have different degree of reflections and absorption of the electromagnetic energy.**
- These differences in reflections and absorption are the reasons for objects to appear distinct in the images.

Image Sensing and Acquisition



a
b
c

FIGURE 2.12
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Image Acquisition Using a Single Sensor

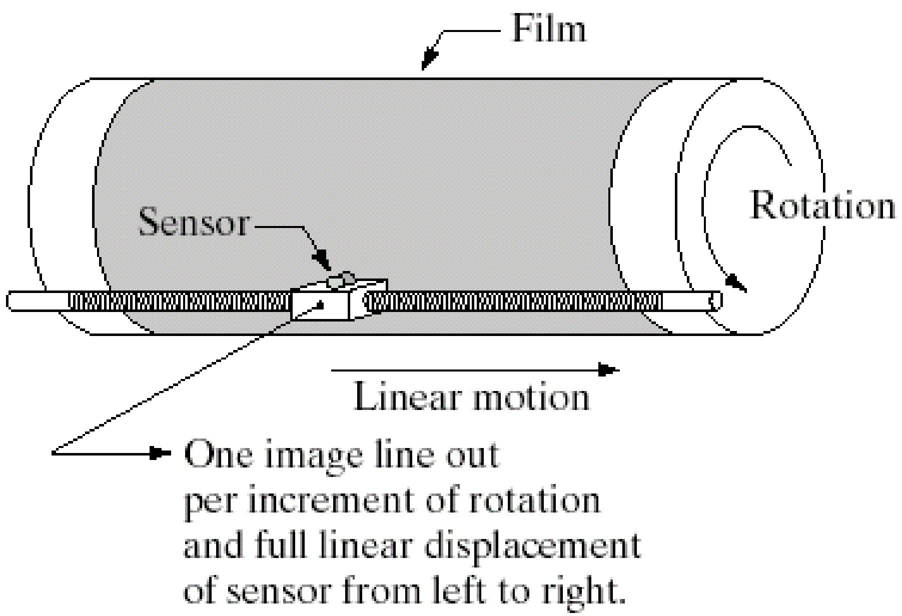


FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.

Image Acquisition Using Sensor Strip

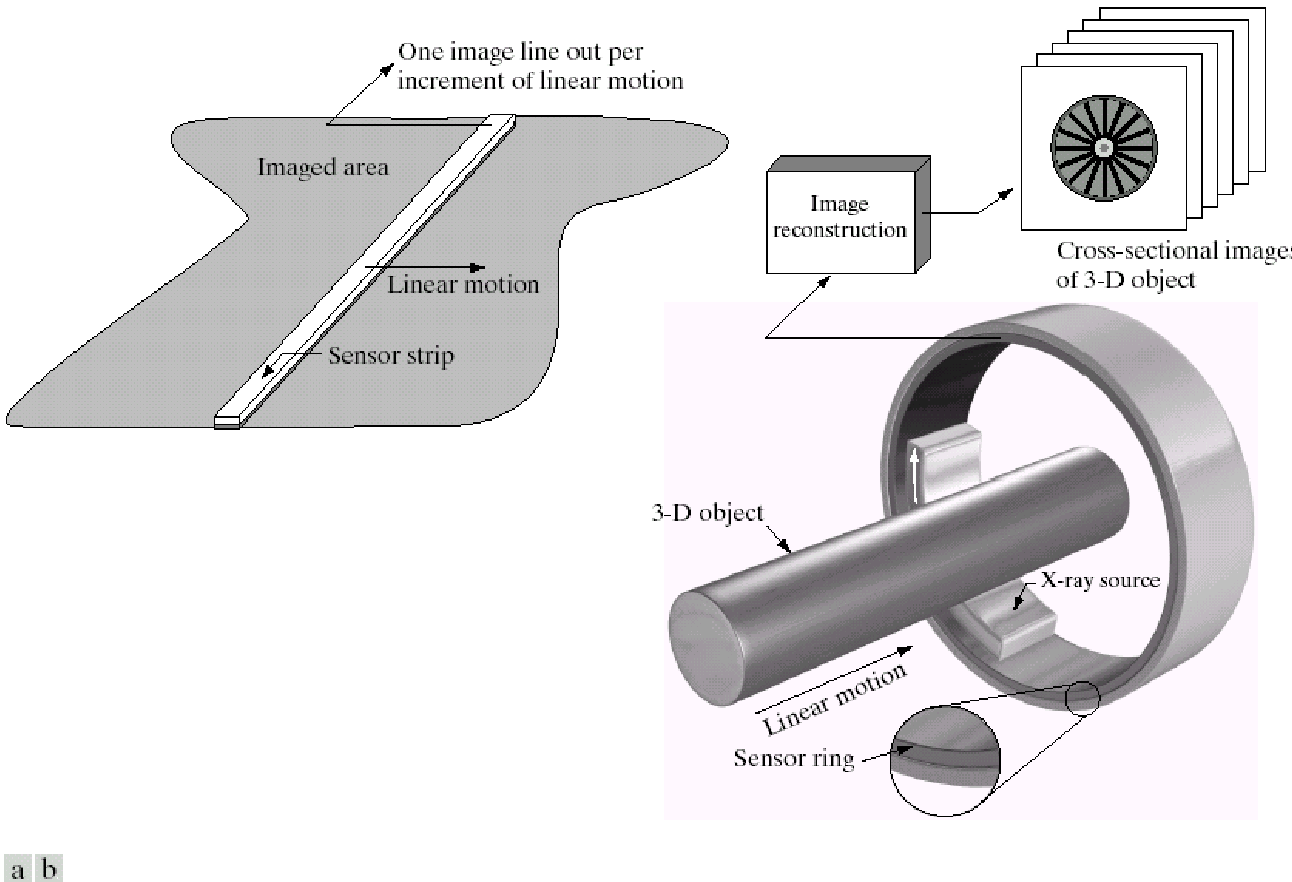
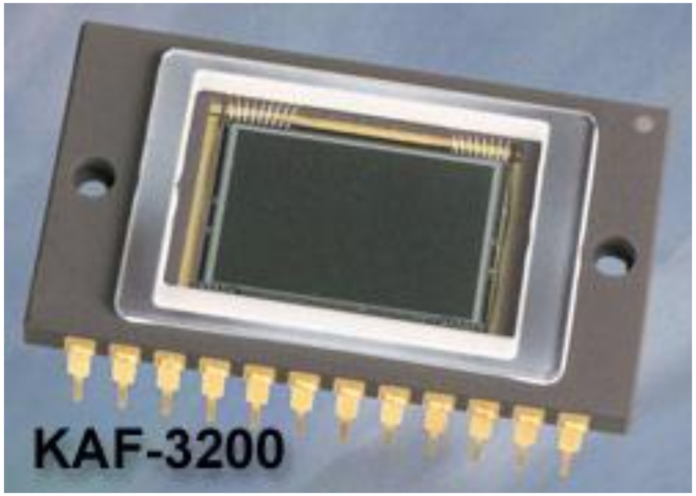


FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

Image Sensors : Array Sensor

Charge-Coupled Device (CCD)



CCD KAF-3200E from Kodak.
(2184 x 1472 pixels,
Pixel size 6.8 microns^2)

- Used for convert a continuous image into a digital image
- Contains an array of light sensors
- Converts photon into electric charges accumulated in each sensor unit

Image Sensor: Inside Charge-Coupled Device

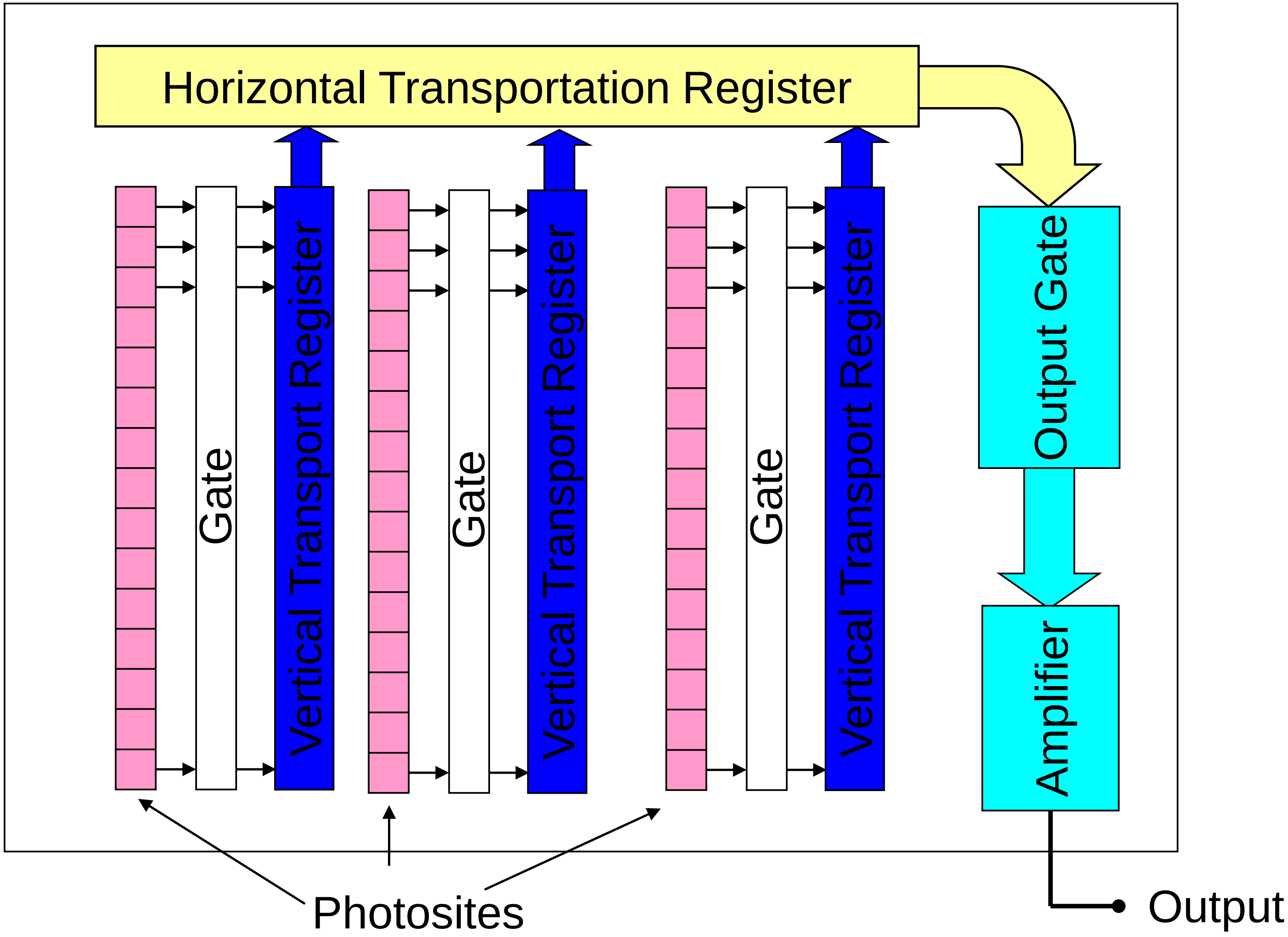


Image Sensor: How CCD works

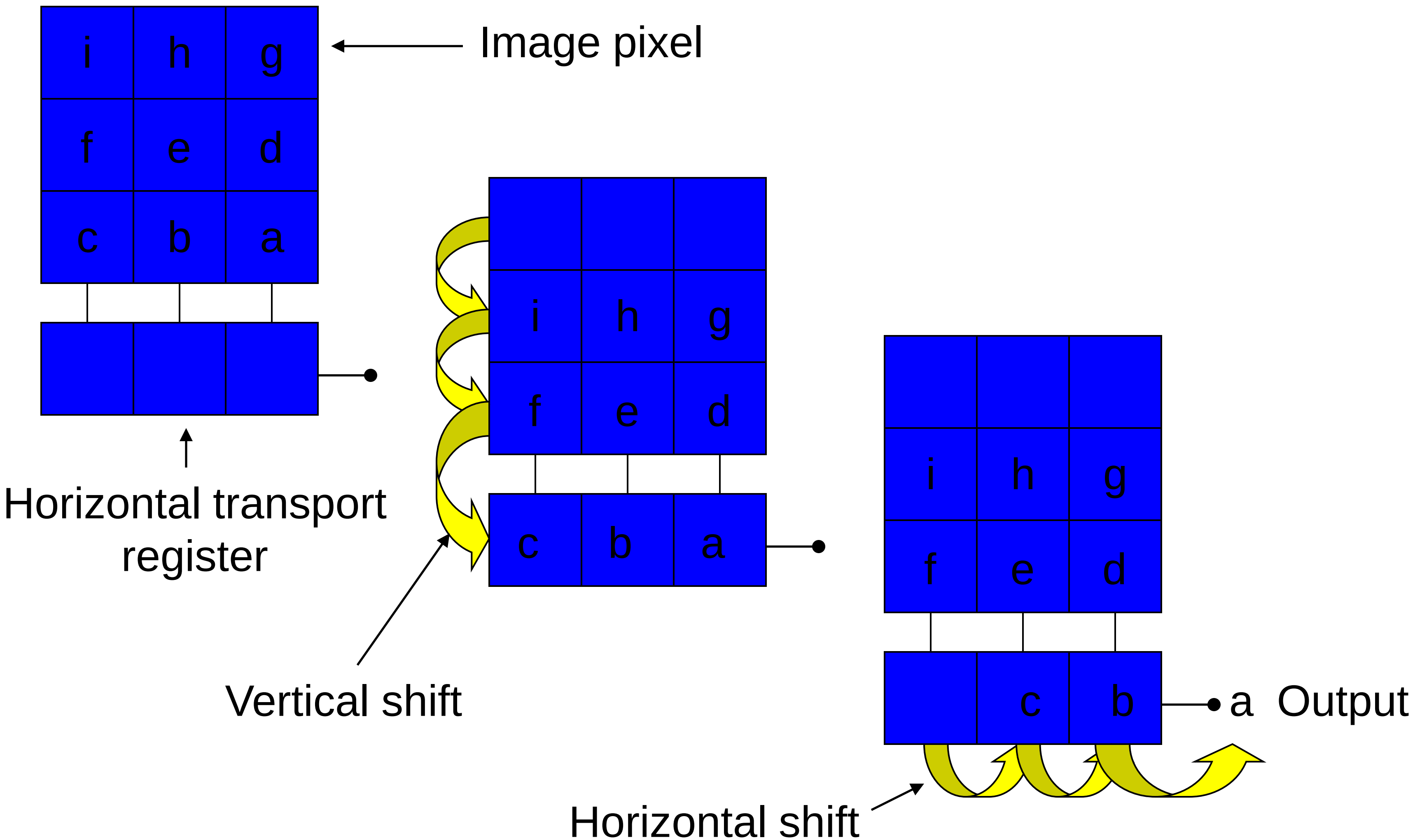


Image Acquisition

Nowadays most visible and near IR electromagnetic imaging is done with 2-dimensional charged-coupled devices (CCDs).

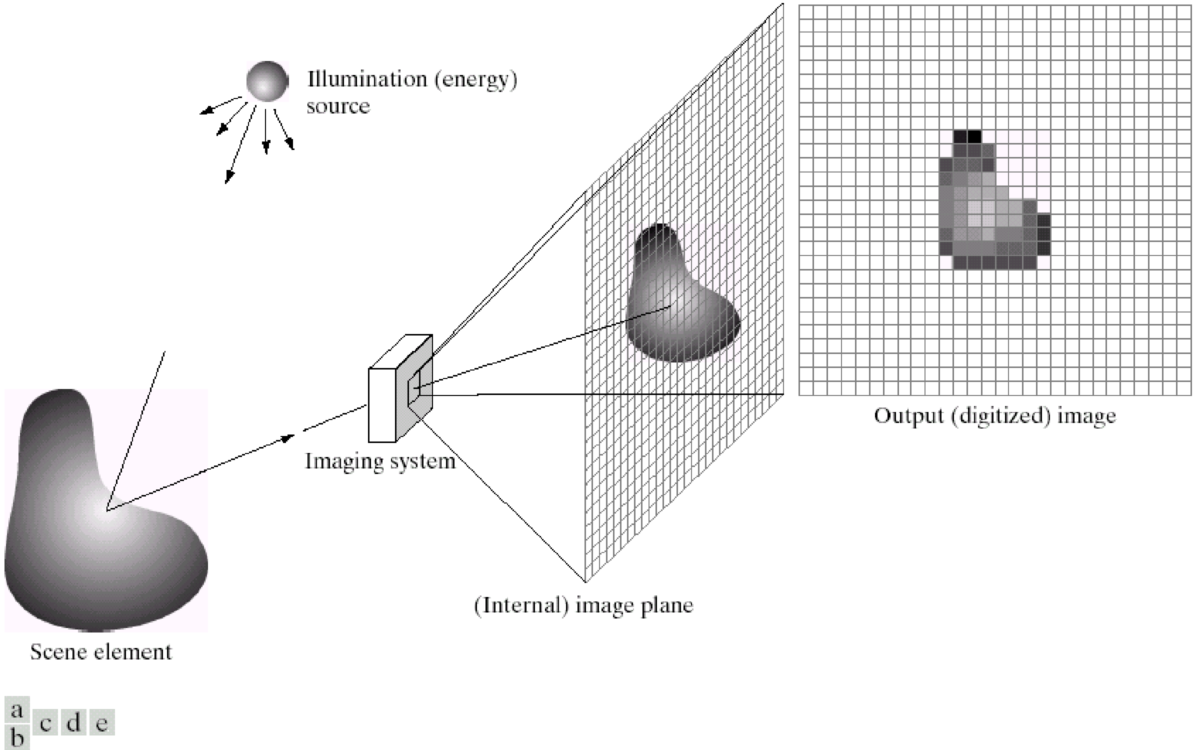


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

A Simple Image Formation Model

Mathematical representation of monochromatic images.

- **Two dimensional function $f(x,y)$, where f is the gray level of a pixel at location x and y.**
- **The values of the function f at different locations are proportional to the energy radiated from the imaged object.**

A Simple Image Formation Model

$0 < f(x,y) < \infty$ Nonzero and Finite

$f(x,y) = i(x,y) * r(x,y)$ *Reflectivity*

$f(x,y) = i(x,y) * t(x,y)$ *Transmissivity*

$0 < i(x,y) < \infty$

$i(x,y)$ - illumination component
$r(x,y)$ – reflectance component

$0 \leq r(x,y)$ and $t(x,y) \leq 1$

A Simple Image Formation Model

$i(x,y)$

: Sun on clear day	90,000 lm/m2
: Sun on cloudy day	10,000 lm/m2
: Full moon	0.1 lm/m2
: Commercial office	1,000 lm/m2

$r(x,y)$

:Black Velvet	0.01
:Stainless Steel	0.65
:Flat-white Wall Paint	0.80
:Silver-plated Metal	0.90
:Snow	0.93

Image Sampling and Quantization

- **The output of most sensors is continuous in amplitude and spatial coordinates.**
- **Converting an analog image to a digital image require sampling and quantization**
- **Sampling: is digitizing the coordinate values**
- **Quantization: is digitizing the amplitude values**

Sampling

- Digitization of the spatial coordinates, sample (x, y) at discrete values of (0, 0), (0, 1), ….

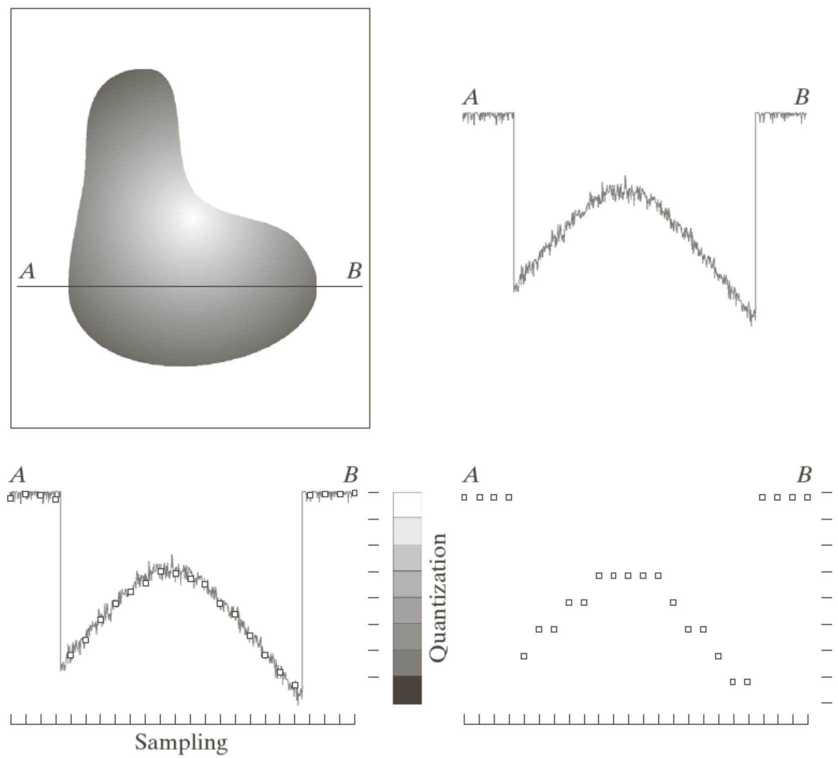
- f(x, y) is 2-D array

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,M-1) \\ f(1,0) & f(1,1) & & f(1,M-1) \\ & & & \\ f(N-1,0) & f(N-1,1) & & f(N-1,M-1) \end{bmatrix}$$

Quantization

- Digitization of the light intensity function
- Each f(i,j) is called a pixel
- The magnitude of f(i,j) is represented digitally with a fixed number of bits - quantization

Image Acquisition

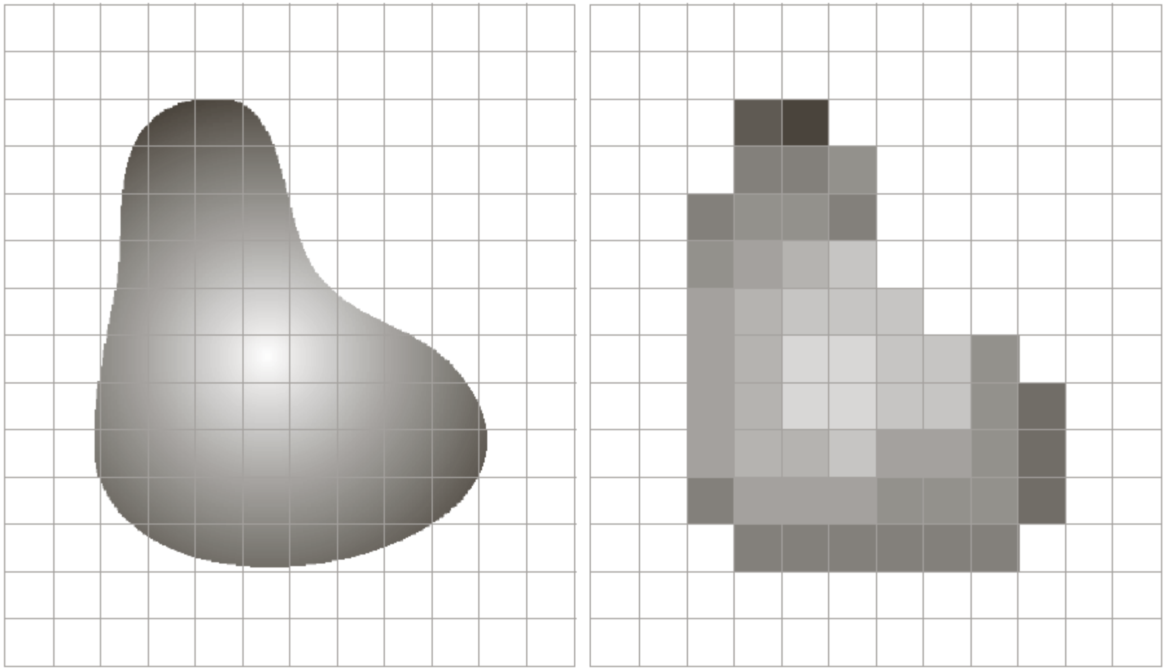


a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Sampling: digitizing the 2-dimensional spatial coordinate values
Quantization: digitizing the amplitude values (brightness level)

Sampling and Quantization



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

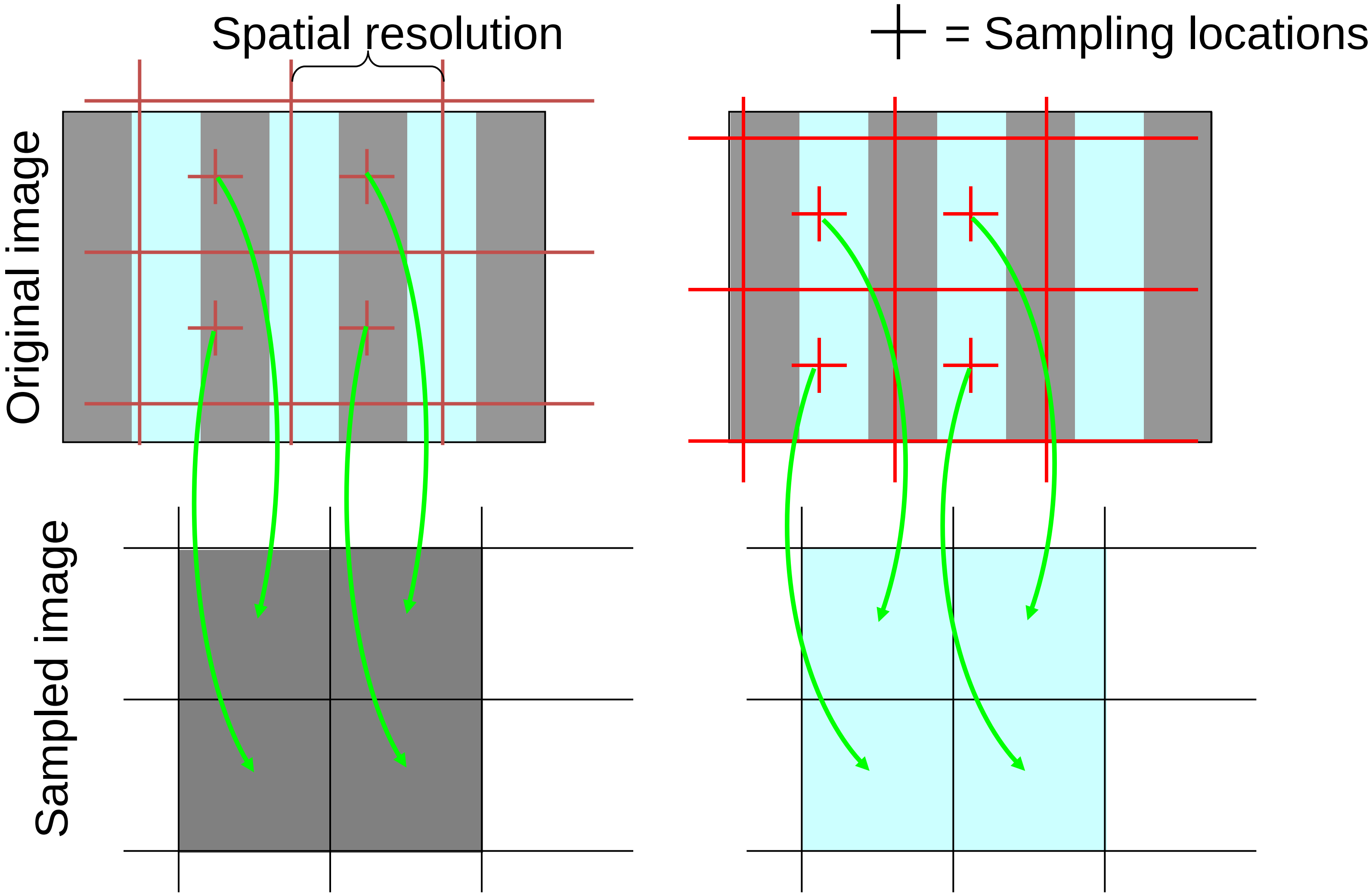
Image sampling: discretize an image in the spatial domain

Spatial resolution / image resolution: pixel size or number of pixels

Sampling and Quantization

- How many samples to take?
  - Number of pixels (samples) in the image
  - Nyquist rate
- How many gray-levels to store?
  - At a pixel position (sample), number of levels of color/intensity to be represented

How to choose the spatial resolution



Under sampling, we lost some image details!

Sampling and Quantization

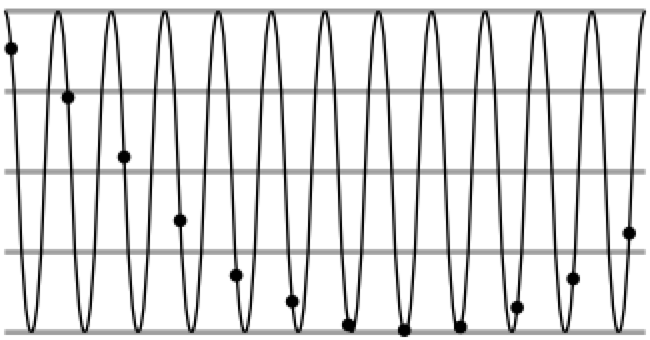
- How many samples to take?

a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Definition of Aliasing

- Aliasing refers to the effect produced when a signal is imperfectly reconstructed from the original signal.
- **Aliasing occurs when a signal is not sampled at a high enough frequency to create an accurate representation. This effect is shown in the following example of a sinusoidal function:**



- **In this example, the dots represent the sampled data and the curve represents the original signal. Because there are too few sampled data points, the resulting pattern produced by the sampled data is a poor representation of the original.** Aliasing is relevant in fields associated with signal processing, such as digital audio, digital photography, and computer graphics.

Aliasing(mixing of two different signals)

- Spatial aliasing in the form of a Moiré pattern.
- In signal processing and related disciplines, aliasing refers to an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. It also refers to the distortion or artifact that results when the signal reconstructed from samples is different from the original continuous signal.
- Aliasing can occur in signals sampled in time, for instance digital audio is referred to as temporal aliasing. Aliasing can also occur in spatially sampled signals, for instance digital images. Aliasing in spatially sampled signals is called spatial aliasing.

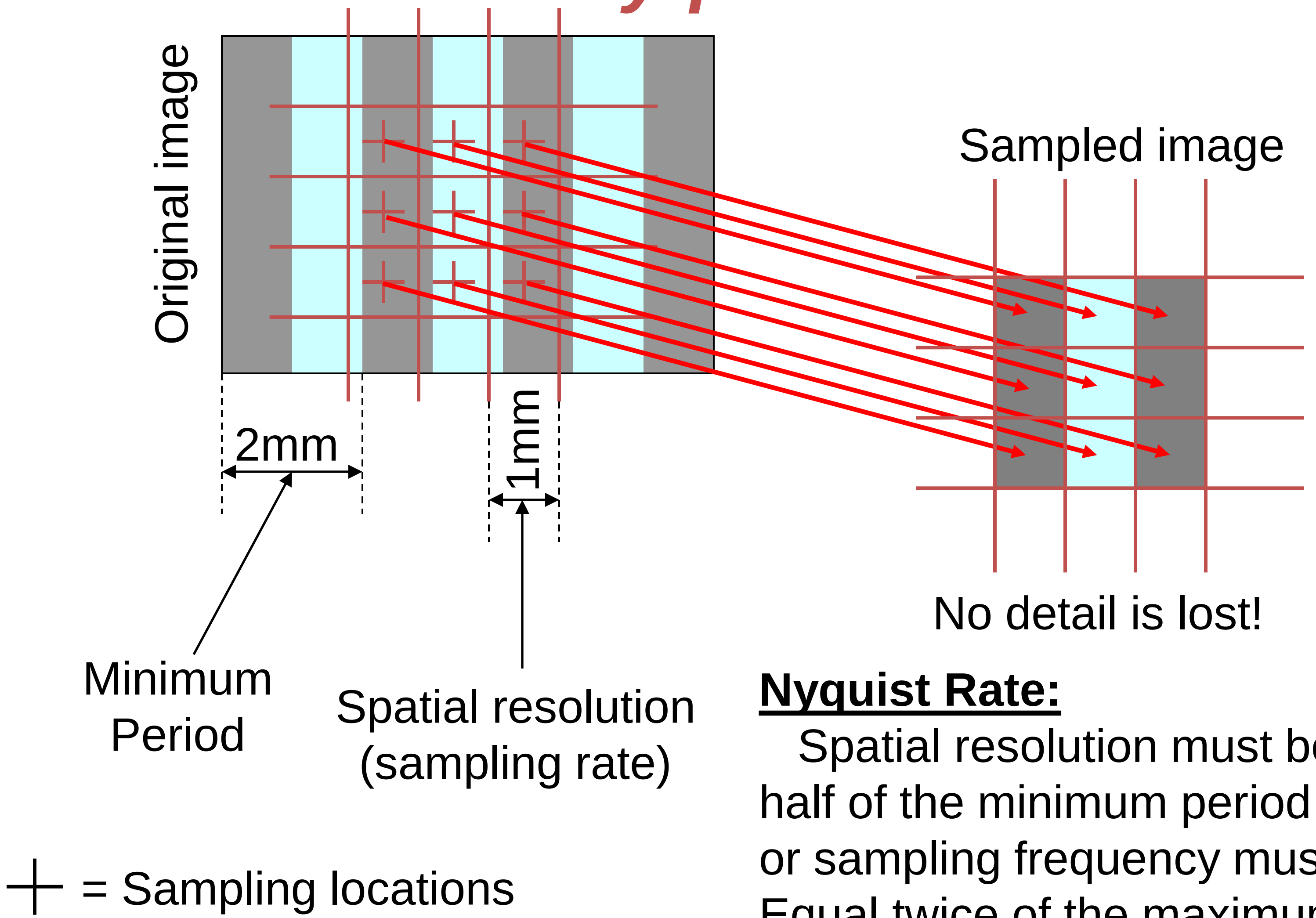
Properly sampled image of brick wall.

Spatial aliasing in the form of a Moiré pattern.

Sampling and Quantization

- How many samples to take?
  - The Nyquist Rate
  - Samples must be taken at a rate that is twice the frequency of the highest frequency component to be reconstructed.
  - Under-sampling: sampling at a rate that is too coarse, i.e., is below the Nyquist rate.
  - Aliasing: artefacts that result from under-sampling.

How to choose the spatial resolution : Nyquist Rate



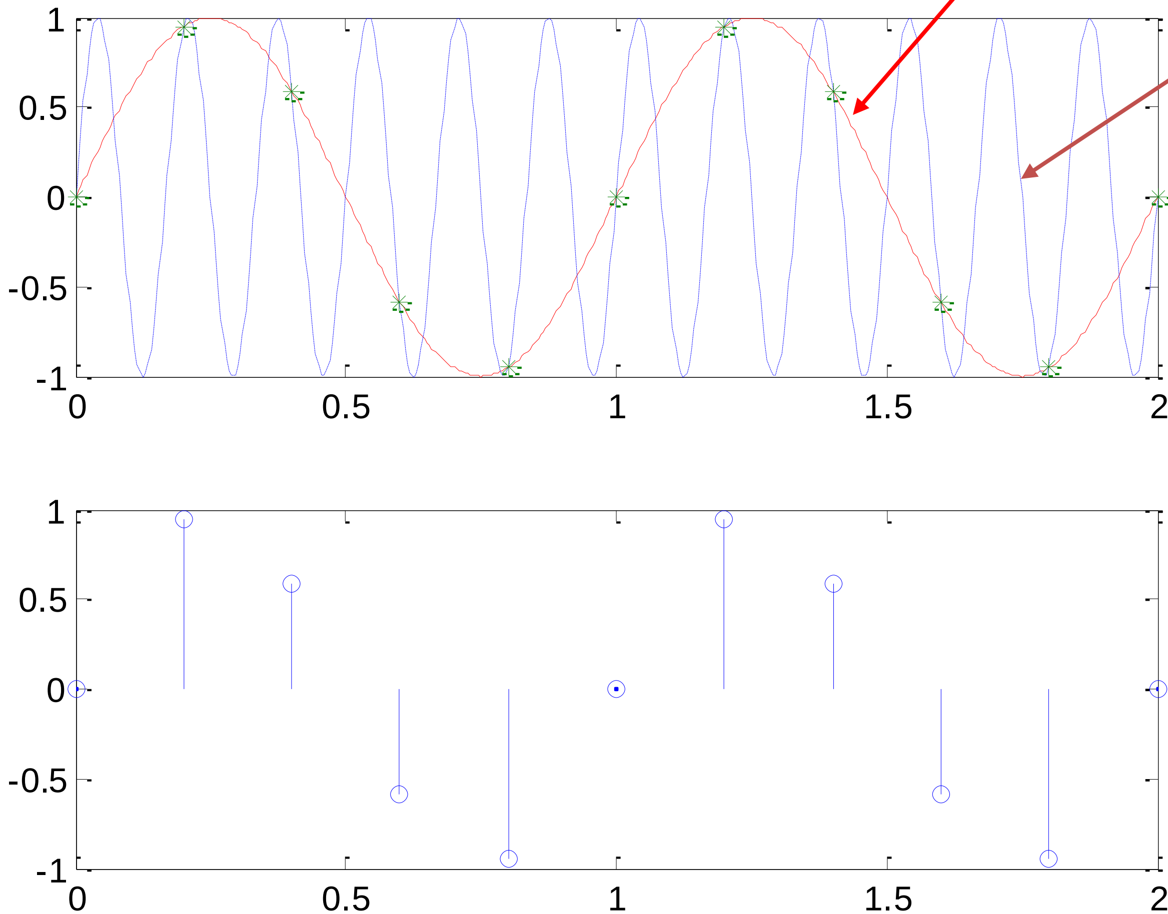
Nyquist Rate:

Spatial resolution must be less or equal half of the minimum period of the image or sampling frequency must be greater or Equal twice of the maximum frequency.

Aliased Frequency

$$x_1(t) = \sin(2\pi t), \qquad f = 1$$

$$x_2(t) = \sin(12\pi t), \qquad f = 6$$



Sampling rate:
5 samples/sec

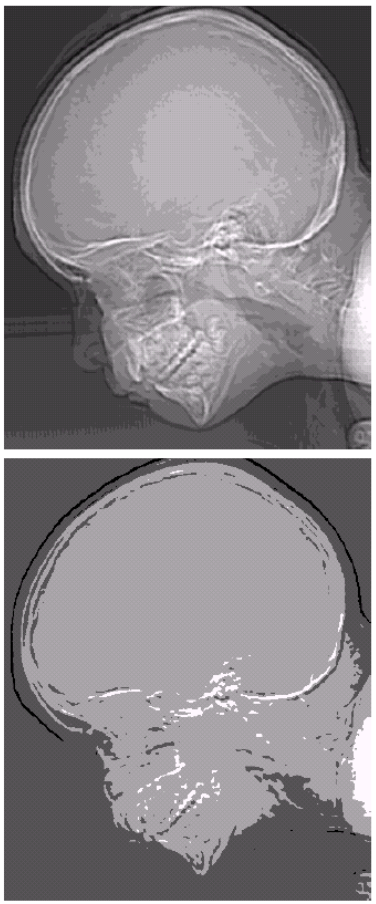
Two different frequencies but the same results !

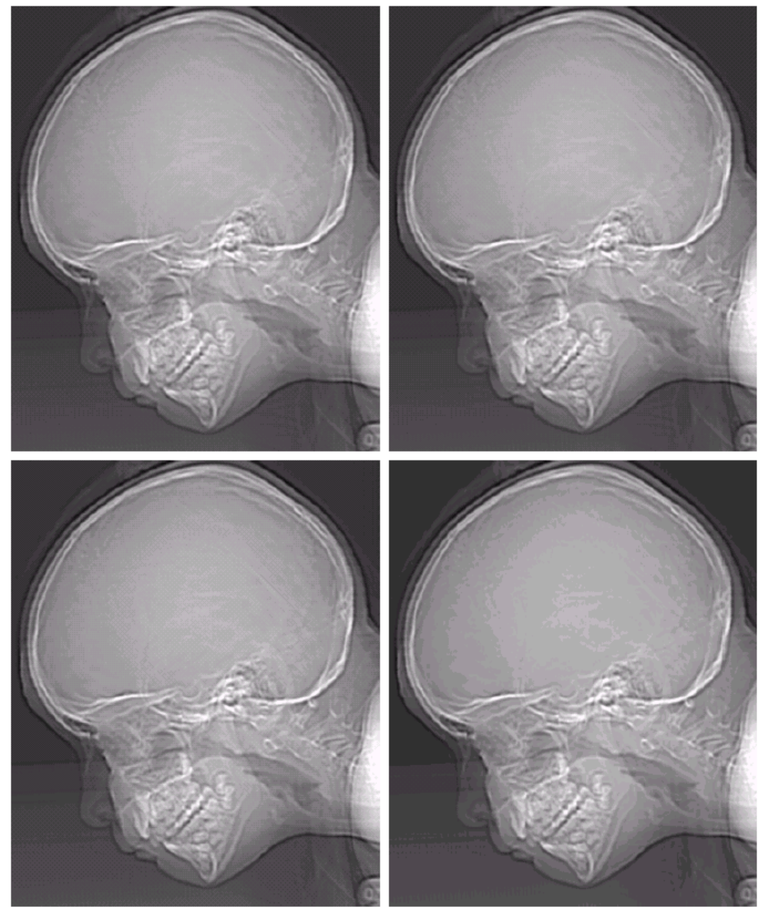
Sampling and Quantization

- How many gray-levels to store?

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

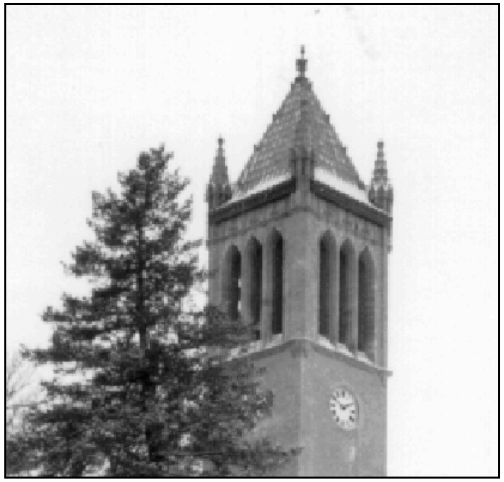




a b
c d

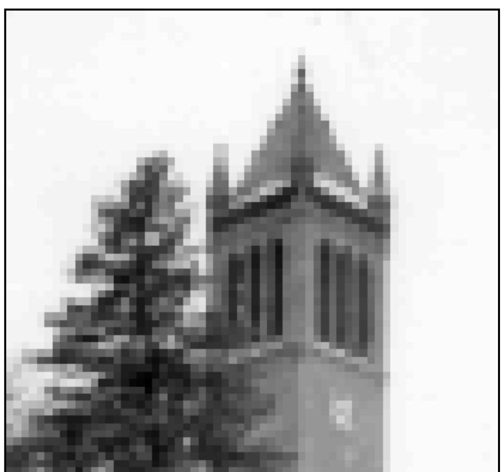
FIGURE 2.21 (a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Effect of Spatial Resolution

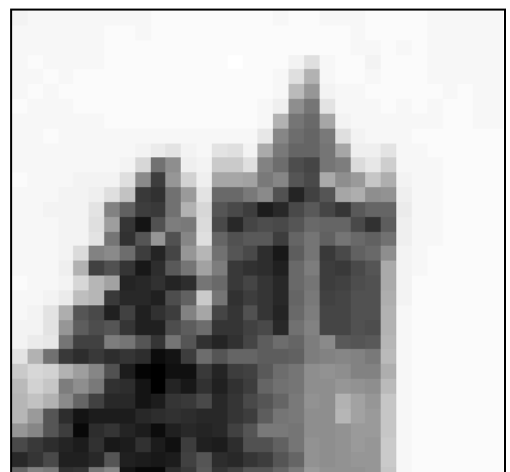


256x256 pixels

128x128 pixels



64x64 pixels



32x32 pixels

Effect of Spatial Resolution

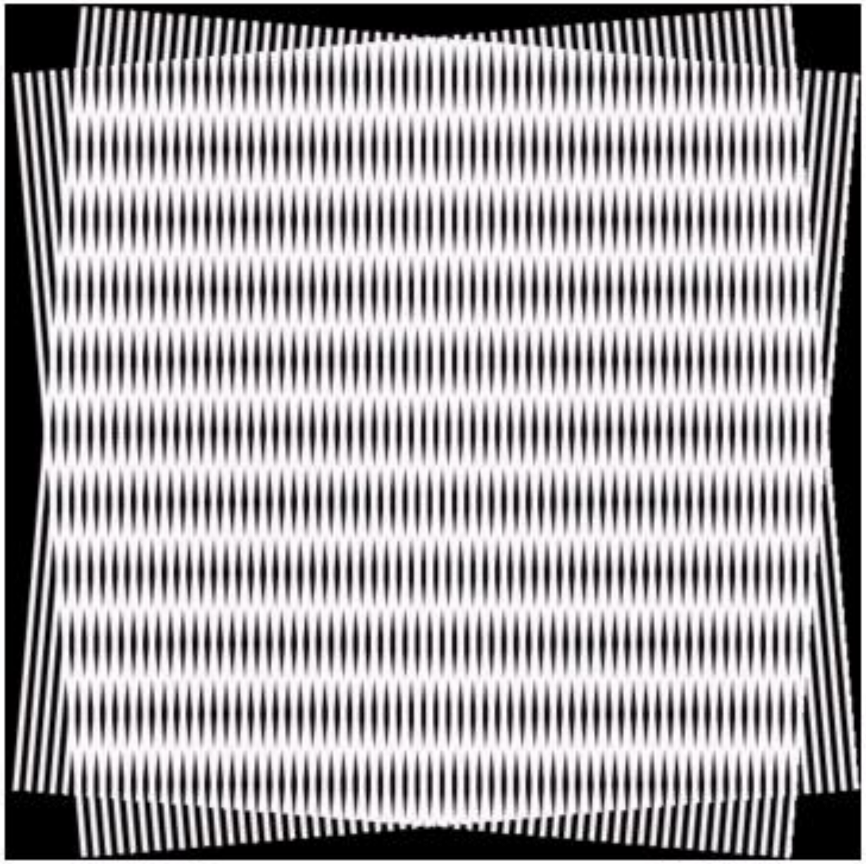
FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

(Images from Rafael C. Gonzalez and Richard Wood, Digital Image Processing, 2nd Edition.

Aliasing and Moiré Pattern

- All signals (functions) can be shown to be made up of a linear combination sinusoidal signals (sines and cosines) of different frequencies.
- For physical reasons, there is a highest frequency component in all real world signals.
- Theoretically,
  - if a signal is sampled at more than twice its highest frequency component, then it can be reconstructed exactly from its samples.
  - But, if it is sampled at less than that frequency (called undersampling), then aliasing will result.
  - This causes frequencies to appear in the sampled signal that were not in the original signal.
  - The Moiré pattern shown in Figure 2.24 is an example. The vertical low frequency pattern is a new frequency not in the original patterns.

Moire Pattern Effect : Special Case of Sampling



Moire patterns occur when frequencies of two superimposed periodic patterns are close to each other.

(Images from Rafael C. Gonzalez and Richard Wood, Digital Image Processing, 2nd Edition.

Aliasing and Moiré Pattern

- Note that subsampling of a digital image will cause undersampling if the subsampling rate is less than twice the maximum frequency in the digital image.
- Aliasing can be prevented if a signal is filtered to eliminate high frequencies so that its highest frequency component will be less than twice the sampling rate.
- Gating function: exists for all space (or time) and has value zero everywhere except for a finite range of space/time. Often used for theoretical analysis of signals. But, a gating signal is mathematically defined and contains unbounded frequencies.
- A signal which is periodic, $x(t) = x(t+T)$ for all t and where T is the period, has a finite maximum frequency component. So it is a bandlimited signal.
- Sampling at a higher sampling rate (usually twice or more) than necessary to prevent aliasing is called oversampling.

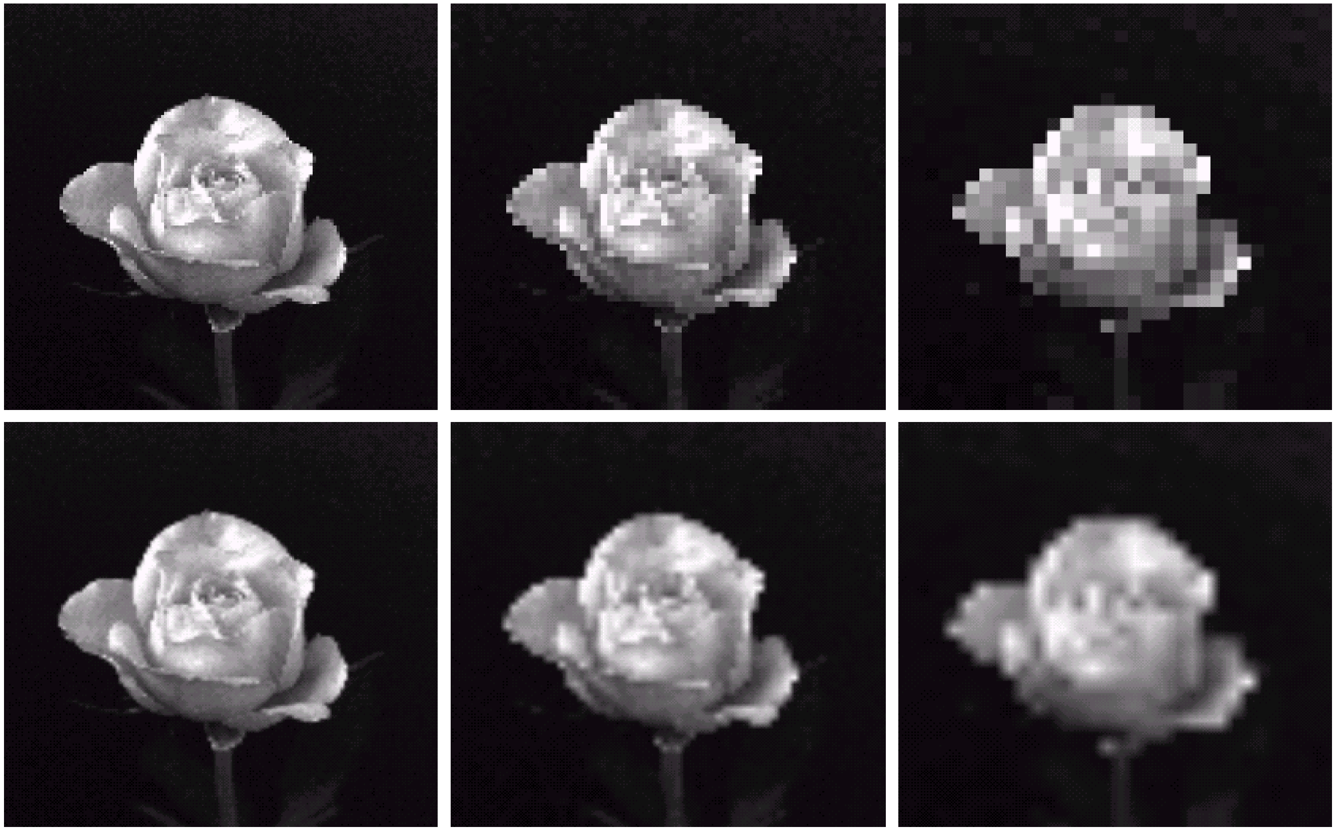
Effect of Spatial Resolution

a	b	c
d	e	f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

(Images from Rafael C. Gonzalez and Richard Wood, Digital Image Processing, 2nd Edition.

Can we increase spatial resolution by interpolation ?



a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Down sampling is an irreversible process.

(Images from Rafael C. Gonzalez and Richard Wood, Digital Image Processing, 2nd Edition.

Image Quantization

Image quantization:

discretize continuous pixel values into discrete numbers

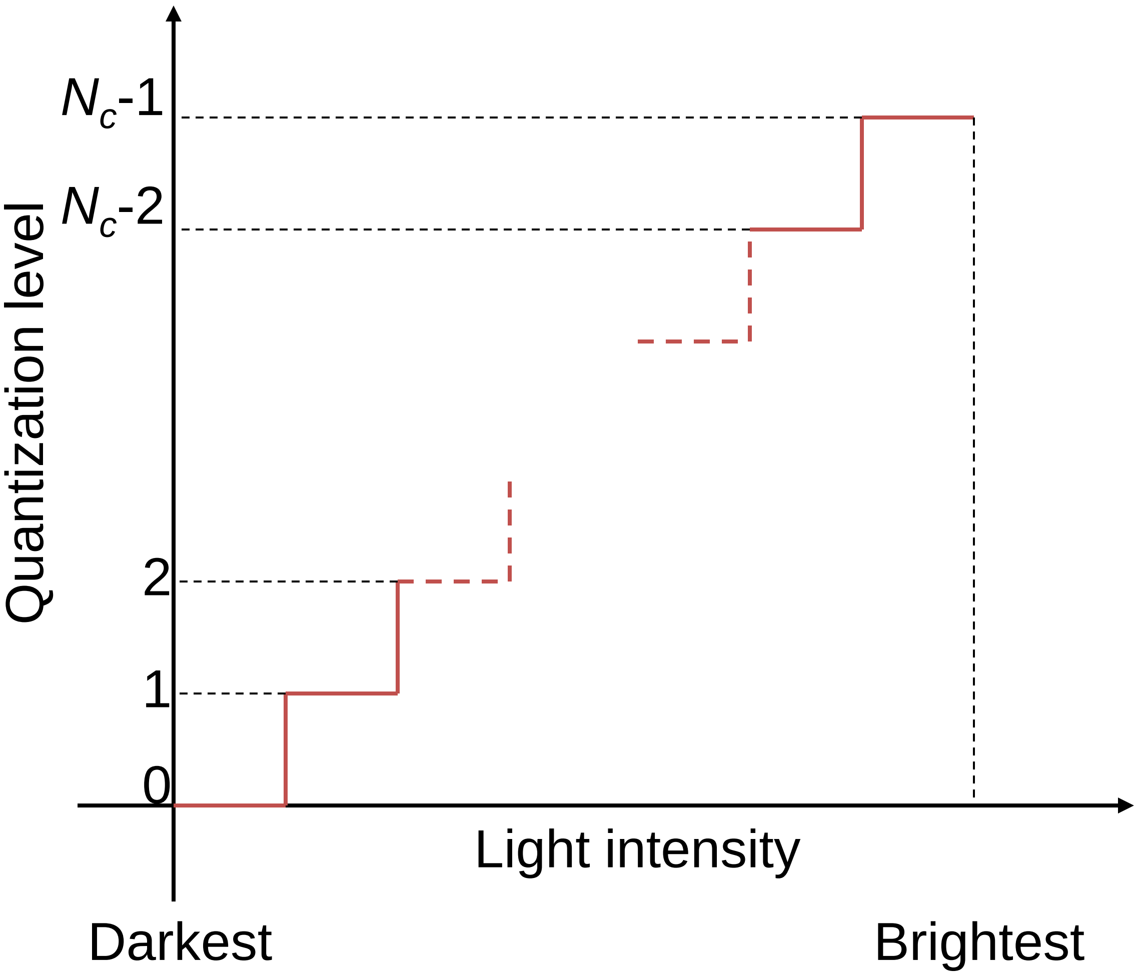
Color resolution/ color depth/ levels:

- No. of colors or gray levels or
- No. of bits representing each pixel value
- No. of colors or gray levels N_c is given by

$$N_c = 2^b$$

where b = no. of bits

Quantization function



Effect of Quantization Levels

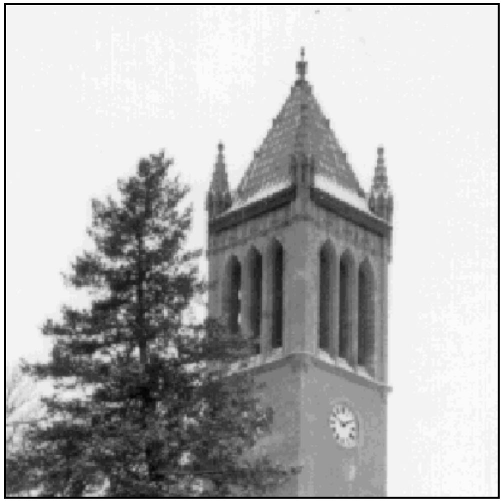
256 levels

128 levels

64 levels

32 levels

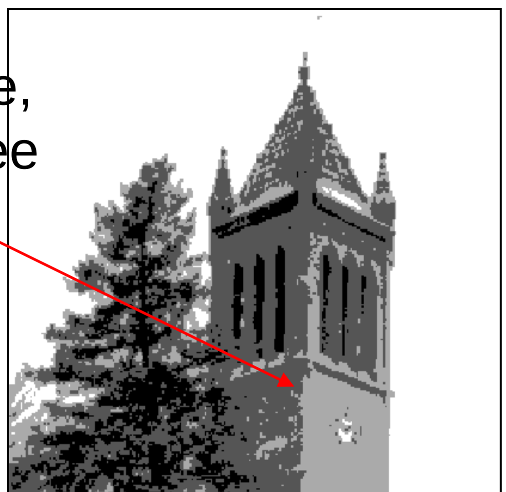
Effect of Quantization Levels (cont.)



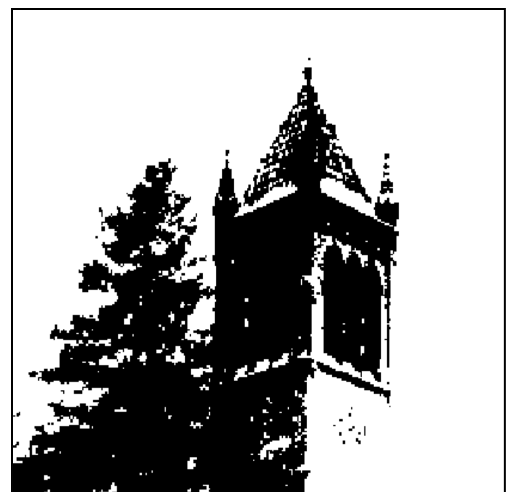
16 levels

8 levels

In this image, it is easy to see false contour.



4 levels



2 levels

How to select the suitable size and pixel depth of images

The word “suitable” is subjective: depending on “subject”.



Low detail image — Lena image

Medium detail image — Cameraman image

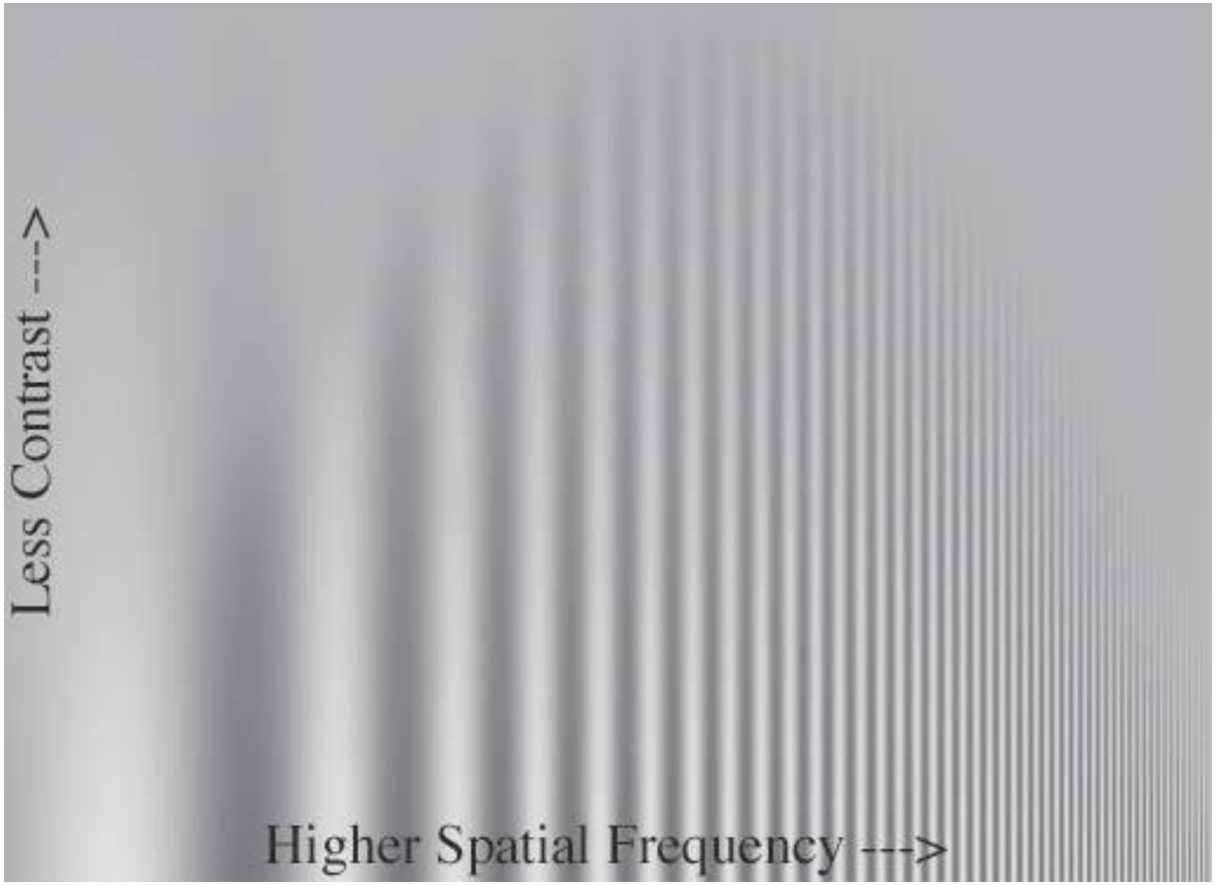
High detail image

To satisfy human mind

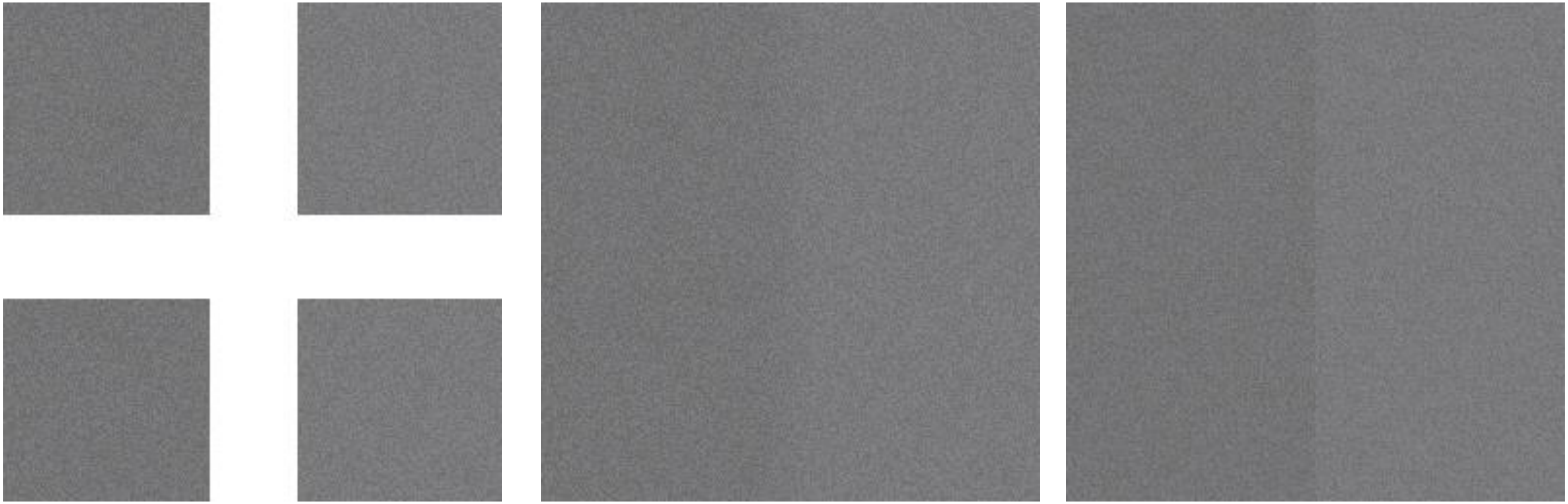
1. For images of the same size, the low detail image may need more pixel depth.
2. As an image size increase, fewer gray levels may be needed.

(Images from Rafael C. Gonzalez and Richard Wood, Digital Image Processing, 2nd Edition.

Human vision: Spatial Frequency vs Contrast



Human vision: Distinguish ability for Difference in brightness

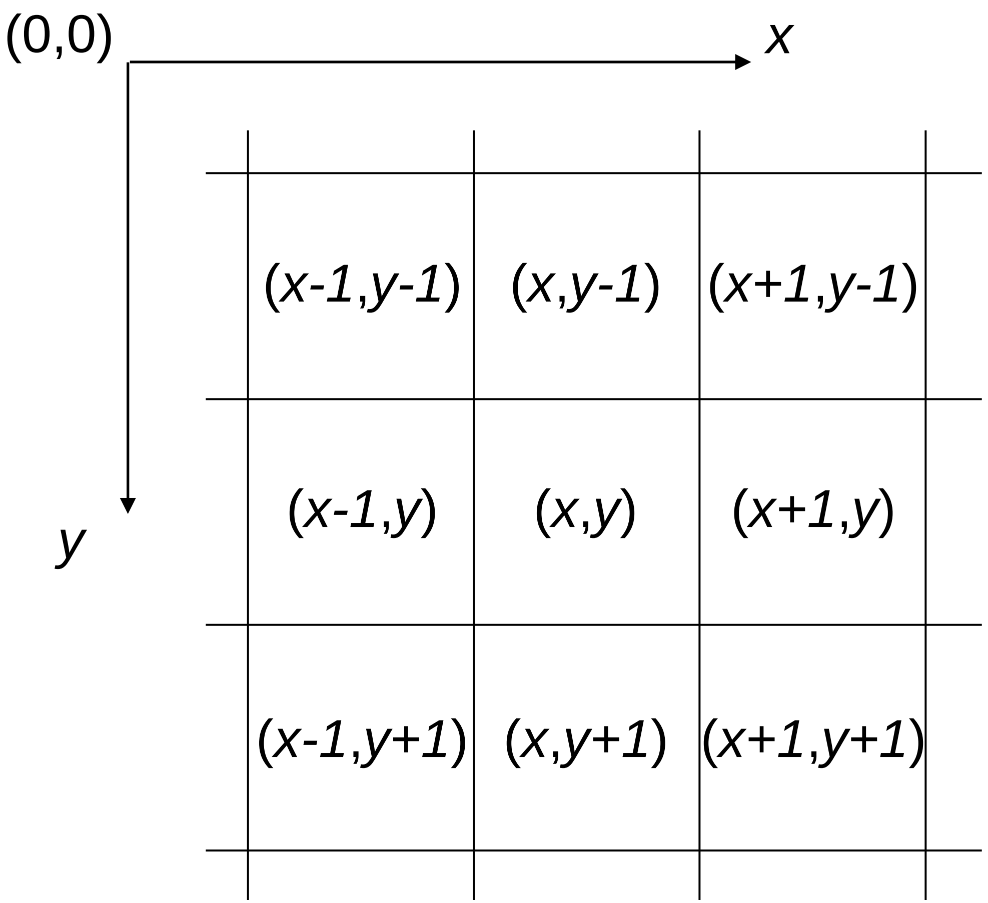


Regions with 5% brightness difference

Sampling Theorem and Aliasing Effect

- *Shannon sampling theorem* states that if a function is sampled at a rate equal to or greater than twice its highest frequency, it is possible to recover completely the original function from its samples.

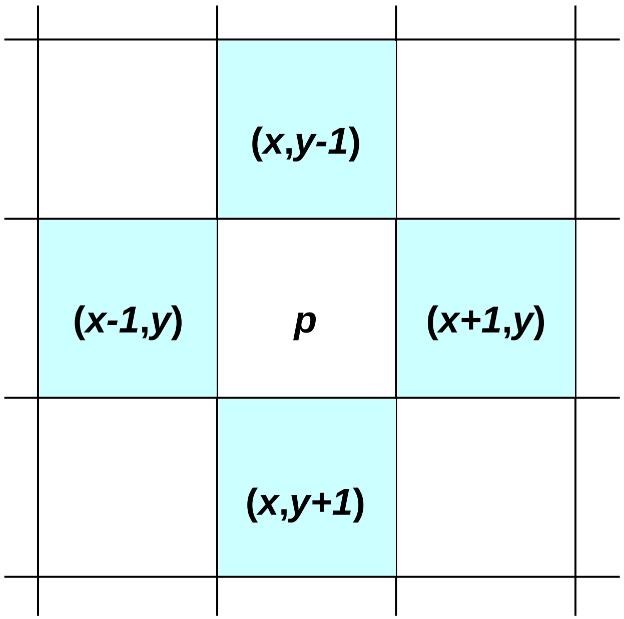
Basic Relationship of Pixels



Conventional indexing method

Neighbors of a Pixel

Neighborhood relation is used to tell adjacent pixels. It is useful for analyzing regions.



4-neighbors of *p*:

$$N_4(p) = \left\{ \begin{array}{c} (x-1,y) \\ (x+1,y) \\ (x,y-1) \\ (x,y+1) \end{array} \right\}$$

4-neighborhood relation considers only vertical and horizontal neighbors.

Note: $q \in N_4(p)$ implies $p \in N_4(q)$

Neighbors of a Pixel (cont.)

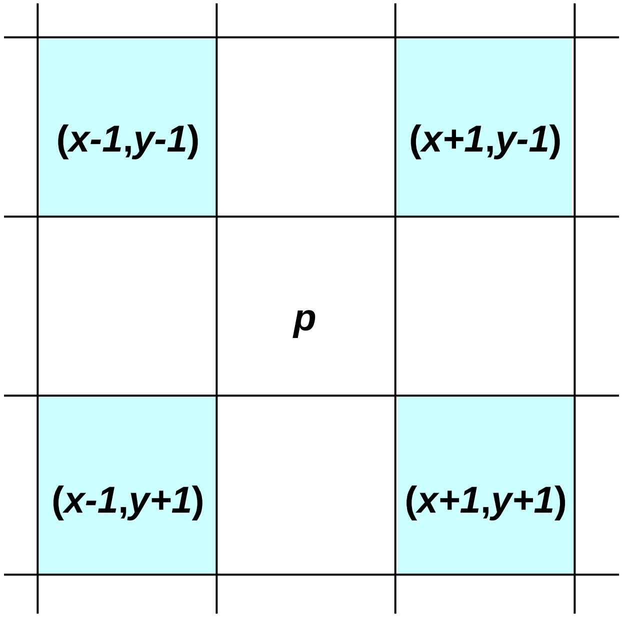
(*x*-*1*,*y*-*1*)	(*x*,*y*-*1*)	(*x*+*1*,*y*-*1*)
(*x*-*1*,*y*)	*p*	(*x*+*1*,*y*)
(*x*-*1*,*y*+*1*)	(*x*,*y*+*1*)	(*x*+*1*,*y*+*1*)

8-neighbors of *p*:

$$N_8(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x,y-1) \\ (x+1,y-1) \\ (x-1,y) \\ (x+1,y) \\ (x-1,y+1) \\ (x,y+1) \\ (x+1,y+1) \end{array} \right\}$$

8-neighborhood relation considers all neighbor pixels.

Neighbors of a Pixel (cont.)



Diagonal neighbors of *p*:

$$N_D(p) = \left\{ \begin{matrix} (x-1,y-1) \\ (x+1,y-1) \\ (x-1,y+1) \\ (x+1,y+1) \end{matrix} \right\}$$

Diagonal -neighborhood relation considers only diagonal neighbor pixels.

Some Basic Relationships Between Pixels

- Neighbors of a pixel
  - There are three kinds of neighbors of a pixel:
    - $N_4(p)$ 4-neighbors: the set of horizontal and vertical neighbors
    - $N_D(p)$ diagonal neighbors: the set of 4 diagonal neighbors
    - $N_8(p)$ 8-neighbors: union of 4-neighbors and diagonal neighbors

	O	
O	X	O
	O	

O		O
	X	
O		O

O	O	O
O	X	O
O	O	O

Some Basic Relationships Between Pixels

- Path:
  - The length of the path
  - Closed path
- Connectivity in a subset *S* of an image
  - Two pixels are connected if there is a path between them that lies completely within *S*.
- Connected component of *S*:
  - The set of all pixels in *S* that are connected to a given pixel in *S*.
- Region of an image
- Boundary, border or contour of a region
- Edge: a path of one or more pixels that separate two regions of significantly different gray levels.

Connectivity

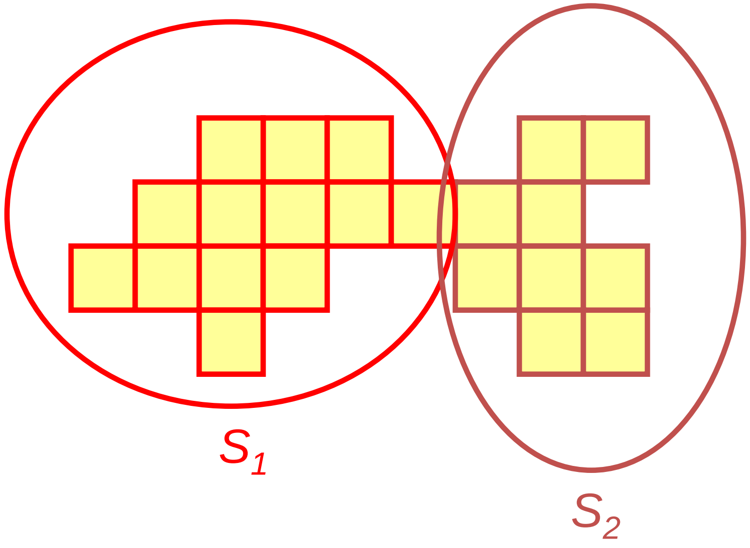
Connectivity is adapted from neighborhood relation. Two pixels are connected if they are in the same class (i.e. the same color or the same range of intensity) and they are neighbors of one another.

For p and q from the same class

- 4-connectivity: p and q are 4-connected if $q \in N_4(p)$
- 8-connectivity: p and q are 8-connected if $q \in N_8(p)$
- mixed-connectivity (m-connectivity):
  p and q are m-connected if $q \in N_4(p)$ or
  $q \in N_D(p)$ and $N_4(p) \cap N_4(q) = \emptyset$

Adjacency

A pixel *p* is *adjacent* to pixel *q* is they are connected.
Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2



We can define type of adjacency: 4-adjacency, 8-adjacency or m-adjacency depending on type of connectivity.

Path

A ***path*** from pixel p at (x,y) to pixel q at (s,t) is a sequence of distinct pixels:

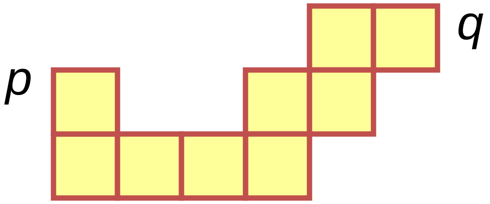
$$(x_0,y_0), (x_1,y_1), (x_2,y_2),\ldots, (x_n,y_n)$$

such that

$$(x_0,y_0) = (x,y) \text{ and } (x_n,y_n) = (s,t)$$

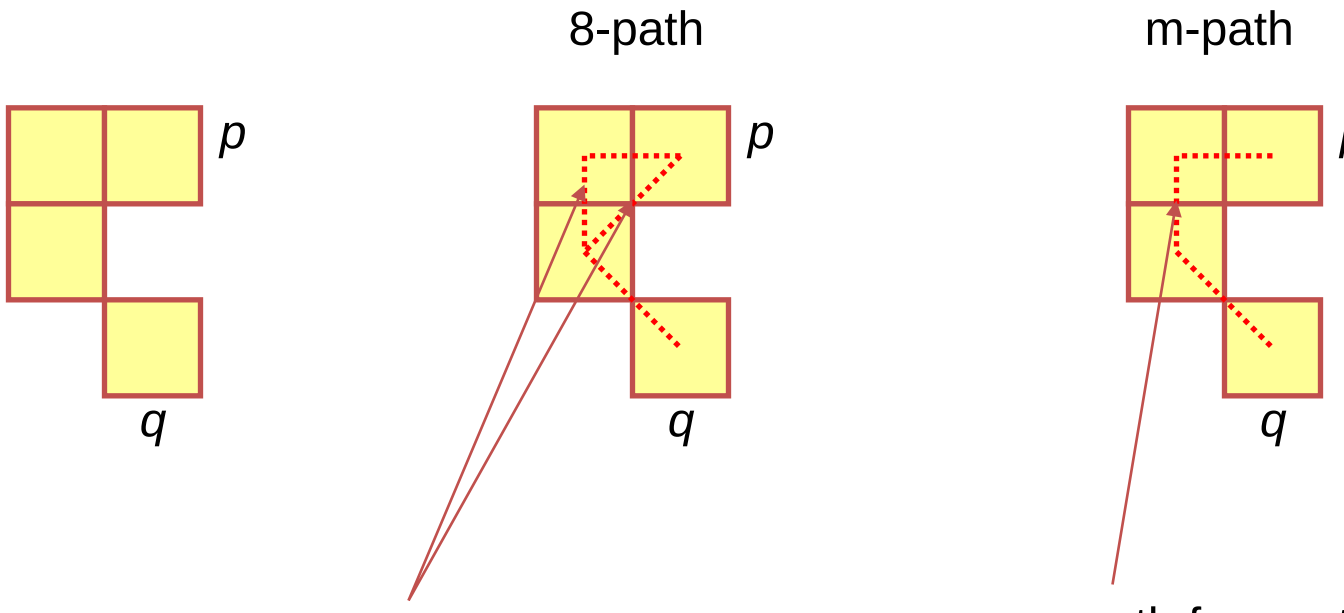
and

$$(x_i,y_i) \text{ is adjacent to } (x_{i-1},y_{i-1}), \qquad i = 1,\ldots,n$$



We can define type of path: 4-path, 8-path or m-path depending on type of adjacency.

Path (cont.)

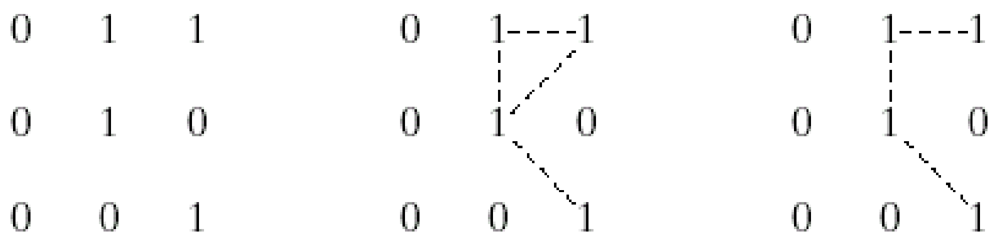


8-path from *p* to *q*
results in some ambiguity

m-path from *p* to *q*
solves this ambiguity

Some Basic Relationships Between Pixels

- An example of adjacency:



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Some Basic Relationships Between Pixels

- Adjacency:
  - Two pixels that are neighbors and have the same grey-level (or some other specified similarity criterion) are adjacent
  - Pixels can be 4-adjacent, diagonally adjacent, 8-adjacent, or m-adjacent.
- *m*-adjacency (mixed adjacency):
  - Two pixels *p* and *q* of the same value (or specified similarity) are *m*-adjacent if either
    - (i) *q* and *p* are 4-adjacent, or
    - (ii) *p* and *q* are diagonally adjacent and do not have any common 4-adjacent neighbors.
    - They cannot be both (i) and (ii).

Distance

For pixel p, q, and z with coordinates (x,y), (s,t) and (u,v), D is a ***distance function*** or ***metric*** if

- $D(p,q) \geq 0$ ($D(p,q) = 0$ if and only if $p = q$)
- $D(p,q) = D(q,p)$
- $D(p,z) \leq D(p,q) + D(q,z)$

Example: Euclidean distance

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

Distance (cont.)

D_4-distance (*city-block distance or* Manhattan) is defined as

$$D_4(p,q) = |x-s| + |y-t|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with $D_4(p) = 1$ is 4-neighbors of p.

Distance (cont.)

D_8-distance (*chessboard distance*) is defined as

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with $D_8(p) = 1$ is 8-neighbors of p.

Zooming and Shrinking Digital Images

- Zooming: increasing the number of pixels in an image so that the image appears larger
  - Nearest neighbor interpolation
    - For example: pixel replication--to repeat rows and columns of an image
  - Bilinear interpolation
    - Smoother
  - Higher order interpolation

- Image shrinking: subsampling

Zooming and Shrinking Digital Images

Nearest neighbor Interpolation (Pixel replication)

Bilinear interpolation

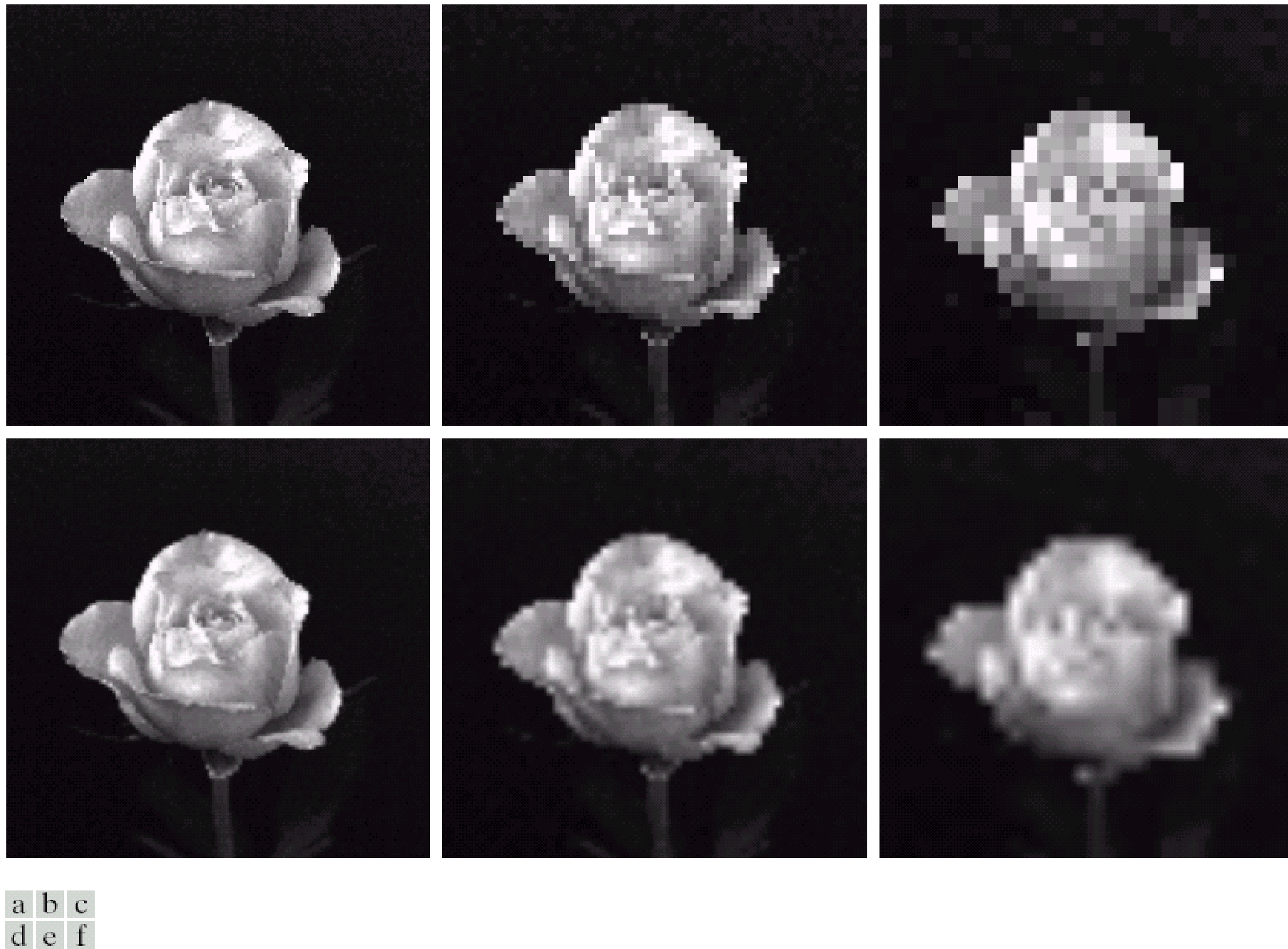


FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Face Recognition

Face Expression

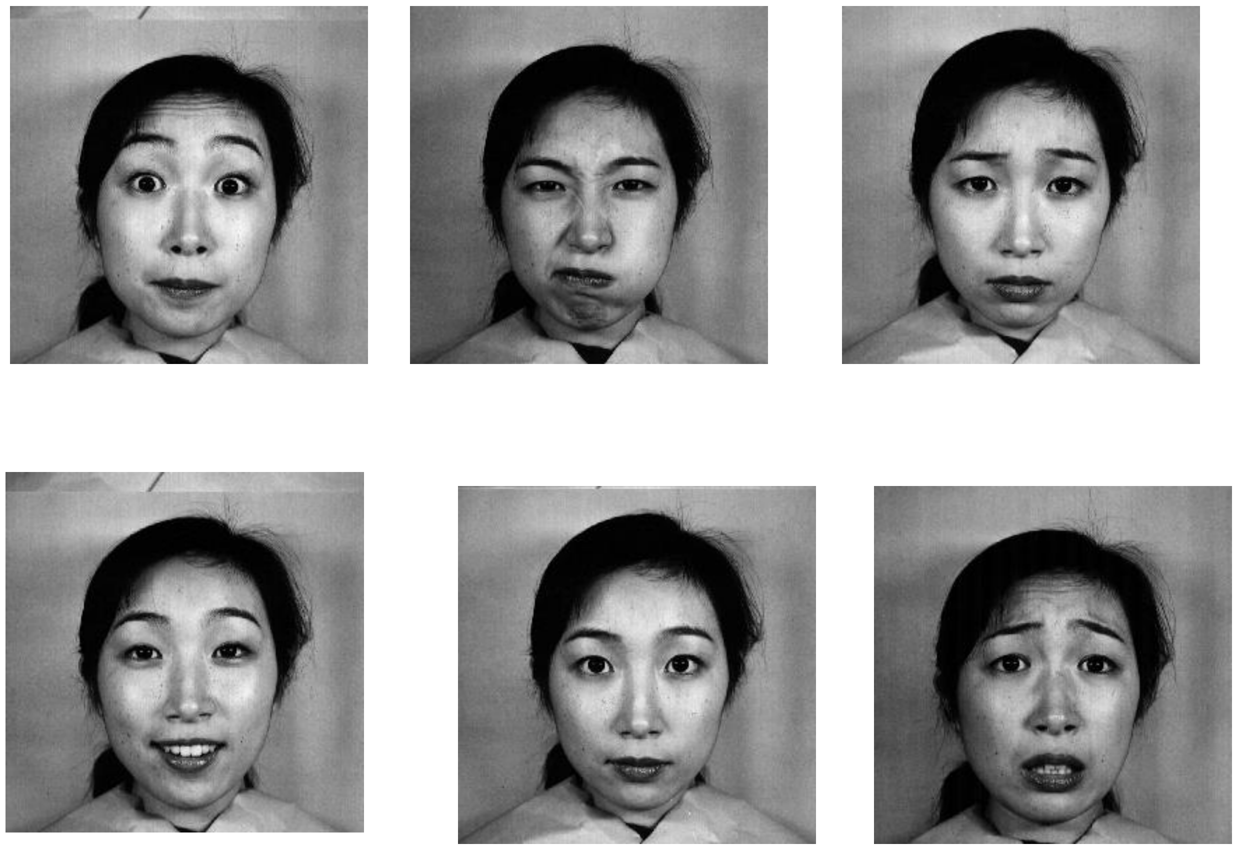
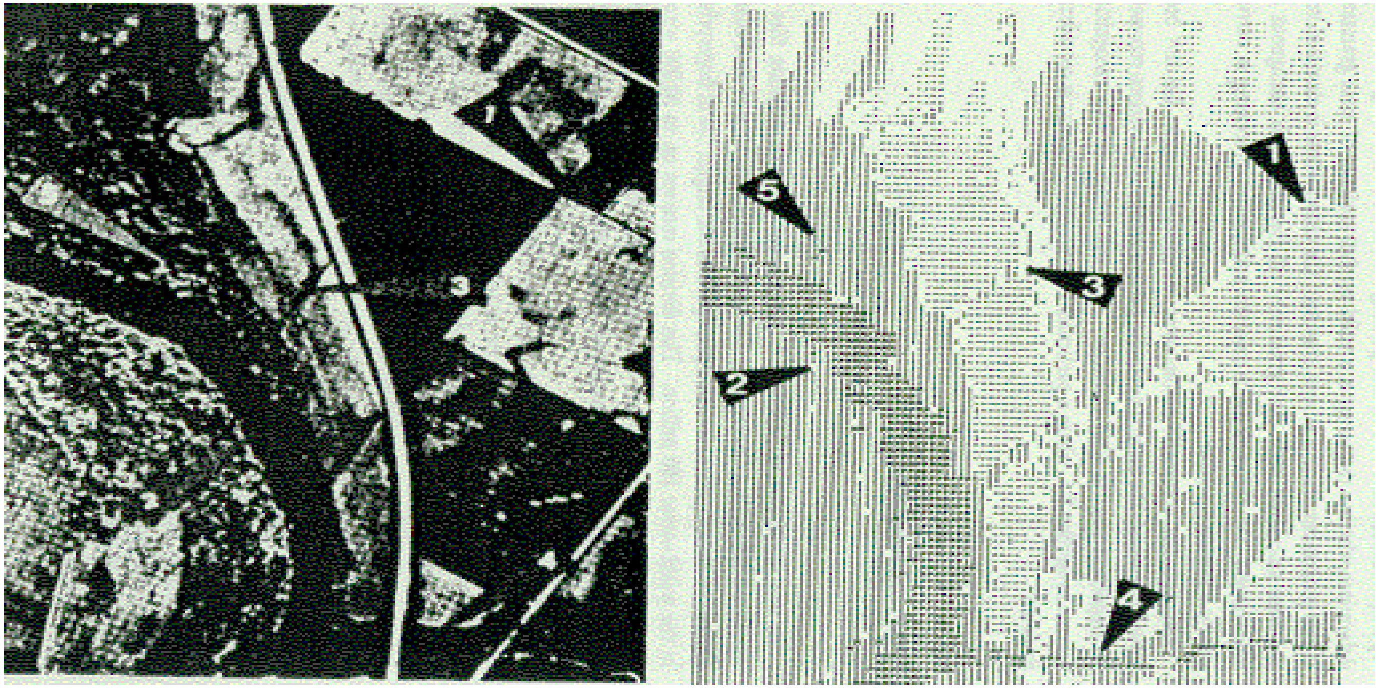


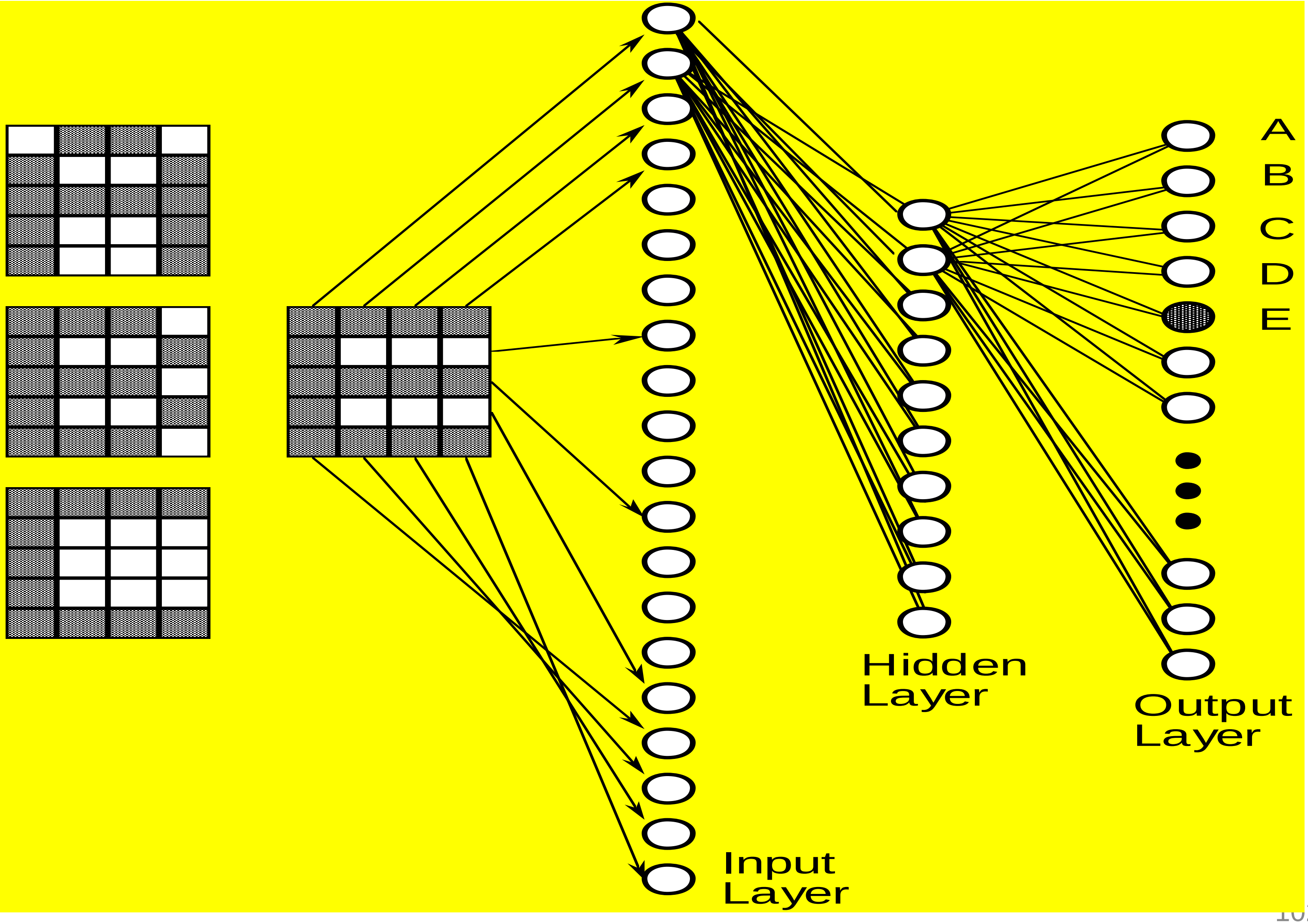
Figure 1 Different Facial Expressions of same person [23]

Classification of Remotely Sensed Data

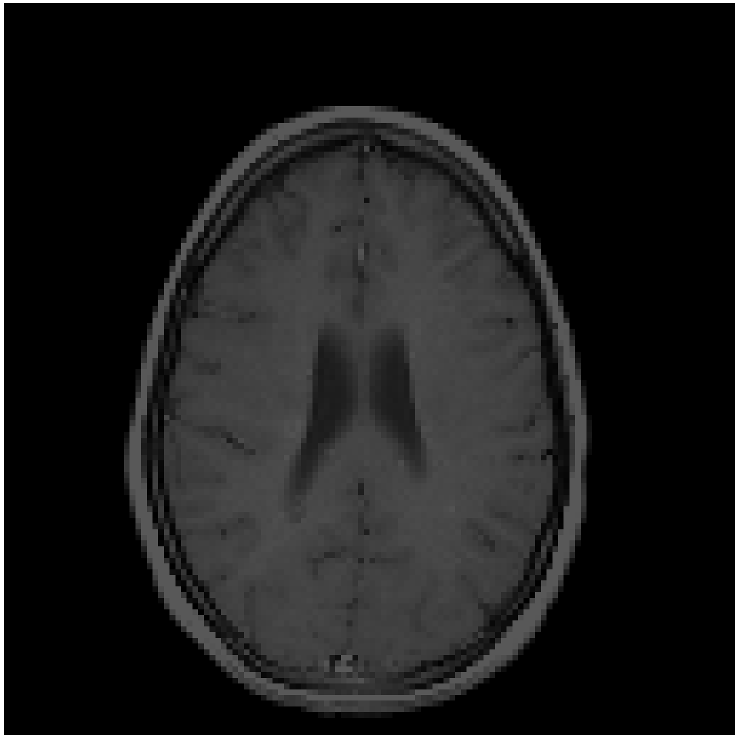


Fingerprint recognition

Optical Character Recognition



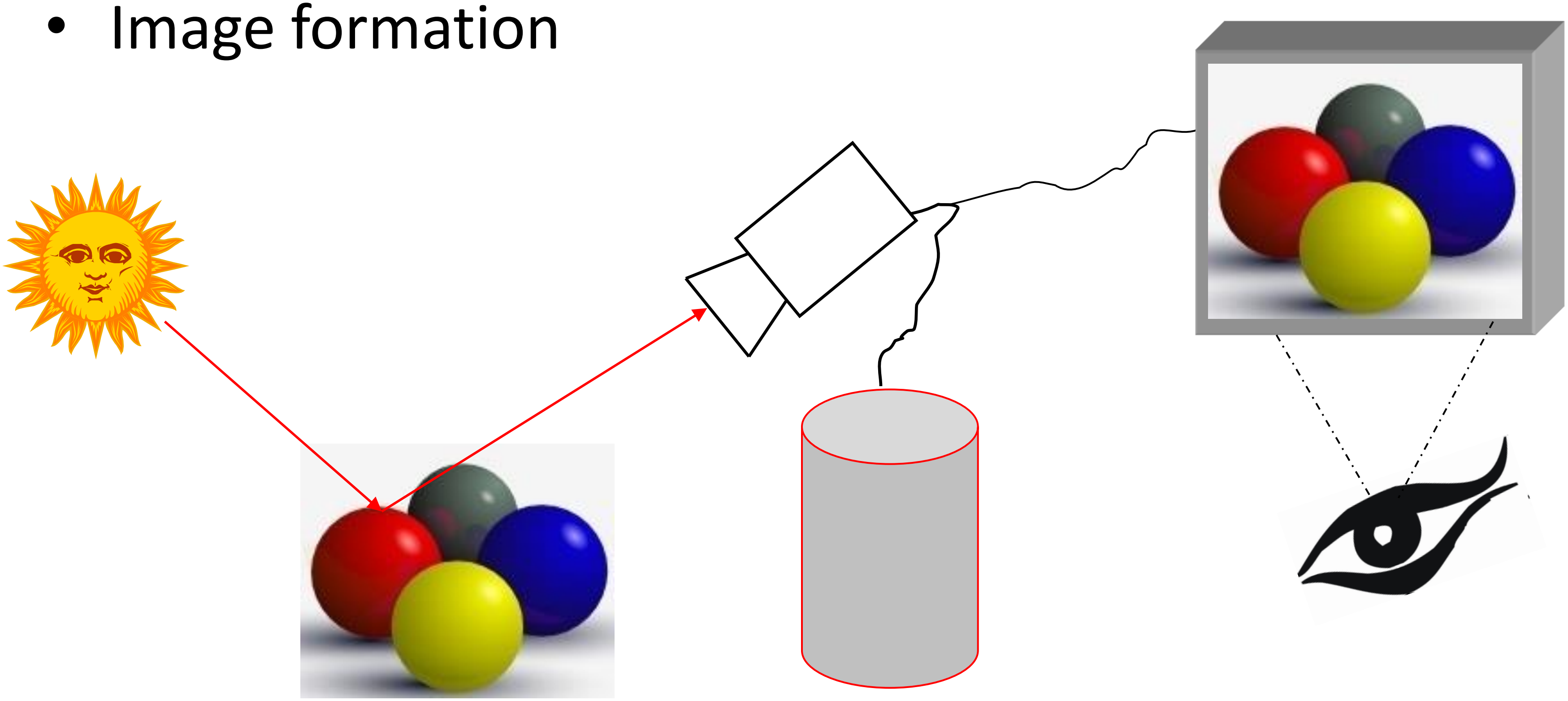
MRI Image



Signature recognition

- Each person's signature is different.
- There are structural similarities which are difficult to quantify.
- One company has manufactured a machine which recognizes signatures to within a high level of accuracy.
  - **Makes forgery even more difficult.**

- Image formation



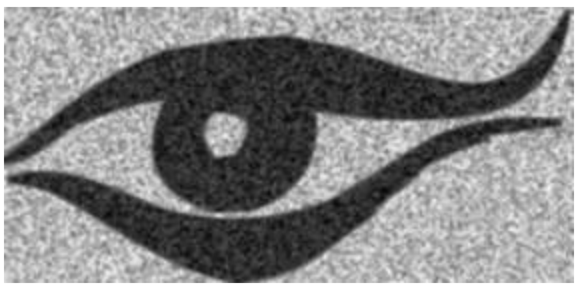
Result of Image processing…

- Image processing theory and practices

Why this is possible?
How ?
Theory
Practice

And much more …

- Edge detection and image segmentation



How ?
Theory
Practice