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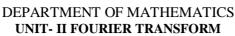
# DEPARTMENT OF MATHEMATICS UNIT- II FOURIER TRANSFORM SINE AND COSINE TRANSFORM

J. Show that e-2/2 les fourier cosine Transform. Self Techprocal ander Fc [f(x)] =√2 ∫ f(x) coe sor dx  $F_c[e^{-\chi^2/2}] = \sqrt{\pi} \int_{e}^{\infty} e^{-\chi^2/2} \cos sx \, dx$ = \\ \frac{2}{\pi} R. P. Of \\ \frac{2}{0} = \frac{2^2}{2} + i \frac{1}{2} \times \\ \text{d} \times \\ \tex = R.P. 0) \[ \frac{2}{\pi} \int^{\infty} = \frac{1}{2} \left[ \frac{2}{\pi} - is \times + (is)^2 - is \frac{2}{\pi} \right] dx = R. P. Q Ja po - (x-is)2 - 32  $=R-P\cdot Q\sqrt{\frac{2}{\pi}}e^{-\frac{Q^2}{2}}\sqrt{\frac{2}{\pi}}e^{-\frac{(2x-1)}{\sqrt{2}}}dx$  $= R. P. Q \sqrt{\frac{2}{\pi}} e^{-\frac{\zeta^2}{2}} \int_{e}^{\infty} e^{-\frac{\xi^2}{2}} dt dx = \sqrt{2} dt$ = R.P. of 200 -t2 dt  $= R.p.ob \frac{a}{\sqrt{\pi}} e^{-s^2/2} \frac{\sqrt{\pi}}{2} \cdot \int e^{\pm^2} dt = \sqrt{2}$  $= e^{-S^2/2}$   $= e^{-S^2/2}$   $\Rightarrow e^{-S^2/2}$  Self lespectal under



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SINE AND COSINE TRANSFORM

Using Auseval's Identity
$$\int_{-\infty}^{\infty} |g(x)|^{2} dx = \int_{-\infty}^{\infty} |f(x)|^{2} dx$$

$$\int_{-\infty}^{\infty} (1-|x|)^{2} dx = \frac{2}{\pi} \int_{-\infty}^{\infty} (\frac{1-\cos s}{g^{2}})^{2} ds$$

$$= \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{(1-\cos s)^{2}}{g^{2}} ds$$

$$= \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{(1-\cos s)^{2}}{g^{2}} ds$$

$$= \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{(2\sin^{2} s)^{2}}{s^{4}} ds$$

$$= \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{(3\sin^{4} s)^{2}}{s^{4}} ds$$

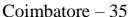
$$= \frac{2}{\pi} \int_{-\infty}^{\infty} (\frac{3\sin^{4} s}{s^{4}})^{2} ds$$

$$= \frac{3}{\pi} \int_{-\infty}^{\infty} (\frac{3\sin^{4} s}{s^{4}})^{2} ds$$

$$= \frac{3}{\pi}$$



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Floblems on 
$$FT$$
:

AJ. Find the Focuses teamsform of  $b(x)$   $f$ 

AJ. Find the Focuses teamsform of  $b(x)$   $f$ 
 $b(x) = \frac{1}{2}$ ;  $|x| \times a$ . Deduce that is  $\frac{\sin t}{2} dt = \frac{\pi}{2}$ 

Soln:

we already tend  $F(c) = \frac{2}{\pi} \left( \frac{3\pi n \, as}{c} \right)$ 
 $\frac{TFT}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(c) e^{-ic\pi} \, dc$ 



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SINE AND COSINE TRANSFORM



Problems on Self neuprocal function:

Show that the function = 2/2 gs Self- 918 Esprecas under fourier transform.

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} e^{iSx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} + iSx dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} + iSx dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} (x^2 - x^2 + x^2) dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} (x - is)^2 + s^2 dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} (x - is)^2 + s^2 dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$= \frac{1}{\sqrt{2\pi}}$$



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Hence

The foweign transform of 
$$e^{-\frac{x^2}{2}}$$
 is  $e^{-\frac{x^2}{2}}$ . Hence  $e^{-\frac{x^2}{2}}$  so self respectat under F.T.

At the find the familes transform of fine = 0

2]. Find the fowerer transform of  $f(x) = e^{-\alpha^2 x e^2}$ .
Solp.:

$$F(S) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{i^2}{2}} \frac{d^2}{2^2} e^{-\frac{i^2}{2}} \frac{d^2}{2^2} e^{-\frac{i^2}{2}} e^{-\frac{i^2}$$

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Put 
$$t = a \times -\frac{is}{sa}$$
  $x = -\infty \Rightarrow t = -\infty$ 

$$dt = a dx$$

$$dx = \frac{dt}{a}$$

$$= \frac{e^{-\frac{s^2}{4a^2}}}{\sqrt{s\pi}} = e^{-\frac{t^2}{a}} dt$$

$$= \frac{e^{-\frac{s^2}{4a^2}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}}} = \frac{e^{-\frac{t^2}{a\sqrt{s\pi}}}}}{\sqrt{a\sqrt{s\pi}}}}$$