Concept of Shear Force and Bending moment in beams:

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beams further. Let us define these terms

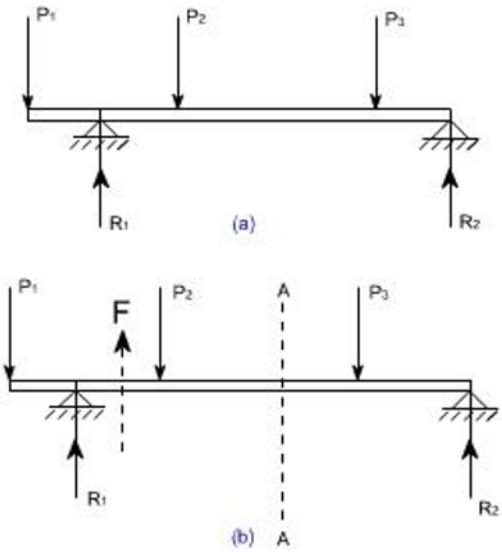


Fig 1

Now let us consider the beam as shown in fig 1(a) which is supporting the loads P_1, P_2, P_3 and is simply supported at two points creating the reactions R_1 and R_2 respectively. Now let us assume that the beam is to divided into or imagined to be cut into two portions at a section AA. Now let us assume that the resultant of loads and reactions to the left of AA is 'F' vertically upwards, and since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be F, acting downwards. This forces 'F' is as a shear force. The shearing force at any x-section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

Therefore, now we are in a position to define the shear force 'F' to as follows:

At any x-section of a beam, the shear force 'F' is the algebraic sum of all the lateral components of the forces acting on either side of the x-section.

Sign Convention for Shear Force:

The usual sign conventions to be followed for the shear forces have been illustrated in figures 2 and 3.

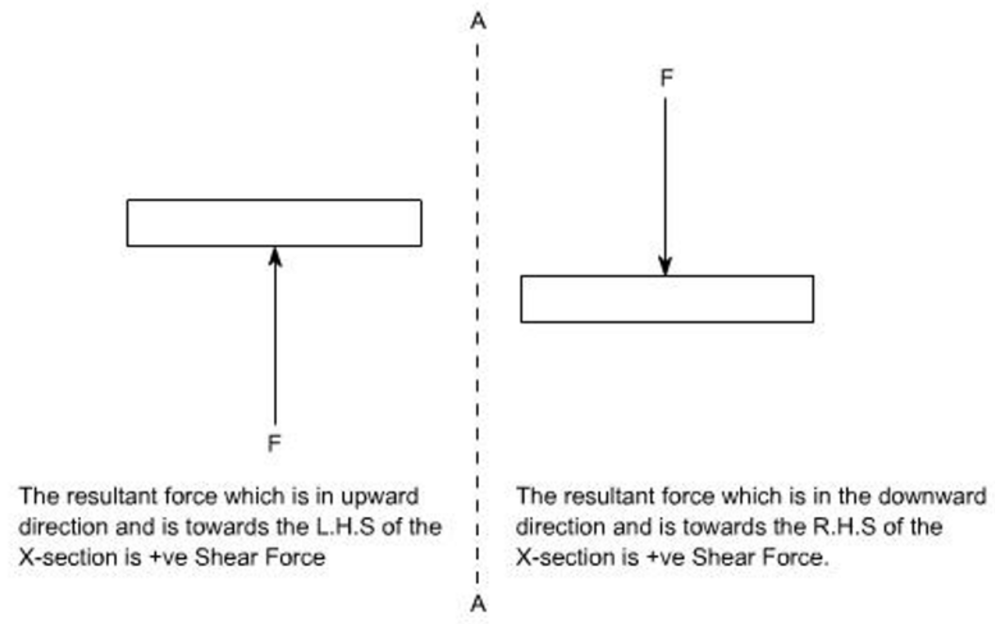


Fig 2: Positive Shear Force

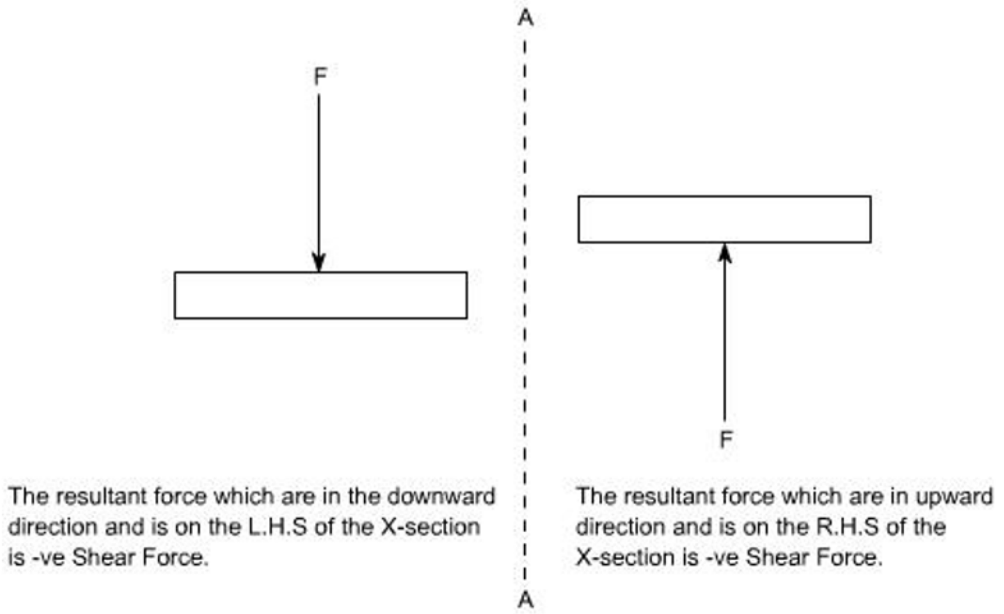


Fig 3: Negative Shear Force

Bending Moment:

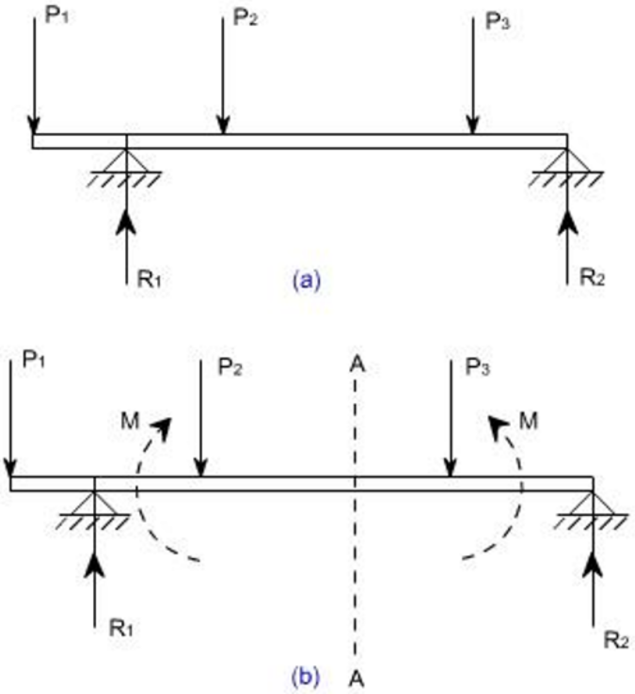


Fig 4

Let us again consider the beam which is simply supported at the two prints, carrying loads P_1, P_2 and P_3 and having the reactions R_1 and R_2 at the supports Fig 4. Now, let us imagine that the beam is cut into two potions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is M in C.W direction, then moment of forces to the right of x-section AA must be 'M' in C.C.W. Then 'M' is called as the Bending moment and is abbreviated as B.M. Now one can define the bending moment to be simply as the algebraic sum of the moments about an x-section of all the forces acting on either side of the section

Sign Conventions for the Bending Moment:

For the bending moment, following sign conventions may be adopted as indicated in Fig 5 and Fig 6.

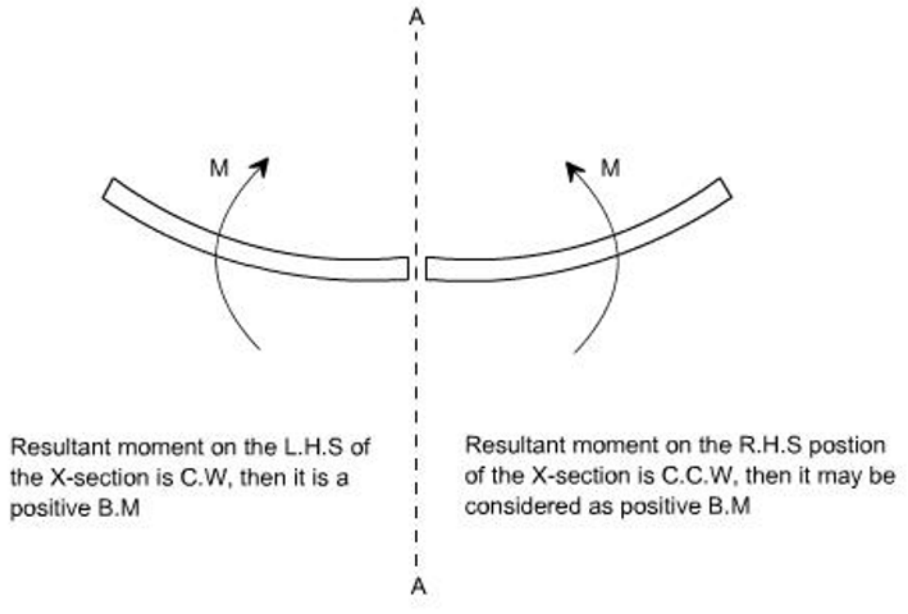


Fig 5: Positive Bending Moment

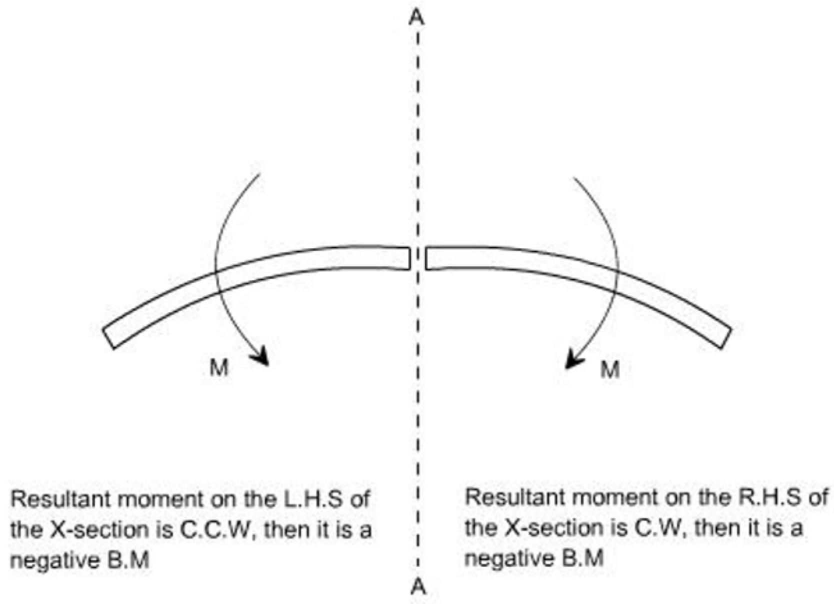


Fig 6: Negative Bending Moment

Some times, the terms ‘Sagging' and Hogging are generally used for the positive and negative bending moments respectively.

Bending Moment and Shear Force Diagrams:

The diagrams which illustrate the variations in B.M and S.F values along the length of the beam for any fixed loading conditions would be helpful to analyze the beam further.

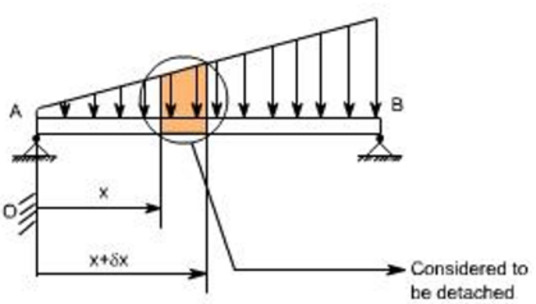
Thus, a shear force diagram is a graphical plot, which depicts how the internal shear force ‘F' varies along the length of beam. If x dentotes the length of the beam, then F is function x i.e. F(x).

Similarly a bending moment diagram is a graphical plot which depicts how the internal be
moment ‘M' varies along the length of the beam. Again M is a function x i.e. M(x).

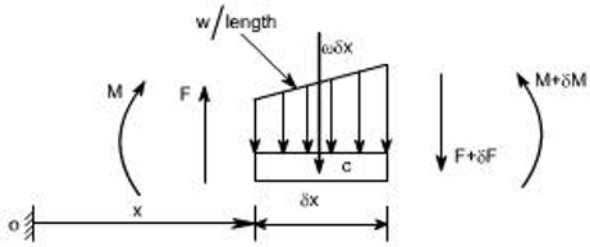
Basic Relationship Between The Rate of Loading, Shear Force and Bending Moment:

The construction of the shear force diagram and bending moment diagrams is greatly simplified if the relationship among load, shear force and bending moment is established.

Let us consider a simply supported beam AB carrying a uniformly distributed load w/length. Let us imagine to cut a short slice of length dx cut out from this loaded beam at distance ‘x' from the origin ‘0'.



Let us detach this portion of the beam and draw its free body diagram.



The forces acting on the free body diagram of the detached portion of this loaded beam are the following

- The shearing force F and F+ δF at the section x and x + δx respectively.

- The bending moment at the sections x and x + δx be M and M + dM respectively.

- Force due to external loading, if ‘w' is the mean rate of loading per unit length then the total loading on this slice of length δx is w. δx, which is approximately acting through the centre ‘c'. If the loading is assumed to be uniformly distributed then it would pass exactly through the centre ‘c'.

This small element must be in equilibrium under the action of these forces and couples.

Now let us take the moments at the point ‘c'. Such that

$$M + F.\frac{\delta x}{2} + (F + \delta F).\frac{\delta x}{2} = M + \delta M$$

$$\Rightarrow F.\frac{\delta x}{2} + (F + \delta F).\frac{\delta x}{2} = \delta M$$

$$\Rightarrow F.\frac{\delta x}{2} + F.\frac{\delta x}{2} + \delta F.\frac{\delta x}{2} = \delta M$$ [Neglecting the product of δF and δx being small quantities]

$$\Rightarrow F.\delta x = \delta M$$

$$\Rightarrow F = \frac{\delta M}{\delta x}$$

Under the limits $\delta x \rightarrow 0$

$$\boxed{F = \frac{dM}{dx}} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

Resolving the forces vertically we get

$$w.\delta x + (F + \delta F) = F$$

$$\Rightarrow w = -\frac{\delta F}{\delta x}$$

Under the limits $\delta x \rightarrow 0$

$$\Rightarrow w = -\frac{dF}{dx} \text{ or } -\frac{d}{dx}\left(\frac{dM}{dx}\right)$$

$$\boxed{w = -\frac{dF}{dx} = -\frac{d^2M}{dx^2}} \qquad \ldots\ldots\ldots\ldots (2)$$

Conclusions: From the above relations,the following important conclusions may be drawn

• From Equation (1), the area of the shear force diagram between any two points, from the basic calculus is the bending moment diagram

$$M = \int F.dx$$

• The slope of bending moment diagram is the shear force,thus

$$F = \frac{dM}{dx}$$

Thus, if F=0; the slope of the bending moment diagram is zero and the bending moment is therefore constant.'

• The maximum or minimum Bending moment occurs where $\frac{dM}{dx} = 0.$

The slope of the shear force diagram is equal to the magnitude of the intensity of the distributed loading at any position along the beam. The –ve sign is as a consequence of our particular choice of sign conventions

Procedure for drawing shear force and bending moment diagram:

Preamble:

The advantage of plotting a variation of shear force F and bending moment M in a beam as a function of ‘x' measured from one end of the beam is that it becomes easier to determine the maximum absolute value of shear force and bending moment.

Further, the determination of value of M as a function of ‘x' becomes of paramount importance so as to determine the value of deflection of beam subjected to a given loading.

Construction of shear force and bending moment diagrams:

A shear force diagram can be constructed from the loading diagram of the beam. In order to draw this, first the reactions must be determined always. Then the vertical components of forces and reactions are successively summed from the left end of the beam to preserve the mathematical sign conventions adopted. The shear at a section is simply equal to the sum of all the vertical forces to the left of the section.

When the successive summation process is used, the shear force diagram should end up with the previously calculated shear (reaction at right end of the beam. No shear force acts through the beam just beyond the last vertical force or reaction. If the shear force diagram closes in this fashion, then it gives an important check on mathematical calculations.

The bending moment diagram is obtained by proceeding continuously along the length of beam from the left hand end and summing up the areas of shear force diagrams giving due regard to sign. The process of obtaining the moment diagram from the shear force diagram by summation is exactly the same as that for drawing shear force diagram from load diagram.

It may also be observed that a constant shear force produces a uniform change in the bending moment, resulting in straight line in the moment diagram. If no shear force exists along a certain portion of a beam, then it indicates that there is no change in moment takes place. It may also further observe that dm/dx= F therefore, from the fundamental theorem of calculus the maximum or minimum moment occurs where the shear is zero. In order to check the validity of the bending moment diagram, the terminal conditions for the moment must be satisfied. If the end is free or pinned, the computed sum must be equal to zero. If the end is built in, the moment computed by the summation must be equal to the one calculated initially for the reaction. These conditions must always be satisfied.

Illustrative problems:

In the following sections some illustrative problems have been discussed so as to illustrate the procedure for drawing the shear force and bending moment diagrams

1. A cantilever of length carries a concentrated load ‘W' at its free end.

Draw shear force and bending moment.

Solution:

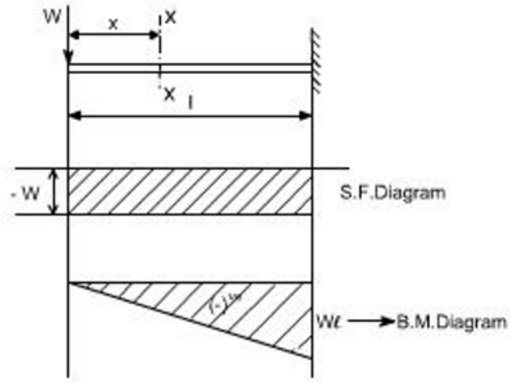
At a section a distance x from free end consider the forces to the left, then F = -W (for all values of x) -ve sign means the shear force to the left of the x-section are in downward direction and therefore negative

Taking moments about the section gives (obviously to the left of the section)

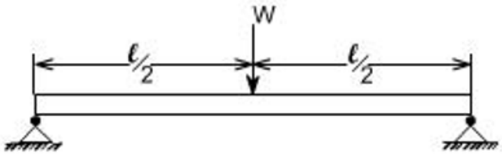
M = -Wx (-ve sign means that the moment on the left hand side of the portion is in the anticlockwise direction and is therefore taken as –ve according to the sign convention)

so that the maximum bending moment occurs at the fixed end i.e. M = -W l

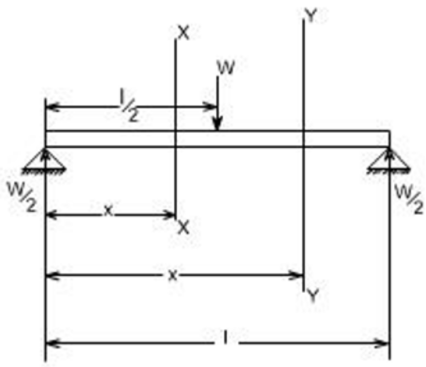
From equilibrium consideration, the fixing moment applied at the fixed end is Wl and the reaction is W. the shear force and bending moment are shown as,



2. Simply supported beam subjected to a central load (i.e. load acting at the mid-way)



By symmetry the reactions at the two supports would be W/2 and W/2. now consider any section X-X from the left end then, the beam is under the action of following forces.

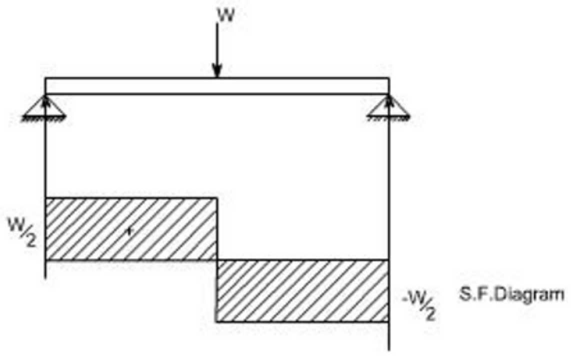


So the shear force at any X-section would be = W/2 [Which is constant upto x < l/2]

If we consider another section Y-Y which is beyond l/2 then

$$S.F_{Y\text{-}Y} = \frac{W}{2} - W = \frac{-W}{2}$$ for all values greater = l/2

Hence S.F diagram can be plotted as,



.For B.M diagram:

If we just take the moments to the left of the cross-section,

$$B.M_{X-X} = \frac{W}{2}x \text{ for } x \text{ lies between } 0 \text{ and } l/2$$

$$B.M_{\text{at } x=\frac{l}{2}} = \frac{W}{2}\,\frac{l}{2} \text{ i.e } B.M \text{ at } x = 0$$

$$= \frac{Wl}{4}$$

$$B.M_{Y-Y} = \frac{W}{2}x - W\left(x - \frac{l}{2}\right)$$

Again

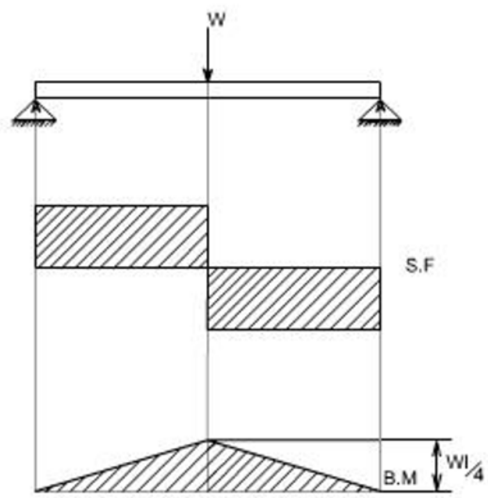
$$= \frac{W}{2}x - Wx + \frac{Wl}{2}$$

$$= -\frac{W}{2}x + \frac{Wl}{2}$$

$$B.M_{\text{at } x=l} = -\frac{Wl}{2} + \frac{Wl}{2}$$

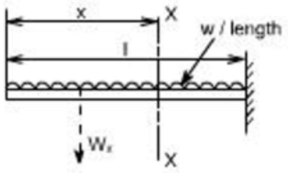
$$= 0$$

Which when plotted will give a straight relation i.e.



It may be observed that at the point of application of load there is an abrupt change in the shear force, at this point the B.M is maximum.

3. A cantilever beam subjected to U.d.L, draw S.F and B.M diagram.



Here the cantilever beam is subjected to a uniformly distributed load whose intensity is given w / length.

Consider any cross-section XX which is at a distance of x from the free end. If we just take the resultant of all the forces on the left of the X-section, then

$S.F_{xx}$ = -Wx for all values of ‘x'. ---------- **(1)**

$S.F_{xx}$ = 0

$S.F_{xx \text{ at } x=1}$ = -Wl

So if we just plot the equation No. (1), then it will give a straight line relation. Bending Moment at X-X is obtained by treating the load to the left of X-X as a concentrated load of the same value acting through the centre of gravity.

Therefore, the bending moment at any cross-section X-X is

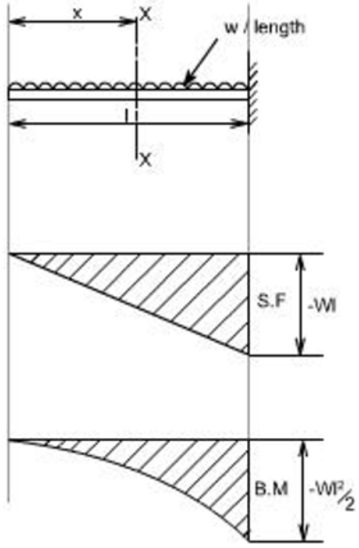
$$B.M_{X\text{-}X} = -\ W x \frac{x}{2}$$
$$= -\ W\frac{x^2}{2}$$

The above equation is a quadratic in x, when B.M is plotted against x this will produces a parabolic variation.

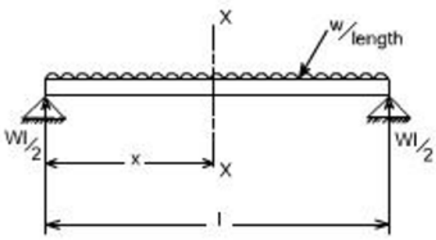
The extreme values of this would be at x = 0 and x = l

$$B.M_{\text{at } x = l} = -\frac{Wl^2}{2}$$
$$= \frac{Wl}{2} - Wx$$

Hence S.F and B.M diagram can be plotted as follows:



4. Simply supported beam subjected to a uniformly distributed load [U.D.L].



The total load carried by the span would be

= intensity of loading x length

= w x l

By symmetry the reactions at the end supports are each wl/2

If x is the distance of the section considered from the left hand end of the beam.

S.F at any X-section X-X is

$$= \frac{Wl}{2} - Wx$$
$$= W\left(\frac{l}{2} - x\right)$$

Giving a straight relation, having a slope equal to the rate of loading or intensity of the loading.

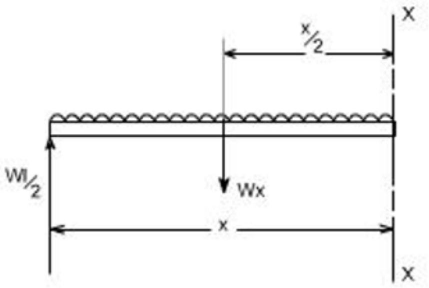
$$S.F_{at\ x=0} = \frac{wl}{2} - wx$$

so at

$S.F_{at\ x=\frac{l}{2}} = 0$ hence the S.F is zero at the centre

$$S.F_{at\ x=l} = -\frac{Wl}{2}$$

The bending moment at the section x is found by treating the distributed load as acting at its centre of gravity, which at a distance of x/2 from the section



$$B.M_{x\text{-}x} = \frac{Wl}{2}x - Wx.\frac{x}{2}$$

so the

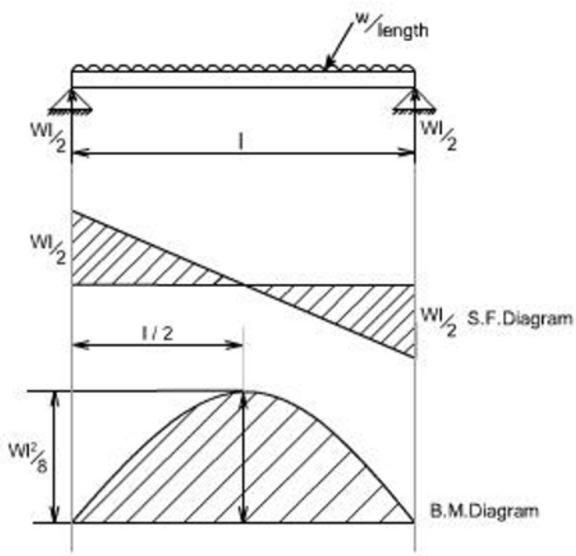
$$= W.\frac{x}{2}(l - 2) \quad \text{........(2)}$$

$B.M_{at\ x=0} = 0$

$B.M_{at\ x=l} = 0$

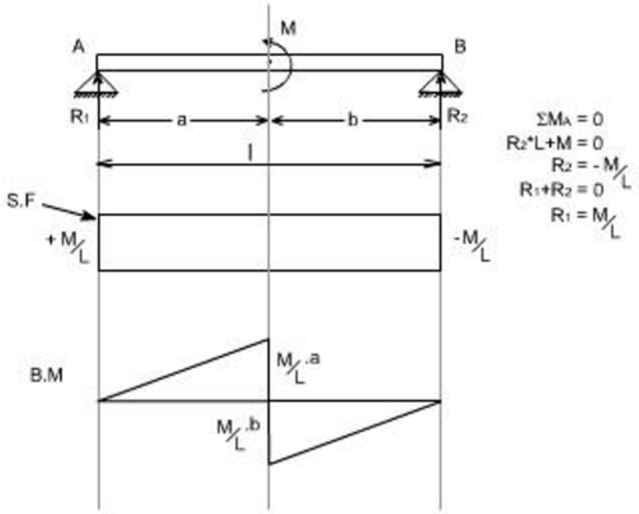
$$B.M\Big|_{at\ x=l} = -\frac{Wl^2}{8}$$

So the equation (2) when plotted against x gives rise to a parabolic curve and the shear force and bending moment can be drawn in the following way will appear as follows:



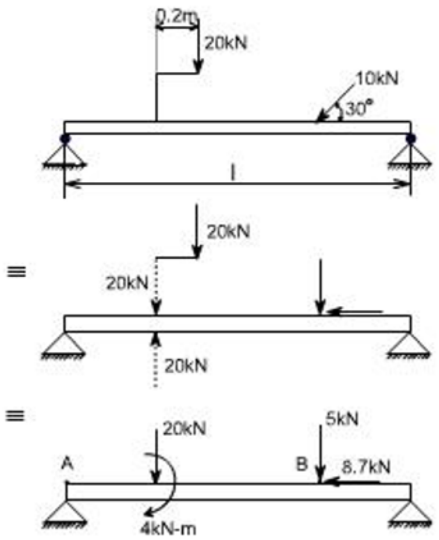
5. Couple.

When the beam is subjected to couple, the shear force and Bending moment diagrams may be d... exactly in the same fashion as discussed earlier.



6. Eccentric loads.

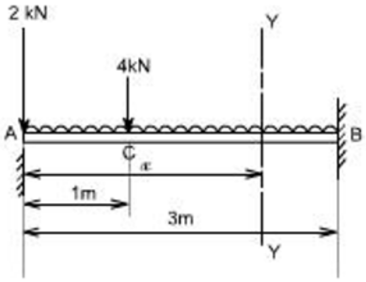
When the beam is subjected to an eccentric loads, the eccentric load are to be changed into a couple/ force as the case may be, In the illustrative example given below, the 20 kN load acting at a distance of 0.2m may be converted to an equivalent of 20 kN force and a couple of 2 kN.m. similarly a 10 kN force which is acting at an angle of 30^0 may be resolved into horizontal and vertical components.The rest of the procedure for drawing the shear force and Bending moment remains the same.



6. Loading changes or there is an abrupt change of loading:

When there is an aabrupt change of loading or loads changes, the problem may be tackled in a systematic way.consider a cantilever beam of 3 meters length. It carries a uniformly distributed load of 2 kN/m and a concentrated loads of 2kN at the free end and 4kN at 2 meters from fixed end.The shearing force and bending moment diagrams are required to be drawn and state the maximum values of the shearing force and bending moment.

Solution



Consider any cross section x-x, at a distance x from the free end

Shear Force at x-x = -2 -2x   0 < x < 1

S.F at x = 0 i.e. at A = -2 kN

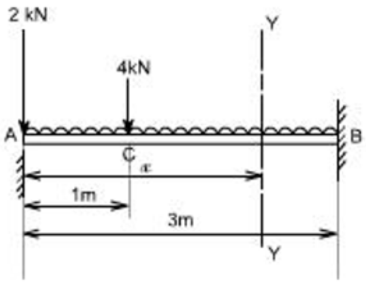
S.F at x = 1 = -2-2 = - 4kN

S.F at C (x = 1) = -2 -2x - 4   Concentrated load

= - 2 - 4 -2x1 kN

= - 8 kN

Again consider any cross-section YY, located at a distance x from the free end



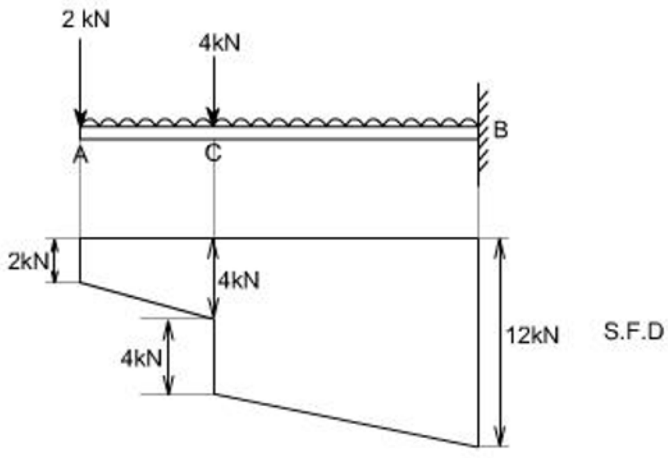
S.F at Y-Y = -2 - 2x - 4   1< x < 3

This equation again gives S.F at point C equal to -8kN

S.F at x = 3 m = -2 -4 -2x3

= -12 kN

Hence the shear force diagram can be drawn as below:



For bending moment diagrams – Again write down the equations for the respective cross sections, as consider above

Bending Moment at xx = -2x - 2x.x/2 valid upto AC

B.M at x = 0 = 0

B.M at x =1m = -3 kN.m

For the portion CB, the bending moment equation can be written for the x-section at Y-Y .

B.M at YY = -2x - 2x.x/2 - 4(x -1)

This equation again gives,

B.M at point C = - 2.1 - 1 - 0 i.e. at x = 1

= -3 kN.m

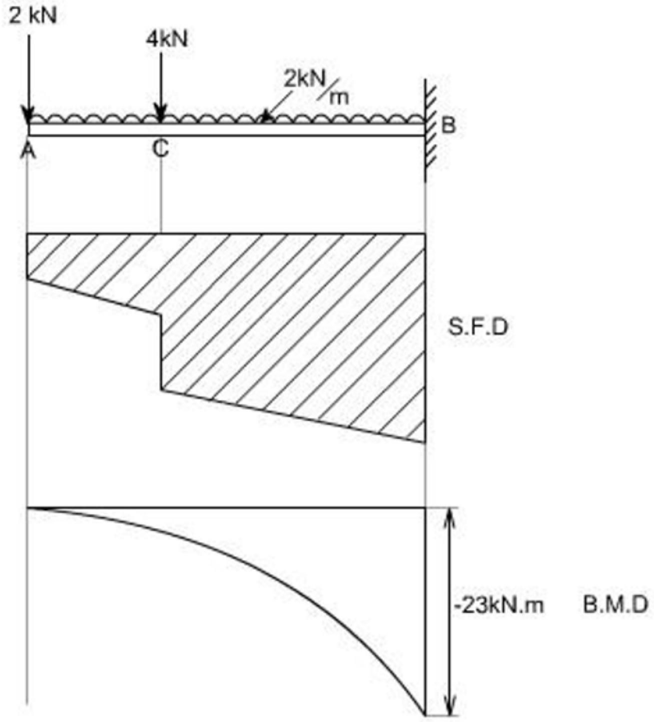
B.M at point B i.e. at x = 3 m

= - 6 - 9 - 8

= - 23 kN-m

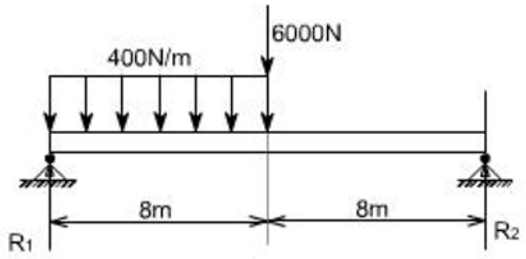
The variation of the bending moment diagrams would obviously be a parabolic curve

Hence the bending moment diagram would be



7. Illustrative Example :

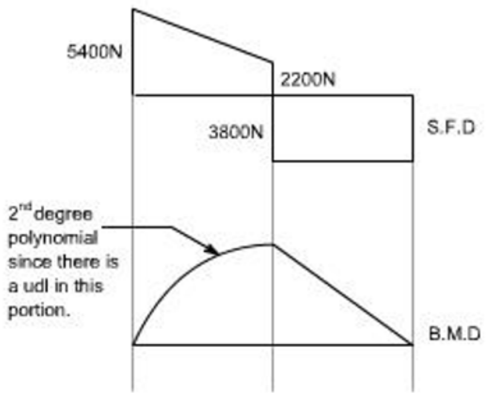
In this there is an abrupt change of loading beyond a certain point thus, we shall have to be careful at the jumps and the discontinuities.



For the given problem, the values of reactions can be determined as

R2 = 3800N and R1 = 5400N

The shear force and bending moment diagrams can be drawn by considering the X-sections at the suitable locations.



8. Illustrative Problem :

The simply supported beam shown below carries a vertical load that increases uniformly from zero at the one end to the maximum value of 6kN/m of length at the other end .Draw the shearing force and bending moment diagrams.

Solution

Determination of Reactions

For the purpose of determining the reactions R1 and R2 , the entire distributed load may be replaced by its resultant which will act through the centroid of the triangular loading diagram.

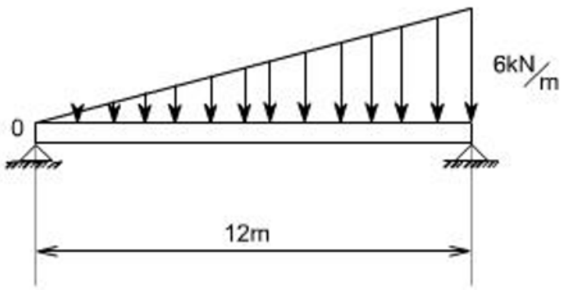
So the total resultant load can be found like this-

Average intensity of loading = (0 + 6)/2

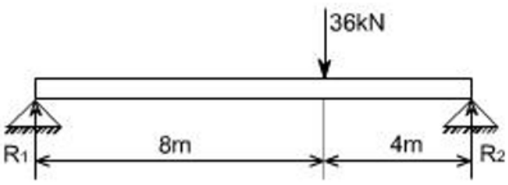
= 3 kN/m

Total Load = 3 x 12

= 36 kN



Since the centroid of the triangle is at a 2/3 distance from the one end, hence 2/3 x 3 = 8 m from the left end support.



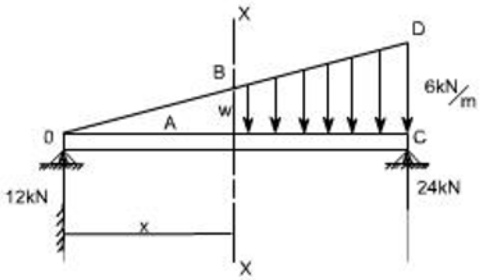
Now taking moments or applying conditions of equilibrium

36 x 8 = R2 x 12

R1 = 12 kN

R2 = 24 kN

Note: however, this resultant can not be used for the purpose of drawing the shear force and bending moment diagrams. We must consider the distributed load and determine the shear and moment at a section x from the left hand end.



Consider any X-section X-X at a distance x, as the intensity of loading at this X-section, is unknown let us find out the resultant load which is acting on the L.H.S of the X-section X-X, hence

So consider the similar triangles

OAB & OCD

$$\frac{w}{6} = \frac{x}{12}$$
$$w = \frac{x}{2} \ k\frac{N}{m}$$

In order to find out the total resultant load on the left hand side of the X-section

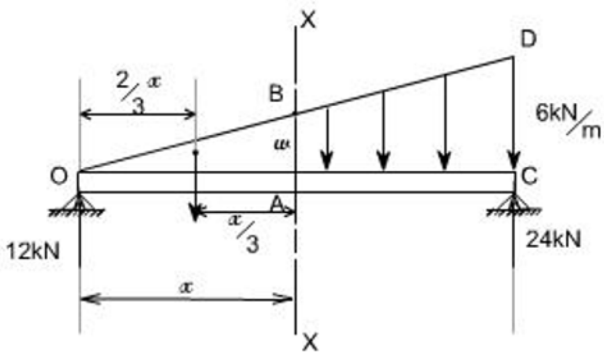
Find the average load intensity

$$= \frac{0 + \frac{x}{2}}{2}$$
$$= \frac{x}{4} \ k\frac{N}{m}$$

Therefore the total load over the length x would be

$$= \frac{x}{4}.x \ \text{kN}$$
$$= \frac{x^2}{4} \ \text{kN}$$

Now these loads will act through the centroid of the triangle OAB. i.e. at a distance 2/3 x from the left hand end. Therefore, the shear force and bending momemt equations may be written as



$$S.F_{at\ xx} = \left(12 - \frac{x^2}{4}\right) \text{ kN}$$

valid for all values of x ....(1)

$$B.M_{at\ xx} = 12\,x - \frac{x^2}{4}.\frac{x}{3}$$

$$B.M_{at\ xx} = 12\,x - \frac{x^3}{12} \text{ kN-m}$$

valid for all values of x ....(2)

$$S.F_{at\ x=0} = 12 \text{ kN}$$

$$S.F_{at\ x=12m} = 12 - \frac{12 \times 12}{4}$$

$$= -24 \text{ kN}$$

In order to find out the point where S.F is zero

$$\left(12 - \frac{x^2}{4}\right) = 0$$

x = 6.92 m (selecting the positive values)

Again

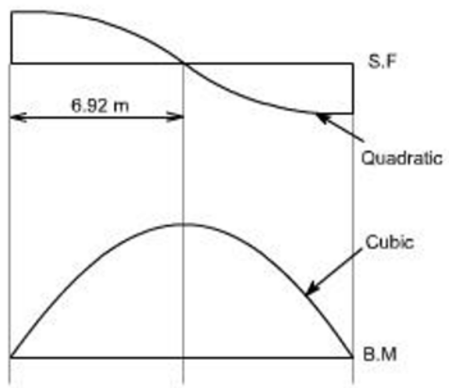
$$B.M_{at\ x=0} = 0$$

$$B.M_{at\ x=12} = 12 \times 12 - \frac{12^3}{12}$$

$$= 0$$

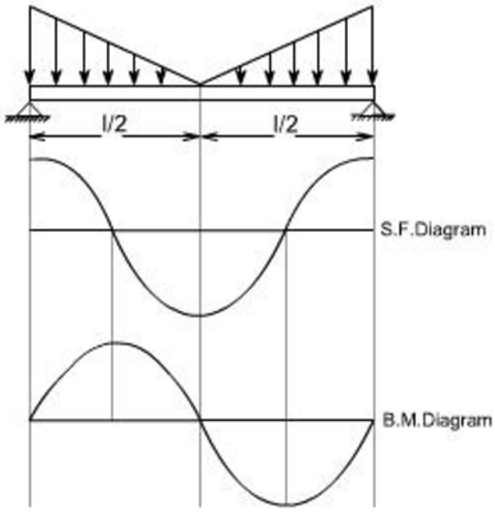
$$B.M_{at\ x=6.92} = 12 \times 6.92 - \frac{6.92^3}{12}$$

$$= 55.42 \text{ kN-m}$$



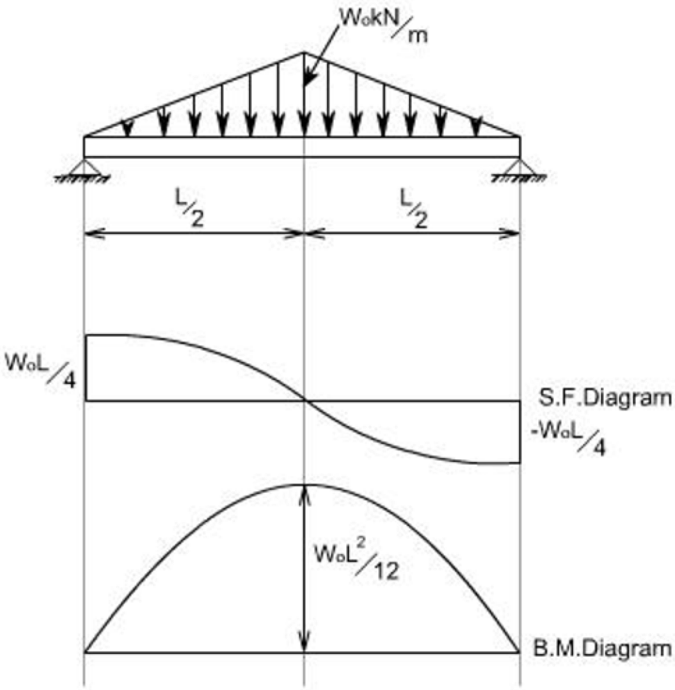
9. Illustrative problem :

In the same way, the shear force and bending moment diagrams may be attempted for the given problem



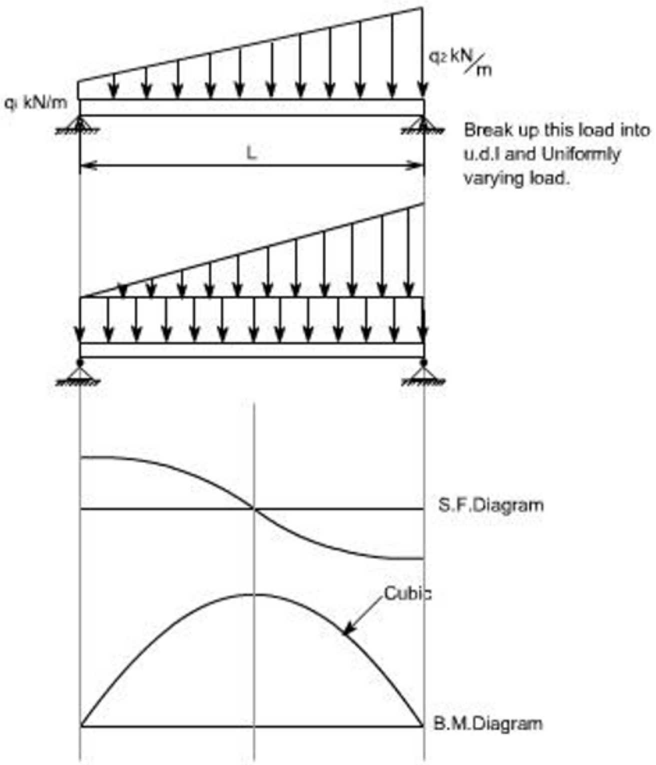
10. Illustrative problem :

For the uniformly varying loads, the problem may be framed in a variety of ways, observe the shear force and bending moment diagrams

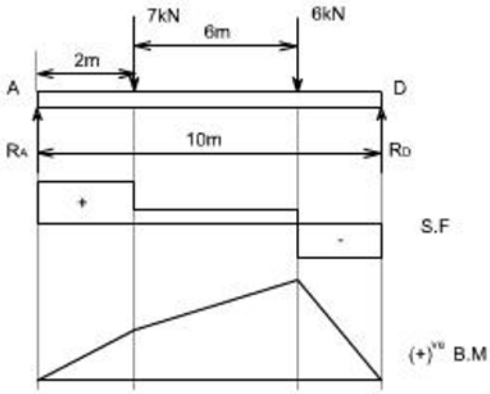


11. Illustrative problem :

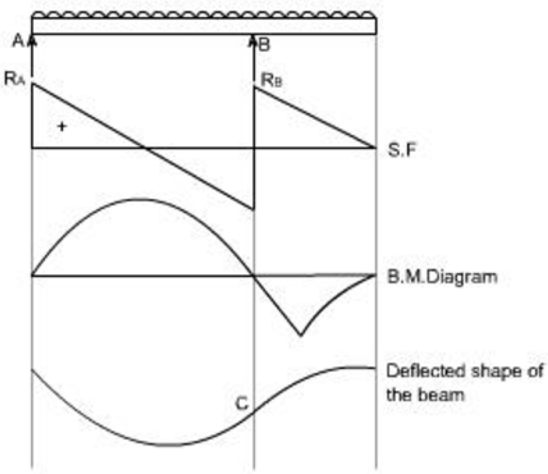
In the problem given below, the intensity of loading varies from q_1 kN/m at one end to the q_2 kN/m at the other end.This problem can be treated by considering a U.d.i of intensity q_1 kN/m over the entire span and a uniformly varying load of 0 to (q_2- q_1)kN/m over the entire span and then super impose teh two loadings.



Point of Contraflexure:

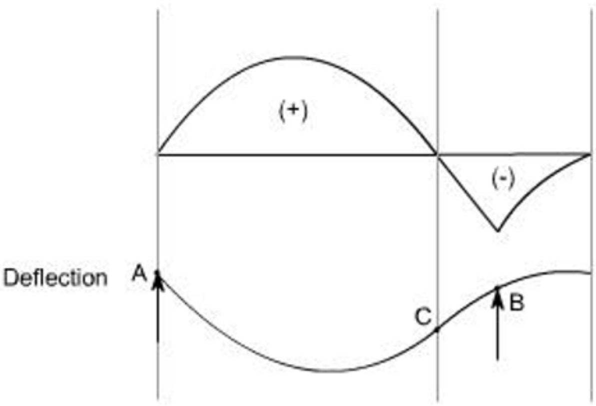


Consider the loaded beam a shown below along with the shear force and Bending moment diagrams for It may be observed that this case, the bending moment diagram is completely positive so that the curvature of the beam varies along its length, but it is always concave upwards or sagging.However if we consider a again a loaded beam as shown below along with the S.F and B.M diagrams, then



It may be noticed that for the beam loaded as in this case,

The bending moment diagram is partly positive and partly negative.If we plot the deflected shape of the beam just below the bending moment



This diagram shows that L.H.S of the beam 'sags' while the R.H.S of the beam 'hogs'

The point C on the beam where the curvature changes from sagging to hogging is a point of contraflexure.

OR

It corresponds to a point where the bending moment changes the sign, hence in order to find the point of contraflexures obviously the B.M would change its sign when it cuts the X-axis therefore to get the points of contraflexure equate the bending moment equation equal to zero.The fibre stress is zero at such sections

Note: there can be more than one point of contraflexure.