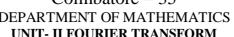
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DEPARTMENT OF MATHEMATICS UNIT- II FOURIER TRANSFORM SINE AND COSINE TRANSFORM

Foweles sine Transform:

The fowlier sine transform of b(DL) is

defined by,

Fs[5] = Fs[8(30)] = [[8(30) 59n 3x dx

The goverise fourtier sine transform of Fg(S)

is given by,

Founday Cosine Transform: The Fowlier cosene transform of fise) is

defened by

E[5] = Fc[8(20)] = [= [8(20) cos son dx

The goverse fourier corsine transform of

FCB) & given by

S) is given by:
$$f(x) = \sqrt{\frac{2}{\pi}} \int_{0}^{\pi} f_{c}[S] \cos Sx \, dc$$



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DEPARTMENT OF MATHEMATICS UNIT- II FOURIER TRANSFORM SINE AND COSINE TRANSFORM

Parseval'à Identity:

Sine Transform:

If F(5) is the fourtier transform of f(30). Then
$$\int_{0}^{\infty} \left[f(x) \right]^{2} dx = \int_{0}^{\infty} \left[f_{S}(S) \right]^{2} dS$$

Cosine Transform:

If
$$F(S)$$
 is the power transform of $f(S)$, then
$$\int \left[f(S) \right]^2 dS = \int \left[F_C(S) \right]^2 dS.$$

I. Find the FST of
$$f(x)$$
 defined as
$$f(x) = \begin{cases} 1 & 9 \\ 0 & 9 \\ 0 & 9 \end{cases} \quad x > 1$$

Soln :

$$f_{S}(S) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} g_{DO} \, dy_{D} \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} g_{DO} \, dx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[-\frac{\cos g_{X}}{g} \right]_{0}^{\infty}$$

$$= \sqrt{\frac{2}{\pi}} \left[-\frac{\cos g_{X}}{g} \right]_{0}^{\infty}$$

Solo:
$$f_{S}(s) = \sqrt{\frac{2}{\pi}} \int_{X}^{\infty} \frac{1}{x} s^{g} n c x d x$$



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UNIT- II FOURIER TRANSFORM SINE AND COSINE TRANSFORM



6] Find the fct of
$$\frac{e^{-ax}}{x}$$
 and hence, find $\frac{e^{-ax}}{x} = \frac{e^{-bx}}{x}$

Fc [8] =
$$\sqrt{\frac{2}{\pi}} \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} \frac{-\alpha x}{x} \cos sx \, dx$$

$$\frac{d}{ds} f_c[s] = \frac{d}{ds} \left[\sqrt{\frac{a}{\pi}} \int_{0}^{\infty} \frac{e^{-ax}}{x} \cos sx \, dx \right]$$

$$= \int_{\overline{M}}^{\overline{Q}} \int_{\overline{X}}^{\infty} \frac{e^{-\alpha x}}{x} \left(-\pi SPnSx\right) dx$$

$$\frac{d}{ds} f_{c}[s] = -\sqrt{\frac{2}{\pi}} \frac{s}{s^{2}+\alpha^{2}} \cdots \int_{0}^{\infty} e^{-\alpha x} s^{n} b^{n} d^{n} d^{n}$$

$$= \frac{b}{\alpha^{2}+b^{2}}$$

$$\int_{0}^{\infty} e^{-ax} \sin bx dx$$

$$= \frac{b}{a^{2} + b^{2}}$$

$$f_{c}[S] = -\sqrt{\frac{2}{\pi}} \int \frac{S}{S^{2} + \alpha^{2}} dS$$

$$=-\sqrt{\frac{2}{\pi}}\frac{1}{2}\int\frac{2c}{c^2+a^2}dc$$

$$F_{c}\left[\frac{e^{-\alpha x}}{n}\right] = -\frac{1}{\sqrt{2\pi}}\log(s^{2}+\alpha^{2})$$



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My
$$E\left[\frac{e^{-bx}}{x}\right] = \frac{1}{\sqrt{2\pi}}\log(s^2+b^2)$$
Nows

$$F_{c} \left[\frac{e^{\alpha x} - e^{-bx}}{x} \right] = F_{c} \left[\frac{e^{-\alpha x}}{x} \right] - F_{c} \left[\frac{e^{-bx}}{x} \right]$$

$$= \frac{-1}{\sqrt{2\pi}} \log (s^{2} + a^{2}) + \frac{1}{\sqrt{2\pi}} \log (s^{2} + b^{2})$$

$$= \frac{1}{\sqrt{2\pi}} \log \left[\frac{s^{2} + b^{2}}{s^{2} + a^{2}} \right]$$

J. Show that e xx/2 is self reapscocal under HW

FCT.

If spind the FST of the function $\frac{e^{-ax}}{x}$ and hence $\frac{e^{-ax}}{x}$ $\frac{e^{-ax}}{x}$

FIND FST OF E-ax, aro

Find the FST of x + a2 47

6] show that e xila 93 Self- ne aprical under