Graph representation:

We can represent a graph in the form of adjacency lists, which are very useful in computer programming.

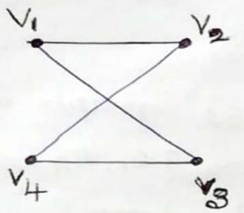
Adjacency matrix of a simple graph:

Let $G = (V, E)$ be a simple graph with 'n' vertices i.e., $v_1, v_2, \ldots, v_n$, its adjacency matrix is denoted by

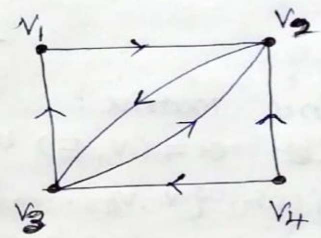
$$A = [a_{ij}] = \begin{cases} 1, & \text{if there exist an edge b/w } v_i \text{ and } v_j \\ 0, & \text{otherwise} \end{cases}$$

I. Find the adjacency matrix and degree of each vertex of the graph gvn. below.

a).



b).



Soln.

Adjacency matrix:

a) $A = [a_{ij}]$

$$= \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

and

$\deg(v_1) = 2$

$\deg(v_2) = 2$

$\deg(v_3) = 2$

$\deg(v_4) = 2$

b) $A = [a_{ij}]$

$$= \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

and

$\deg(v_1) = 1$ (Sum of the entries in the 1st row

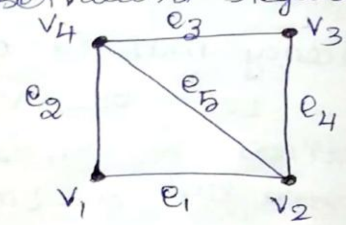
$\deg(v_2) = 1$

$\deg(v_3) = 2$

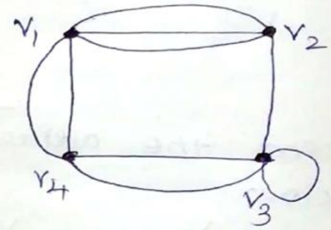
$\deg(v_4) = 2$

Hw

1. Find the adjacency matrix of the following graph G. Hence find the degree of each vertex and also find A^2 & A^3. What is your observations regarding the entries in A^2 and A^3.



2. Obtain Adjacency matrix to represent the pseudograph



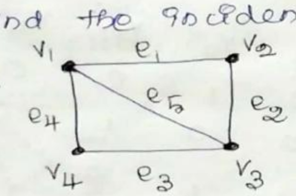
Incidence matrix:

Let $G = (V, E)$ be an undirected graph with n vertices $\{v_1, v_2, \dots, v_n\}$ and m edges $\{e_1, e_2, \dots, e_m\}$. Then the $(n \times m)$ matrix $B = [b_{ij}]$ where

$$b_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ incident on } v_i \\ 0 & \text{otherwise} \end{cases}$$

Find the incident matrix for the following graph.

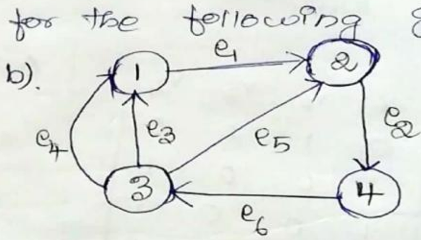
a).



Soln. Incidence matrix

$$B = [b_{ij}] = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

b).



$$B = [b_{ij}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

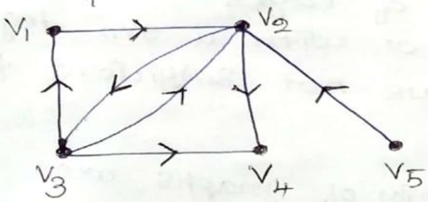
Path matrix:

If $G = (V, E)$ be a simple diagraph in which $|V| = n$ and the nodes of G are assumed to be ordered. An $n \times n$ matrix P whose elts. are given by,

$$P_{ij} = \begin{cases} 1 & \text{if there exists a path from } V_i \text{ to } V_j \\ 0 & \text{otherwise} \end{cases}$$

is called the path matrix (reachability matrix) of the graph G.

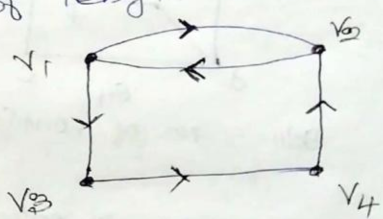
1. Find path matrix of



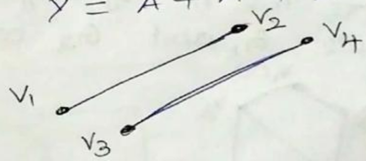
$$P = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

HW.

2. Consider the following digraph. Find the No. of possible elementary paths of length 3 from vertex $V_1 - V_2$.



3. Find the adjacency matrix of following graph also find $Y = A + A^2 + A^3 + A^4$



Problems:

1. Let $\delta(G)$ and $\Delta(G)$ denotes minimum and maximum degrees of all the vertices of G resly. Then show that for a non directed graph G.

$$\delta(G) \le \frac{2|E|}{|V|} \le \Delta(G)$$