## CHAPTER 12

## MIXING

Mixing is the dispersing of components, one throughout the other. It occurs in innumerable instances in the food industry and is probably the most commonly encountered of all process operations. Unfortunately, it is also one of the least understood. There are, however, some aspects of mixing which can be measured and which can be of help in the planning and designing of mixing operations.

## CHARACTERISTICS OF MIXTURES

Ideally, a mixing process begins with the components, grouped together in some container, but still separate as pure components. Thus, if small samples are taken throughout the container, almost all samples will consist of one pure component. The frequency of occurrence of the components is proportional to the fractions of these components in the whole container. As mixing then proceeds, samples will increasingly contain more of the components, in proportions approximating to the overall proportions of the components in the whole container. Complete mixing could then be defined as that state in which all samples are found to contain the components in the same proportions as in the whole mixture. Actually, this state of affairs would only be attained by some ordered grouping of the components and would be a most improbable result from any practical mixing process. Another approach can then be made, defining the perfect mixture as one in which the components in samples occur in proportions whose statistical chance of occurrence is the same as that of a statistically random dispersion of the original components. Such dispersion represents the best that random mixing processes can do.

## MEASUREMENT OF MIXING

The assessment of mixed small volumes, which can be taken or sampled for measurement, is what mixing measurement is all about. Sample compositions move from the initial state to the mixed state, and measurements of mixing must reflect this.

The problem at once arises, what size of sample should be chosen? To take extreme cases, if the sample is so large that it includes the whole mixture, then the sample composition is at once mean composition and there remains no mixing to be done. At the other end of the scale, if it were possible to take samples of molecular size, then every sample would contain only one or other of the components in the pure state and no amount of mixing would make any difference. Between these lie all of the practical sample sizes, but the important point is that the results will depend upon sample size.

In many practical mixing applications, process conditions or product requirements prescribe suitable sample sizes. For example, if table salt is to contain $1 \%$ magnesium carbonate, the addition of 10 kg of magnesium carbonate to 990 kg of salt ensures, overall, that this requirement has been met. However, if the salt is to be sold in 2 kg packets, the practical
requirement might well be that each packet contains 20 g of magnesium carbonate with some specified tolerance, and adequate mixing would have to be provided to achieve this. A realistic sample size to take from this mixture, containing 1000 kg of mixture, would be 2 kg . As mixing proceeds, greater numbers of samples containing both components appear and their composition tends towards $99 \%$ salt and $1 \%$ magnesium carbonate.

It can be seen from this discussion that the deviation of the sample compositions from the mean composition of the overall mixture represents a measure of the mixing process. This deviation decreases as mixing progresses. A satisfactory way of measuring the deviation is to use the statistical term called the standard deviation. This is the mean of the sum of the squares of the deviations from the mean, and so it gives equal value to negative and positive deviation and increasingly greater weight to larger deviations because of the squaring. It is given by:

$$
\begin{equation*}
s^{2}=1 / \mathrm{n}\left[\left(x_{1}-\bar{x}\right)^{2}+\left(x_{2}-\bar{x}\right)^{2}+\quad \ldots .+\left(x_{\mathrm{n}}-\bar{x}\right)^{2}\right] \tag{12.1}
\end{equation*}
$$

where $s$ is the standard deviation, n is the number of samples taken, $x_{1}, x_{2}, \ldots x_{\mathrm{n}}$, are the fractional compositions of component $X$ in the $1,2 \ldots$ n samples and $\bar{x}$ is the mean fractional composition of component X in the whole mixture.

Using eqn. (12.1) values of $s$ can be calculated from the measured sample compositions, taking the n samples at some stage of the mixing operation. Often it is convenient to use $s^{2}$ rather than $s$, and $s^{2}$ is known as the variance of the fractional sample compositions from the mean composition.

EXAMPLE 12.1. Mixing salt and magnesium carbonate
After a mixer mixing 99 kg of salt with 1 kg of magnesium carbonate had been working for some time, ten samples, each weighing 20 g , were taken and analysed for magnesium carbonate. The weights of magnesium carbonate in the samples were: $0.230,0.172,0.163$, $0.173,0.210,0.182,0.232,0.220,0.210,0.213 \mathrm{~g}$. Calculate the standard deviation of the sample compositions from the mean composition.

Fractional compositions of samples, that is the fraction of magnesium carbonate in the sample, are respectively:
$0.0115,0.0086,0.0082,0.0087,0.0105,0.0091,0.0116,0.0110,0.0105,0.0107 \quad(x)$
Mean composition of samples, overall $=1 / 100=0.01$
Deviations of samples from mean, (0.0115-0.01), (0.0086-0.01), etc.
$(x-\bar{x})$

$$
\begin{aligned}
s^{2} & =1 / n\left[\left(x_{1}-\bar{x}\right)^{2}+\left(x_{2}-\bar{x}\right)^{2}+\ldots .+\left(x_{\mathrm{n}}-\bar{x}\right)^{2}\right] \\
s^{2} & =1 / 10\left[(0.0115-0.01)^{2}+(0.0086-0.01)^{2}+\ldots\right] \\
& =2.250 \times 10^{-6} \\
s & =\underline{1.5 \times 10^{-3}}
\end{aligned}
$$

At some later time samples were found to be of fractional compositions: $0.0113,0.0092$, $0.0097,0.0108,0.0104,0.0098,0.0104,0.0101,0.0094,0.0098$, giving:

$$
s \quad=3.7 \times 10^{-7}
$$

showing the reducing standard deviation. With continued mixing the standard deviation diminishes further.

The process of working out the differences can be laborious, and often the standard deviation can be obtained more quickly by making use of the mathematical relationship, proof of which will be found in any textbook on statistics:

$$
\begin{aligned}
s^{2} & =1 / \mathrm{n}\left[\sum\left(x_{1}^{2}\right)-\sum(\bar{x})^{2}\right] \\
& =1 / \mathrm{n}\left[\sum\left(x_{1}^{2}\right)-\mathrm{n}(\bar{x})^{2}\right] \\
& =1 / \mathrm{n}\left[\sum\left(x_{1}^{2}\right)\right]-(\bar{x})^{2}
\end{aligned}
$$

## PARTICLE MIXING

If particles are to be mixed, starting out from segregated groups and ending up with the components randomly distributed the expected variances $\left(s^{2}\right)$ of the sample compositions from the mean sample composition can be calculated.

Consider a two component mixture, consisting of a fraction $p$ of component $P$ and a fraction $q$ of component $Q$. In the unmixed state, virtually all the small samples taken will consist either of pure $P$ or of pure $Q$. From the overall proportions, if a large number of samples are taken, it would be expected that a proportion $p$ of the samples would contain pure component $P$. That is their deviation from the mean composition would be $(1-p)$, as the sample containing pure $P$ has a fractional composition 1 of component $P$. Similarly, a proportion $q$ of the samples would contain pure $Q$, that is, a fractional composition 0 in terms of component $P$ and a deviation ( $0-p$ ) from the mean.

Summing these in terms of fractional composition of component $P$, and remembering that $p+q=1$ :
for n samples

$$
\begin{align*}
s_{0}{ }^{2} & =1 / \mathrm{n}\left[p \mathrm{n}(1-p)^{2}+(1-p) \mathrm{n}(0-p)^{2}\right] \\
& =p(1-p) \tag{12.2}
\end{align*}
$$

When the mixture has been thoroughly dispersed, it is assumed that the components are distributed through the volume in accordance with their overall proportions. The probability that any particle picked at random will be component $Q$ will be $q$, and $(1-q)$ that it is not $Q$. Extending this to samples containing $N$ particles, it can be shown, using probability theory, that:

$$
\begin{equation*}
\mathrm{s}_{\mathrm{r}}{ }^{2}=p(1-p) / N=\mathrm{s}_{\mathrm{o}}{ }^{2} / N . \tag{12.3}
\end{equation*}
$$

This assumes that all the particles are equally sized and that each particle is either pure $P$ or pure $Q$. For example, this might be the mixing of equal sized particles of sugar and milk powder. The subscripts 0 and $r$ have been used to denote the initial and the random values of $s_{2}$, and inspection of the formulae, eqn. (12.2) and eqn. (12.3), shows that in the mixing process the value of $s^{2}$ has decreased from $p(1-p)$ to $1 / N$ th of this value. It has been suggested that intermediate values between $\mathrm{s}_{\mathrm{o}}{ }^{2}$ and $\mathrm{s}_{\mathrm{r}}{ }^{2}$ could be used to show the progress of mixing. Suggestions have been made for a mixing index, based on this, for example:

$$
\begin{equation*}
(M) \quad=\left(\mathrm{s}_{0}{ }^{2}-\mathrm{s}^{2}\right) /\left(\mathrm{s}_{\mathrm{o}}{ }^{2}-\mathrm{s}_{\mathrm{r}}^{2}\right) \tag{12.4}
\end{equation*}
$$

which is so designed that $(M)$ goes from 0 to 1 during the course of the mixing process. This measure can be used for mixtures of particles and also for the mixing of heavy pastes.

EXAMPLE 12.2. Mixing of yeast into dough
For a particular bakery operation, it was desired to mix dough in 95 kg batches and then at a later time to blend in 5 kg of yeast. For product uniformity it is important that the yeast be well distributed and so an experiment was set up to follow the course of the mixing. It was desired to calculate the mixing index after 5 and 10 min mixing.
Sample yeast compositions, expressed as the percentage of yeast in 100 g samples, were found to be:
$\begin{array}{lllllllllllll}\text { After } 5 \mathrm{~min} & 0.0 & 16.5 & 3.2 & 2.2 & 12.6 & 9.6 & 0.2 & 4.6 & 0.5 & 8.5\end{array}$
$\begin{array}{lllllllllllll}\text { After } 10 \mathrm{~min} & & 3.4 & 8.3 & 7.2 & 6.0 & 4.3 & 5.2 & 6.7 & 2.6 & 4.3 & 2.0\end{array}$

Fractional compositions:
$5 \mathrm{~min} \quad 0.00 .1650 .0320 .0220 .1260 .096 \quad 0.0020 .046 \quad 0.0050 .085$
$\begin{array}{lllllllllllllllllll}10 \mathrm{~min} & 0.034 & 0.083 & 0.072 & 0.06 & 0.043 & 0.052 & 0.067 & 0.026 & 0.043 & 0.020\end{array}$
$\mathrm{s}^{2} \quad=1 / \mathrm{n}\left[\sum\left(x_{1}{ }^{2}\right)\right]-(\bar{x})^{2}$
Therefore $\mathrm{s}_{5}{ }^{2}=3.0 \times 10^{-3}=0.3 \times 10^{-2}$
$s_{10^{2}}=3.8 \times 10^{-4}=0.38 \times 10^{-2}$
The value of $\quad s_{0}{ }^{2} \quad=0.05 \times 0.95=4.8 \times 10^{-2}$
and $\mathrm{s}_{r}{ }^{2} \approx 0$ as the number of "particles" in a sample is very large,
From equation $12.4(M) \quad=\left(\mathrm{s}_{\mathrm{o}}{ }^{2}-\mathrm{s}^{2}\right) /\left(\mathrm{s}_{\mathrm{o}}{ }^{2}-\mathrm{s}_{\mathrm{r}}{ }^{2}\right)$

$$
\begin{aligned}
(M)_{5} & =(4.8-0.3) /(4.8-0) \\
& =\underline{0.93} \\
(M)_{10} & =(4.8-0.04) /(4.8-0) \\
& =\underline{0.99} .
\end{aligned}
$$

## Mixing of Widely Different Quantities

The mixing of particles varying substantially in size or in density presents special problems, as there will be gravitational forces acting in the mixer which will tend to segregate the particles into size and density ranges. In such a case, initial mixing in a mixer may then be followed by a measure of (slow gravitational) un-mixing and so the time of mixing may be quite critical.

Mixing is simplest when the quantities that are to be mixed are roughly in the same proportions. In cases where very small quantities of one component have to be blended uniformly into much larger quantities of other components, the mixing is best split into stages, keeping the proportions not too far different in each stage. For example, if it were required to add a component such that its final proportions in relatively small fractions of the product are 50 parts per million, it would be almost hopeless to attempt to mix this in a single stage. A possible method might be to use four mixing stages, starting with the added component in the first of these at about $30: 1$. In planning the mixing process it would be wise
to take analyses through each stage of mixing, but once mixing times had been established it should only be necessary to make check analyses on the final product.

EXAMPLE 12.3. Vitamin addition to powdered cereal
It is desired to mix vitamin powder at the level of $10^{-3} \%$ by weight into a 1 tonne batch of powdered cereal. Two double-cone blenders are available, one (L) with a capacity of 100 to 500 kg powder and another ( S ) with a capacity of 1 to 10 kg . Both will mix adequately in 10 min so long as the minor constituent constitutes not less than $10 \%$. Suggest a procedure for the mixing.

Total weight vitamin needed $=1000 \times 10^{-5} \mathrm{~kg}=10 \mathrm{~g}$
Divide into two, $2 \times 5 \mathrm{~g}$ as two final 500 kg batches will be needed.
Then

1. Hand blend 5 g and 50 g cereal $\rightarrow$ mixture (1), then (1) and 945 g cereal, mixture $\rightarrow$ (2). This may need analytical checking to set up a suitable hand-mixing procedure.
2. Take (2) +9 kg cereal and mix in mixer $(\mathrm{S}) \rightarrow$ mixture (3).
3. Take (3) +90 kg cereal and mix in $(\mathrm{L}) \rightarrow$ mixture (4).
4. Take (4) +400 kg cereal and mix in $(\mathrm{L}) \rightarrow$ mixture (5) - product.
5. Repeat, with other 5 g vitamin and 500 kg cereal, steps (1) to (4).

## Rates of Mixing

Once a suitable measure of mixing has been found, it becomes possible to discuss rates of accomplishing mixing. It has been assumed that the mixing index ought to be such that the rate of mixing at any time under constant working conditions, such as in a well designed mixer working at constant speed, ought to be proportional to the extent of mixing remaining to be done at that time.
That is,

$$
\begin{equation*}
\mathrm{d} M / \mathrm{d} t=K[(1-(M))] \tag{12.5}
\end{equation*}
$$

where $(M)$ is the mixing index and $K$ is a constant, and on integrating from $t=0$ to $t=t$ during which $(M)$ goes from 0 to $(M)$,
or

$$
\begin{align*}
& {[(1-(M))]=\mathrm{e}^{-K t}} \\
& (M)=1-\mathrm{e}^{-K t} \tag{12.6}
\end{align*}
$$

This exponential relationship, using $(M)$ as the mixing index, has been found to apply in many experimental investigations at least over two or three orders of magnitude of ( $M$ ). In such cases, the constant $K$ can be related to the mixing machine and to the conditions and it can be used to predict, for example, the times required to attain a given degree of mixing.

EXAMPLE 12.4. Blending starch and dried vegetables for a soup mix
In a batch mixer, blending starch and dried, powdered vegetables for a soup mixture, the initial proportions of dried vegetable to starch were 40:60. The variance of the sample compositions measured in terms of fractional compositions of starch was found to be 0.0823 after 300s of mixing. For how much longer should the mixing continue to reach the specified maximum sample composition variance of 0.02 ?
Assume that the starch and the vegetable particles are of approximately the same physical size and that a sample contains 24 particles.

Taking the fractional content of dried vegetables to be $p=0.4$,

$$
\begin{array}{ll}
(1-p) & = \\
\mathrm{s}_{0}{ }^{2} & =(1-0.4)=0.6 \\
& 0.4 \times 0.6=0.24
\end{array}
$$

from eqn. (12.3)
$\mathrm{s}_{\mathrm{r}}{ }^{2} \quad=\quad \mathrm{s}_{\mathrm{o}}{ }^{2} / N=0.24 / 24=0.01$
Substituting in eqn. (12.4) we have:
For $\mathrm{s}^{2}=0.00823$,

$$
\begin{aligned}
(M) \quad & =\left(\mathrm{s}_{\mathrm{o}}{ }^{2}-\mathrm{s}^{2}\right) /\left(\mathrm{s}_{\mathrm{o}}{ }^{2}-\mathrm{s}_{\mathrm{r}}^{2}\right) \\
& =(0.24-0.0823) /(0.24-0.01) \\
& =0.685
\end{aligned}
$$

Substituting in eqn.12.6

$$
\begin{array}{ll}
(M)=1-\mathrm{e}^{-K t} \\
\mathrm{e}^{-300 K} & =1-0.685=0.315 \\
-300 K & =-1.155 \\
K & =3.85 \times 10^{-3}
\end{array}
$$

For $\mathrm{s}^{2}=0.02$,
so

$$
\begin{aligned}
(M) & =(0.24-0.02) /(0.24-0.01) \\
& =0.957 \\
\mathrm{e}^{-0.00385 t} & =1-0.957=0.043 \\
-0.00385 t & =-3.147 \\
t & =817 \mathrm{~s}, \text { say } 820 \mathrm{~s},
\end{aligned}
$$

The additional mixing time would be $820 \mathrm{~s}-300 \mathrm{~s}=520 \mathrm{~s}$

## Energy Input in Mixing

Quite substantial quantities of energy can be consumed in some types of mixing, such as in the mixing of plastic solids. There is no necessary connection between energy consumed and the progress of mixing: to take an extreme example there could be shearing along one plane in a sticky material, then recombining to restore the original arrangement, then repeating which would consume energy but accomplish no mixing at all. However, in well-designed mixers energy input does relate to mixing progress, though the actual relationship has normally to be determined experimentally. In the mixing of flour dough using high-speed mixers, the energy consumed, or the power input at any particular time, can be used to determine the necessary mixing time. This is a combination of mixing with chemical reaction as flour components oxidize during mixing in air which leads to increasing resistance to shearing and so to increased power being required to operate the mixer.

EXAMPLE 12.5. Mixing time for bread dough
In a particular mixer used for mixing flour dough for breadmaking, it has been found that mixing can be characterized by the total energy consumed in the mixing process and that
sufficient mixing has been accomplished when 8 watt hours of energy have been consumed by each kg of dough.
Such a mixer, handling 2000 kg of dough, is observed just after starting to be consuming 80 amperes per phase, which rises steadily over 10s to 400 amperes at which level it then remains effectively constant. The mixer is driven by a 440 -volt, 3 -phase electric motor with a power factor $(\cos \phi)$ ) of 0.89 , and the overall mechanical efficiency between motor and mixing blades is $75 \%$. Estimate the necessary mixing time.

Power to the motor $=\sqrt{3} E I \cos \phi)$ where $I$ is the current per phase and $E$ and $\cos \phi$ in this case are 440 volts and 0.89 , respectively.
In first 10 s , $\quad I_{\text {average }}=1 / 2(80+400)=240$ amperes.

$$
\begin{aligned}
& \text { Energy consumed in first } 10 \mathrm{~s} \\
&=\text { power } \times \text { time } \\
&=0.75[\sqrt{ } 3 \times 440 \times 240 \times 0.89] \times 10 / 3600 \\
&=339 \mathrm{watt} \text { hours } \\
&=339 / 2000 \\
&=0.17 \mathrm{~Wh} \mathrm{~kg}^{-1} .
\end{aligned}
$$

Energy still needed after first 10s

$$
\begin{aligned}
& =(8-0.17) \\
& =7.83 \mathrm{Whkg}^{-1} .
\end{aligned}
$$

Additional time needed, at steady 400 ampere current

$$
\begin{aligned}
t \quad & =\text { Energy/Power } \\
& =(7.83 \times 2000 \times 3600) /(0.75[\sqrt{ } 3 \times 440 \times 400 \times 0.89]) \\
& =277 \mathrm{~s} .
\end{aligned}
$$

Total time $\quad=277+10 \mathrm{~s}=287 \mathrm{~s}$
$=4.8 \mathrm{~min}$.
In actual mixing of dough, the power consumed would decrease slowly and more or less uniformly, but only by around $10-15 \%$ over the mixing process.

## LIQUID MIXING

Food liquid mixtures could in theory be sampled and analysed in the same way as solid mixtures but little investigational work has been published on this, or on the mixing performance of fluid mixers. Most of the information that is available concerns the power requirements for the most commonly used liquid mixer - some form of paddle or propeller stirrer. In these mixers, the fluids to be mixed are placed in containers and the stirrer is rotated. Measurements have been made in terms of dimensionless ratios involving all of the physical factors that influence power consumption. The results have been correlated in an equation of the form

$$
\begin{equation*}
(\mathrm{Po})=K(\mathrm{Re})^{n}(\mathrm{Fr})^{m} \tag{12.7}
\end{equation*}
$$

where $(\operatorname{Re})=\left(D^{2} N \rho / \mu\right),(\mathrm{Po})=\left(P / D^{5} N^{3} \rho\right)$ and this is called the Power number (relating drag forces to inertial forces), $(\mathrm{Fr})=\left(D N^{2} / \mathrm{g}\right)$ and this is called the Froude number (relating inertial forces to those of gravity); $D$ is the diameter of the propeller, $N$ is the rotational frequency of the propeller (rev/sec), $\rho$ is the density of the liquid, $\mu$ is the viscosity of the liquid and $P$ is the power consumed by the propeller.

Notice that the Reynolds number in this instance, uses the product $D N$ for the velocity, which differs by a factor of $\pi$ from the actual velocity at the tip of the propeller.

The Froude number correlates the effects of gravitational forces and it only becomes significant when the propeller disturbs the liquid surface. Below Reynolds numbers of about 300, the Froude number is found to have little or no effect, so that eqn. (12.7) becomes:

$$
\begin{equation*}
(\mathrm{Po})=K(\mathrm{Re})^{n} \tag{12.8}
\end{equation*}
$$

Experimental results from the work of Rushton are shown plotted in Fig. 12.1.


Figure 12.1 Performance of propeller mixers
Adapted from Rushton (1950)

Unfortunately, general formulae have not been obtained, so that the results are confined to the particular experimental propeller configurations that were used. If experimental curves are available, then they can be used to give values for $n$ and $K$ in eqn. (12.8) and the equation then used to predict power consumption. For example, for a propeller, with a pitch equal to the diameter, Rushton gives $n=-1$ and $K=41$.

In cases in which experimental results are not already available, the best approach to the prediction of power consumption in propeller mixers is to use physical models, measure the factors, and then use eqn. (12.7) or eqn. (12.8) for scaling up the experimental results.

EXAMPLE 12.6. Blending vitamin concentrate into molasses
Vitamin concentrate is being blended into molasses and it has been found that satisfactory mixing rates can be obtained in a small tank 0.67 m diameter, height 0.75 m , with a propeller,
0.33 m diameter rotating at $450 \mathrm{rev} \mathrm{min}^{-1}$. If a large-scale plant is to be designed which will require a tank 2 m diameter, what will be suitable values to choose for tank depth, propeller diameter and rotational speed, if it is desired to preserve the same mixing conditions as in the smaller plant? What would be the power requirement for the motor driving the propeller? Assume that the viscosity of molasses is $6.6 \mathrm{Nsm}^{-2}$ and its density is $1520 \mathrm{kgm}^{-3}$.

Use the subscripts S for the small tank and L for the larger one. To preserve geometric similarity the dimensional ratios should be the same in the large tank as in the small.

Given that the full-scale tank is three times larger than the model, Diameter of large tank $=D_{\mathrm{L}}=3 D_{\mathrm{S}}=3 \times 0.67=2.01$

Depth of large tank $\quad=H_{\mathrm{L}}=3 H_{\mathrm{S}}=3 \times 0.75=2.25 \mathrm{~m}$
and
Propeller diameter in the large tank $=D_{\mathrm{L}}=3 D_{\mathrm{S}}=3 \times 0.33=1 \mathrm{~m}$
For dynamic similarity, $(\operatorname{Re})_{\mathrm{L}}=(\operatorname{Re})_{\mathrm{S}}$

$$
\begin{aligned}
\left(D^{2} N \rho / \mu\right)_{\mathrm{L}} & =\left(D^{2} N \rho / \mu\right)_{\mathrm{S}} \\
D_{\mathrm{L}}{ }^{2} N_{\mathrm{L}} & =D_{\mathrm{s}}{ }^{2} N_{\mathrm{S}} \\
& \\
N_{\mathrm{L}} & =(1 / 3)^{2} \times 450 \\
& =50 \mathrm{rev}^{2} \mathrm{~min}^{-1} \\
& =0.83 \mathrm{rev} \mathrm{sec}^{-1} \\
& =\text { speed of propeller in the large tank. }
\end{aligned}
$$

For the large tank

$$
(\mathrm{Re})=\left(D_{\mathrm{L}}{ }^{2} N_{\mathrm{L}} \rho / \mu\right)
$$

so

$$
\begin{aligned}
(\operatorname{Re}) & =\left(1^{2} \times 0.83 \times 1520\right) / 6.6 \\
& =191
\end{aligned}
$$

Eqn. (12.8) is applicable, and assuming that $K=41$ and $\mathrm{n}=-1$, we have

$$
\begin{array}{rll} 
& (\mathrm{Po}) & =\left(P / D^{5} N^{3} \rho\right) \\
\text { But } & (\mathrm{Po}) & =41(\mathrm{Re})^{-1} \\
\text { Therefore } \quad\left(P / D^{5} N^{3} \rho\right) & =41 / 191 \\
P & =\left(41 \times 1^{5} \times(0.83)^{3} \times 1520\right) /(191) \\
& & =186 \mathrm{Js}^{-1}
\end{array}
$$

And since 1 horsepower $=746 \mathrm{Js}^{-1}$
Required motor $=\underline{186 / 746}$, say $1 / 4$ horsepower.

Apart from deliberate mixing, liquids in turbulent flow or passing through equipment such as pumps are being vigorously mixed. By planning such equipment in flow lines, or by ensuring turbulent flow in pipelines, liquid mixing may in many instances be satisfactorily accomplished as a by product of fluid transport.

## MIXING EQUIPMENT

Many forms of mixers have been produced from time to time but over the years a considerable degree of standardization of mixing equipment has been reached in different branches of the food industry. Possibly the easiest way in which to classify mixers is to divide them according to whether they mix liquids, dry powders, or thick pastes.

## Liquid Mixers

For the deliberate mixing of liquids, the propeller mixer is probably the most common and the most satisfactory. In using propeller mixers, it is important to avoid regular flow patterns such as an even swirl round a cylindrical tank, which may accomplish very little mixing. To break up these streamline patterns, baffles are often fitted, or the propeller may be mounted asymmetrically. Various baffles can be used and the placing of these can make very considerable differences to the mixing performances. It is tempting to relate the amount of power consumed by a mixer to the amount of mixing produced, but there is no necessary connection and very inefficient mixers can consume large amounts of power.

## Powder and Particle Mixers

The essential feature in these mixers is to displace parts of the mixture with respect to other parts. The ribbon blender, for example, shown in Fig. 12.2(a) consists of a trough in which rotates a shaft with two open helical screws attached to it, one screw being right-handed and the other left-handed. As the shaft rotates sections of the powder move in opposite directions and so particles are vigorously displaced relative to each other.


Figure12.2 Mixers(a) ribbon blender, (b) double-cone mixer

A commonly used blender for powders is the double-cone blender in which two cones are mounted with their open ends fastened together and they are rotated about an axis through their common base. This mixer is shown in Fig. 12.2(b).

## Dough and Paste Mixers

Dough and pastes are mixed in machines that have, of necessity, to be heavy and powerful. Because of the large power requirements, it is particularly desirable that these machines mix with reasonable efficiency, as the power is dissipated in the form of heat, which may cause substantial heating of the product. Such machines may require jacketing of the mixer to remove as much heat as possible with cooling water.

Perhaps the most commonly used mixer for these very heavy materials is the kneader which employs two contra-rotating arms of special shape, which fold and shear the material across a cusp, or division, in the bottom of the mixer. The arms are of so-called sigmoid shape as indicated in Fig. 12.3.


Figure 12.3 Kneader

They rotate at differential speeds, often in the ratio of nearly $3: 2$. Developments of this machine include types with multiple sigmoid blades along extended troughs, in which the blades are given a forward twist and the material makes its way continuously through the machine.

Another type of machine employs very heavy contra-rotating paddles, whilst a modern continuous mixer consists of an interrupted screw which oscillates with both rotary and reciprocating motion between pegs in an enclosing cylinder. The important principle in these machines is that the material has to be divided and folded and also displaced, so that fresh surfaces recombine as often as possible.

## SUMMARY

1. Mixing can be characterized by analysis of sample compositions and then calculation of the standard deviation, s , of these from the mean composition of the whole mixture,
where

$$
s^{2}=1 / n\left[\left(x_{1}-\bar{x}\right)^{2}+\left(x_{2}^{2}-\bar{x}\right)^{2}+\quad\left(x_{n}-\bar{x}\right)^{2}\right]
$$

2. An index of mixing is given by:

$$
(M)=\left(\mathrm{s}_{\mathrm{o}}{ }^{2}-\mathrm{s}^{2}\right) /\left(\mathrm{s}_{\mathrm{o}}{ }^{2}-\mathrm{s}_{\mathrm{r}}^{2}\right)
$$

3. A mixing index $M$ can often be related to the time of mixing by

$$
[1-(M)]=\mathrm{e}^{-K t}
$$

4. The power consumed in liquid mixing can be expressed by a relationship of the form

$$
\mathrm{Po}=K(\mathrm{Re})^{n}(\mathrm{Fr})^{m}
$$

## PROBLEMS

1. Analysis of the fat content of samples from a chopped-meat mixture in which the overall fat content was $15 \%$ gave the following results, expressed as percentages:

### 23.410 .416 .419 .630 .47 .6

For this mixture, estimate the value of $s^{2}, s_{\mathrm{o}}{ }^{2}$, and $s_{\mathrm{r}}{ }^{2}$, if the samples are 5 g and the fat and meat are in 0.1 g particles.

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\left(0.68 \times 10^{-2}, 12.8 \times 10^{-2}, 0.256 \times 10^{-2}\right)
$$

2. If it is found that the mixture in problem 1 is formed from the initial separate ingredients of fat and lean meat after mixing for 10 min estimate the value of the mixing index after a further 5 min of mixing.
( $0.966,0.994$ )
3. For a liquid mixer in which a propeller stirrer, 0.3 m in diameter, is rotating at $300 \mathrm{rev} / \mathrm{min}$ in water estimate the power required to operate the stirrer. The tank is 1.2 m in diameter.
4. If the same assembly as in problem 3 is to be used to stir olive oil estimate the power needed.
5. A blended infant feed powder is to include a nutrient supplement, supplied as a powder similar to the bulk feed, incorporated at 15 parts per million. The plant is to produce 50 tonnes per day and has powder mixers with capacities of $3,0.5$ and 0.05 tonnes, which will mix efficiently down to one-sixth of their nominal capacity. Suggest a mixing schedule that should lead to the required dispersion of the supplement.
