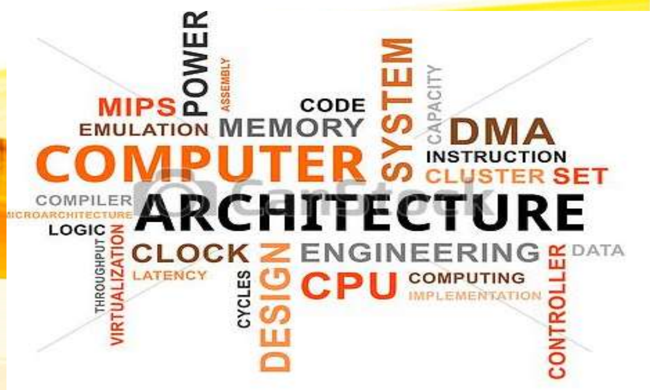
UNIT II
ARITHMETIC OPERATIONS

Addition and subtraction of signed numbers – Design of fast adders – Multiplication of positive numbers - Signed operand multiplication- fast multiplication – **Integer division** – Floating point numbers and operations



Recap the previous Class

Introduction

- Division is more complex than multiplication.
- *Example*: Typical values in Pentium-3 processor.

– Not easy to construct high-speed dividers.

- The ratios have not changed much in later processors.

Instruction	Latency	Cycles / Issue
Load / Store	3	1
Integer Multiply	4	1
Integer Divide	36	36
Floating-point Add	3	1
Floating-point Multiply	5	2
Floating-point Divide	38	38

The Process of Integer Division

- In integer division, a ***divisor*** M and a ***dividend*** D are given.
- The objective is to find a third number Q, called the ***quotient***,

  such that **D = Q x M + R** where R is the ***remainder*** such that $0 \le R < M$.
- The relationship D = Q x M suggests that there is a close correspondence between division and multiplication.
  - Dividend, quotient and divisor correspond to product, multiplicand and multiplier, respectively.

- One of the simplest division methods is the **sequential digit-by-digit algorithm** similar to that used in pencil-and-paper methods.

```
                          0 1 1 0      Quotient Q = Q0Q1Q2Q3
Divisor M      1 1 0 | 1 0 0 1 0 1    Dividend D = R0
                       1 1 0          Q0.M        (Does not go; Q0 = 0)
                      -------------
                       1 0 0 1 0 1    R1
                      -  1 1 0        Q1.2^-1.M   (Does go; Q1 = 1)
                      -------------
                         0 1 1 0 1    R2
                      -    1 1 0      Q2.2^-2.M   (Does go; Q2 = 1)
                      -------------
                           0 0 0 1    R3
                             1 1 0    Q3.2^-3.M   (Does not go; Q3 = 0)
                      -------------
                             0 0 1    R4 = Remainder R
```

$D = 37 = (1\,0\,0\,1\,0\,1)_2$

$M = 6 = (1\,1\,0)_2$

Quotient $Q = 6$

Remainder $R = 1$

- Machine implementation:
  - For hardware implementation, it is more convenient to shift the partial remainder to the left relative to a fixed divisor; thus

$$R_{i+1} = 2R_i - Q_i.M \quad (instead\ of\ R_{i+1} = R_i - Q_i.2^{-i}.M)$$

  - The final partial remainder is the required remainder shifted to the left, so that $R = 2^{-3}.R_4$

Machine implementation

Divisor M			Quotient Q
1 1 0	1 0 0 1 0 1	Dividend = $2R_0$	
Do not subtract	1 1 0	$Q_0.M$	0
	1 0 0 1 0 1	R_1	
	1 0 0 1 0 1 0	$2R_1$	
	1 1 0	$Q_1.M$	0 1
	0 1 1 0 1 0	R_2	
	0 1 1 0 1 0 0	$2R_2$	
	1 1 0	$Q_2.M$	0 1 1
	0 0 0 1 0 0	R_3	
	0 0 0 1 0 0 0	$2R_3$	
Do not subtract	1 1 0	$Q_3.M$	0 1 1 0
	0 0 1 0 0 0	$R_4 = 2^3.R$	

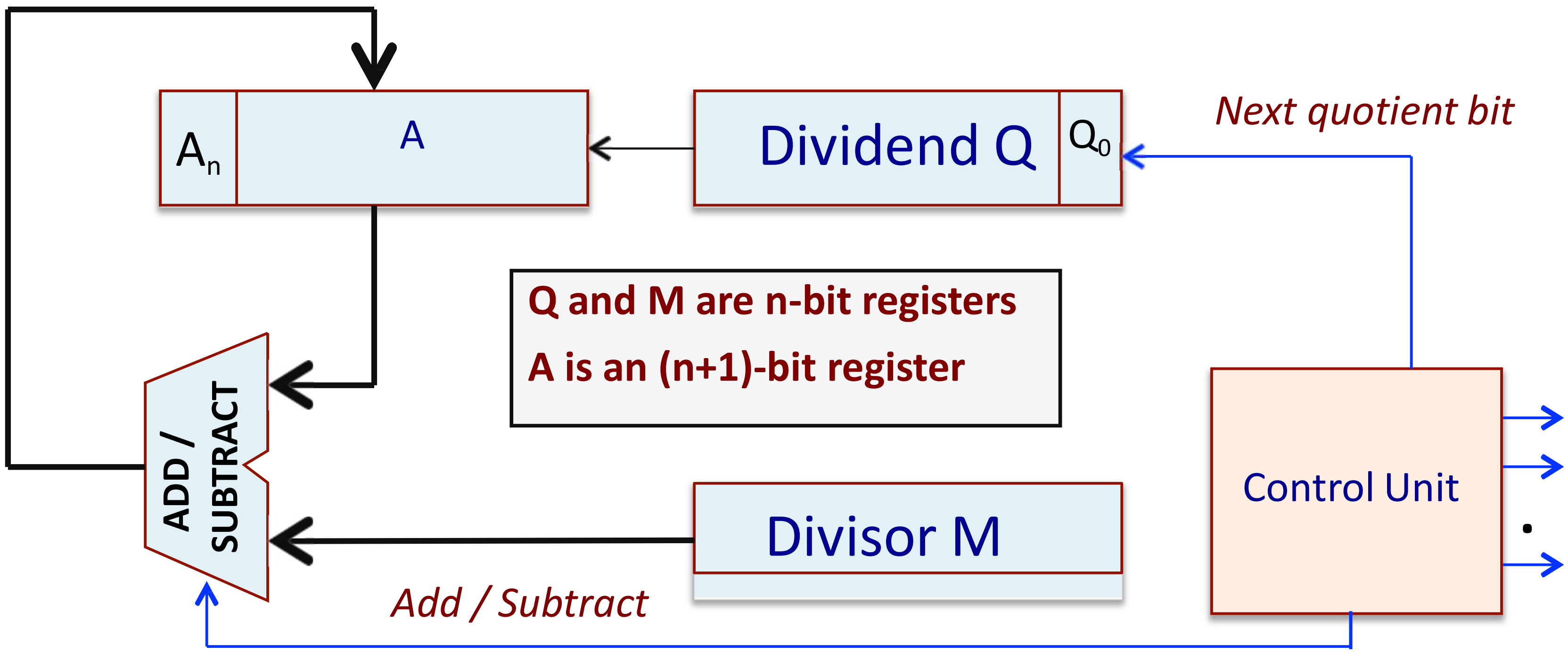
$D = 37 = (1\,0\,0\,1\,0\,1)_2$

$M = 6 = (1\,1\,0)_2$

Quotient Q = 6

Remainder R = 1

Restoring Division: The Data Path



Basic Steps

Repeat the following steps n times:

a) Shift the dividend one bit at a time starting into register A.

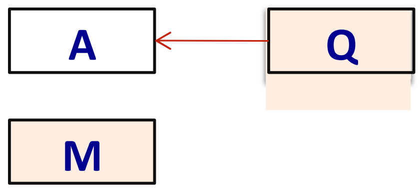
b) Subtract the divisor M from this register A (***trial subtraction***).

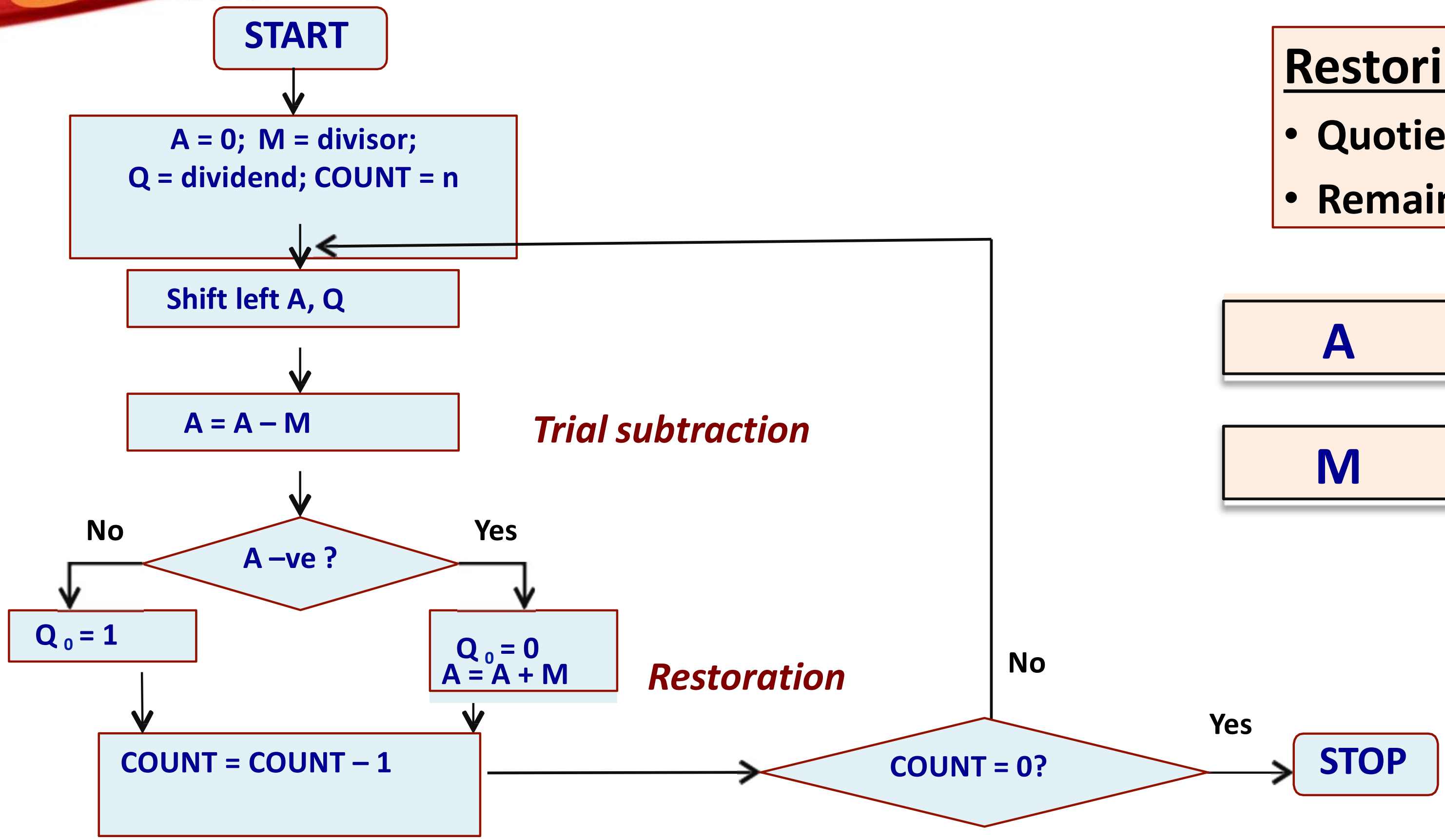
c) If the result is negative (***i.e. not going***):

- Add the divisor M back into the register A (***i.e. restoring back***).
- Record 0 as the next quotient bit.

d) If the result is positive:

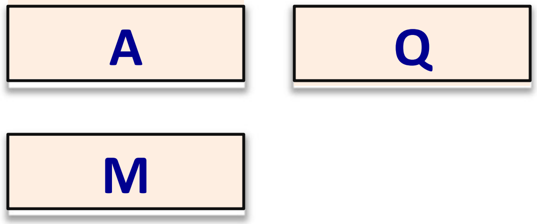
- Do not restore the intermediate result.
- Record 1 as the next quotient bit.





Restoring Division

- Quotient in Q
- Remainder in A



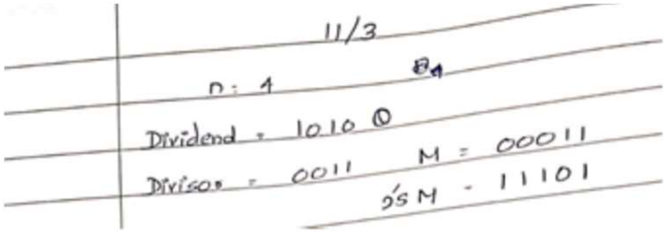
A Simple Example: 8/3 for 4-bit representation (n=4)

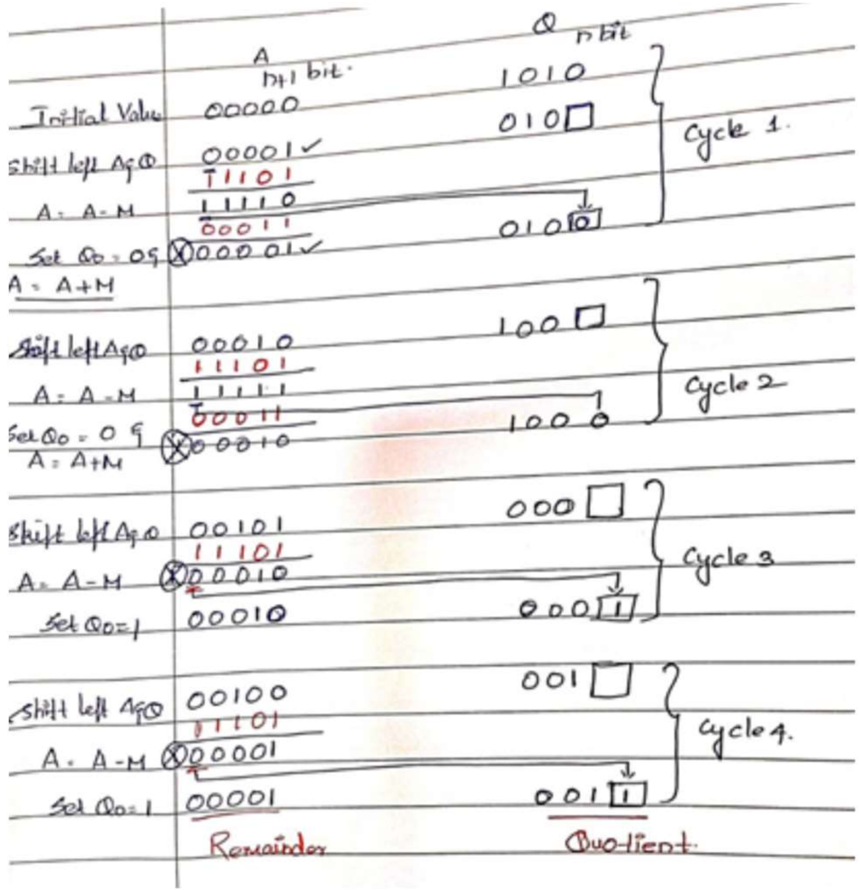
Initially:	0 0 0 0 0	1 0 0 0
	0 0 0 1 1	
Shift:	0 0 0 0 1	0 0 0 –
Subtract:		
Set Q_0:	1 1 1 1 0	
Restore:	0 0 0 1 1	
	0 0 0 0 1	0 0 0 0
Shift:	0 0 0 1 0	0 0 0 –
Subtract:		
Set Q_0:	1 1 1 1 1	
Restore:	0 0 0 1 1	
	0 0 0 1 0	0 0 0 0

Shift:	0 0 1 0 0	0 0 0 –
Subtract:		
Set Q_0:	0 0 0 0 1	
	0 0 0 0 0	0 0 0 1
Shift:	0 0 0 1 0	0 0 1 –
Subtract:		
Set Q_0:	1 1 1 1 1	
Restore:	0 0 0 1 1	
	0 0 0 1 0	0 0 1 0

Remainder	Quotient
00010 = 2	0010 = 2

Perform the restoring division algorithm for the number 11_{10} / 3_{10}



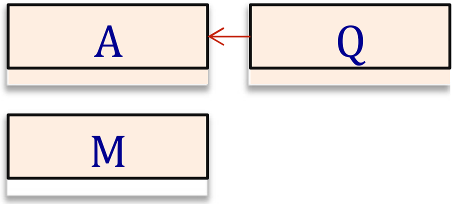


Example : $7_{10} / 3_{10}$

A	Q	M = 0011	
0000	0111	Initial Value	
0000	1110	Shift	First cycle
1101		Subtract	
0000	1110	Restore	
0001	1100	Shift	Secon cycl
1110		Subtract	
0001	1100	Restore	
0011	1000	Shift	Third cycle
0000		Subtract	
0000	1001	Set Q_0 = 1	
0001	0010	Shift	Fourt cycl
1110		Subtract	
Remainder 0001	Quotient 0010	Restore	

Non-Restoring Division

- The performance of restoring division algorithm can be improved by exploiting the following observation.
  - In restoring division, what we do actually is:
    - If A is positive, we shift it left and subtract M. That is, we compute 2A – M.
    - If A is negative, we restore is by doing A+M, shift it left, and then subtract M.
      - That is, we compute 2(A + M) – M = 2A + M.
  - We can accordingly modify the basic division algorithm by eliminating the restoring step ***NON-RESTORING DIVISION*.**



Shift left means multiplying by 2.

Basic steps in non-restoring division:

a)Start by initializing register A to 0, and repeat steps (b)-(d) *n* times.

b)If the value in register A is positive,

- Shift A and Q left by one bit position.
- Subtract M from A.

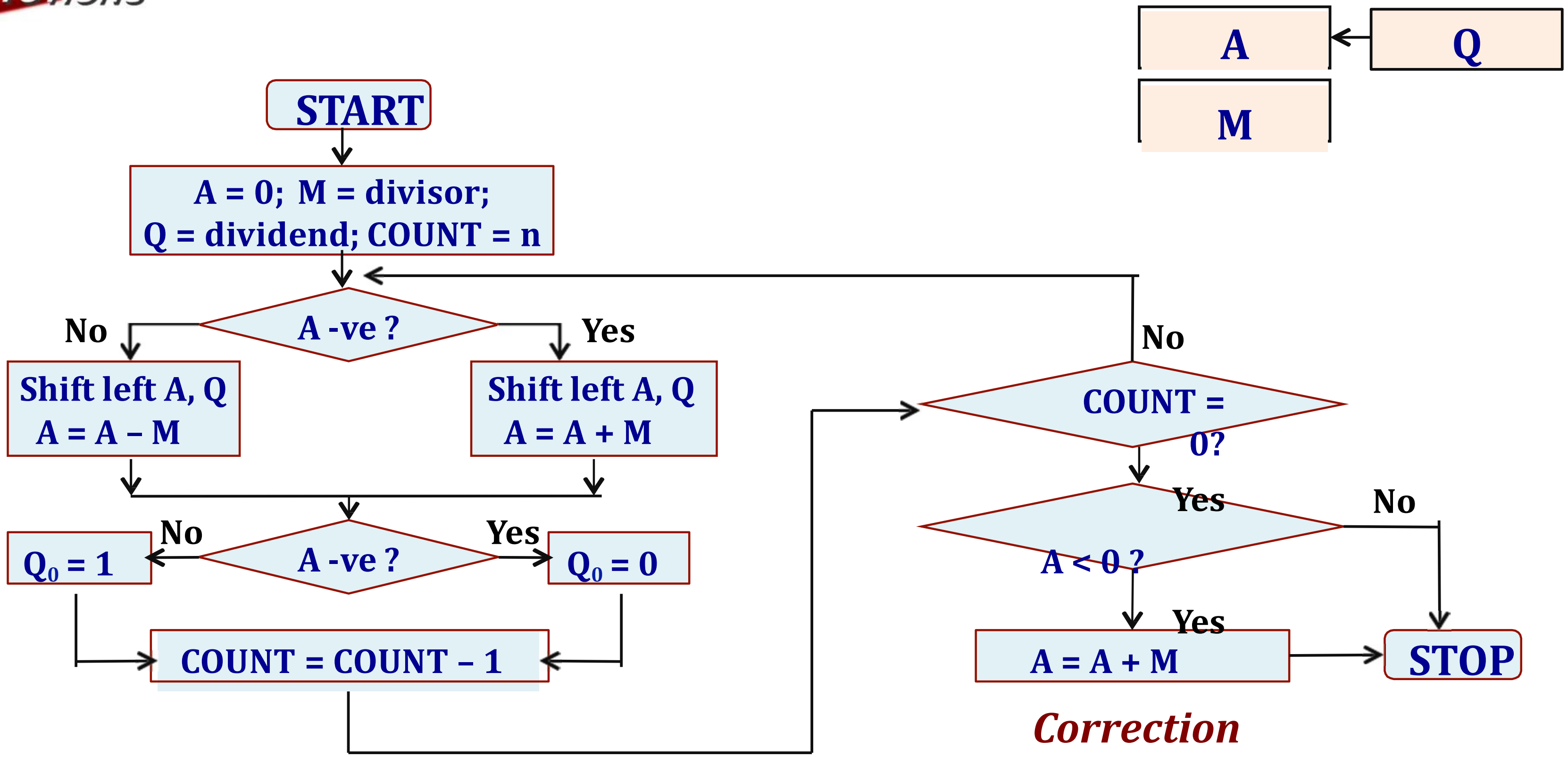
c) If the value in register A is negative,

- Shift A and Q left by one bit position.
- Add M to A.

c)If A is positive, set $Q_0 = 1$; else, set $Q_0 = 0$.

d)If A is negative, add M to A as a final corrective step.

Non-Restoring Division



A Simple Example: 8/3 for n=4

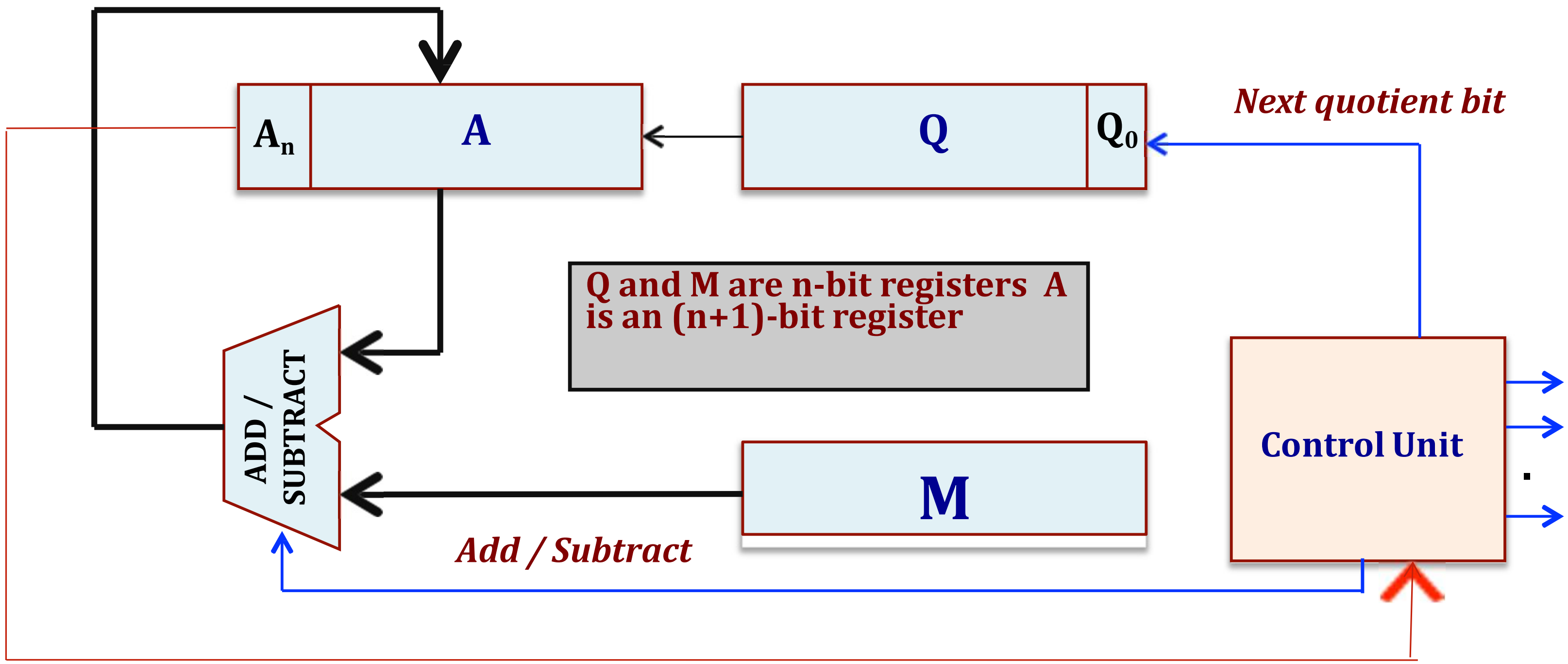
```
Initially:      0 0 0 0 0     1 0 0 0

Shift:          0 0 0 0 1     0 0 0 -
Subtract:   -     0 0 1 1
Set Q0:         1 1 1 1 0     0 0 0 0
Shift:          1 1 1 0 0     0 0 0 -
Add:              0 0 1 1
Set Q0:         1 1 1 1 1     0 0 0 0
Shift:          1 1 1 1 0     0 0 0 -
Add:              0 0 1 1
Set Q0:         0 0 0 0 1     0 0 0 1
```

```
Shift:          0 0 0 1 0     0 0 1 -
Subtract:   -     0 0 1 1
Set Q0:         1 1 1 1 1     0 0 1 0
Correction Add:
                1 1 1 1 1
                0 0 0 1 1
                0 0 0 1 0
```

Quotient
0010 = 2

Remainder
00010 = 2

Data Path for Non-Restoring Division



TEXT BOOK

Carl Hamacher, Zvonko Vranesic and Safwat Zaky, “Computer Organization”, McGraw-Hill, 6th Edition 2012.

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THANK YOU