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DEPARTMENT OF MATHEMATICS UNIT- II FOURIER TRANSFORM **PROPERTIES**

Proposition of Fowers Transform, FST and FCT:

J. IPnear Proposty:

$$\Rightarrow$$
 F[a](xx) + bg(xx)] = a f[f(xx)] + b f[g(xx)] evhous a and b proof:

$$F\left[\alpha_{f}(x) + bg(x)\right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[\alpha_{f}(x) + bg(x)\right] e^{iSx} dx$$

$$= \frac{a}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{iSx} dx + b \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) e^{iSx} dx$$

$$= a + [8 log) + p + [8 log)$$

$$FST = FS \left[a \left(sx \right) + b g(x) \right] = a F_S \left[\left(sx \right) \right] + b F_S \left[g(x) \right]$$

$$F_{S}\left[a_{1}(x)+b_{1}(x)\right] = \sqrt{\frac{2}{\pi}} \int \left[a_{1}(x)+b_{1}(x)\right] 3^{n} 5x dx$$

$$= a \sqrt{\frac{2}{\pi}} \int (x) 3^{n} 5x dx + b \sqrt{\frac{2}{\pi}} \int (x) 3^{n} 5x dx$$

$$= a F_{S}[g(AV)] + b F_{S}[g(AV)]$$

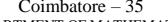
2] change of scale property:

For any non-rocco real a,
$$F[\{(axo)\} = \frac{1}{4a!}F[\frac{3}{a}]$$
, are



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Now,
$$F[f(axi)] = \frac{1}{\sqrt{a\pi}} \int_{-\infty}^{\infty} f(axi) e^{i6x} dxi$$

Put
$$\pm = ax$$

$$\frac{dt}{dx} = a$$

$$\Rightarrow dx = \frac{dt}{a}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} \frac{1}{3}(\pm) e^{-\frac{1}{3}(\pm)a} dt$$

i).
$$F[g(x-a)] = e^{ias} F(s)$$

ii). $F[e^{iax}g(x)] = F(s+a)$

$$F[\{(x)] = \frac{\sqrt{2\pi}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(x) e^{i6x} dx$$

i). NOW,
$$F[[[8](x-a)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [(x-a) e^{i6x} dx$$

Put
$$\pm = \alpha - \alpha \mid \alpha = -\infty \Rightarrow \pm = -\infty$$

 $dt = d\alpha \mid \alpha = \infty \Rightarrow \pm = \infty$

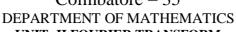
$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} f(\pm) e^{iS(\pm+\alpha)} dt$$

$$= \frac{e^{iS\alpha}}{\sqrt{2\pi}} \int_{0}^{\infty} f(\pm) e^{iS\pm} dt = e^{i\alpha S} F(S)$$



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$$= \frac{1}{18\pi} \int_{\infty}^{\infty} f(x) = \frac{1}{18\pi} \int_{\infty}^{\infty} f(x) e^{i\theta x} dx$$

$$= \frac{1}{18\pi} \int_{\infty}^{\infty} f(x) e^{i\theta x} dx$$

H. modulation Peroposity:

If
$$F(S)$$
 is the Fourier transform of $f(S)$ then $F[f(S)] = \frac{1}{2} \left[F(S+a) + F(S-a) \right]$

peoof: $F[\{(x)\} = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} f(x) e^{iSx} dx$

Now,
$$F[8(x)\cos ax] = \frac{1}{\sqrt{x\pi}} \int_{-\infty}^{\infty} 8(x)\cos ax e^{isx} dx$$

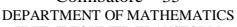
$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \sqrt[3]{x} \left[\frac{e^{i\alpha x} + e^{-i\alpha x}}{2} \right] e^{i\alpha x} dx$$

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{3(x)} \left[e^{i(x+a)x} + e^{i(x-a)x} \right] dx$$



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$$=\frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}(s+a)x\right]dx+\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{0}^{\infty}(s-a)x$$

$$= \frac{1}{2} \left[F(S+a) + F(S-a) \right]$$

$$= \frac{1}{2} \left[F(S+a) + F(S-a) \right]$$

$$= \frac{1}{2} \left[\frac{1}{2$$

$$F_{S}[[[3/2]] = \frac{1}{2}[F_{S}(9+\alpha) + F_{S}(5-\alpha)]$$

Ploop

Now,
$$F_S[g(x)\cos ax] = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} g(x)\cos ax \sin 6x \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_{B}^{\infty} \beta(x) \frac{3\pi}{A} \cos \frac{\alpha x}{A} dx$$

$$= \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \delta(x) \frac{1}{2\pi} \left[\frac{S9n}{A+B} + \frac{S9n}{A+B} + \frac{S9n}{A+B} \right] dx$$