SNS COLLEGE OF TECHNOLOGY
Coimbatore-35
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECE351 – IMAGE PROCESSING AND COMPUTER VISION

III B.E. ECE / V SEMESTER

UNIT 1 – DIGITAL IMAGE FUNDAMENTALS AND TRANSFORMS

TOPIC – IMAGE TRANSFORMS-PROPERTIES OF 2D DFT

IMAGE TRANSFORMS

Transformation

Transformation is a function. A function that maps one set to another set after performing some operations.

Digital Image Processing system

in digital image processing, we will develop a system that whose input would be an image and output would be an image too. And the system would perform some processing on the input image and gives its output as an processed image. It is shown below.

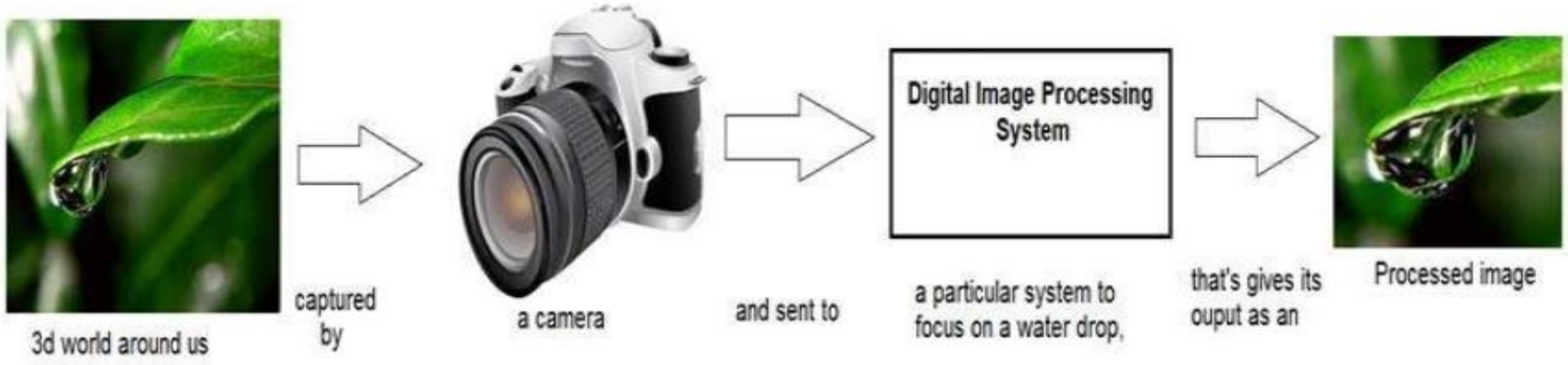


IMAGE TRANSFORMS

Consider this equation

$G(x,y) = T\{ f(x,y) \}$

In this equation,

$F(x,y)$ = input image on which transformation function has to be applied.

$G(x,y)$ = the output image or processed image.

T is the transformation function.

This relation between input image and the processed output image can also be represented as.

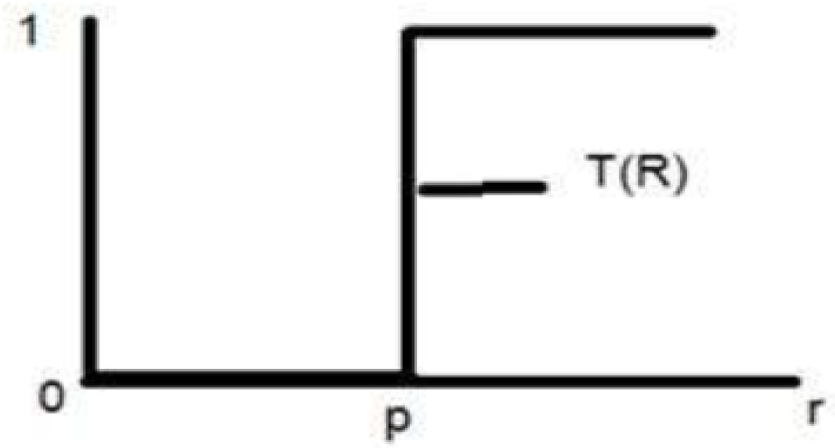
$s = T(r)$

where r is actually the pixel value or gray level intensity of $f(x,y)$ at any point. And s is the pixel value or gray level intensity of $g(x,y)$ at any point.

IMAGE TRANSFORMS

Examples

Consider this transformation function.



Lets take the point r to be 256, and the point p to be 127. Consider this image to be a one bpp image. That means we have only two levels of intensities that are 0 and 1. So in this case the transformation shown by the graph can be explained as.

All the pixel intensity values that are below 127 (point p) are 0, means black. And all the pixel intensity values that are greater then 127, are 1, that means white. But at the exact point of 127, there is a sudden change in transmission, so we cannot tell that at that exact point, the value would be 0 or 1.

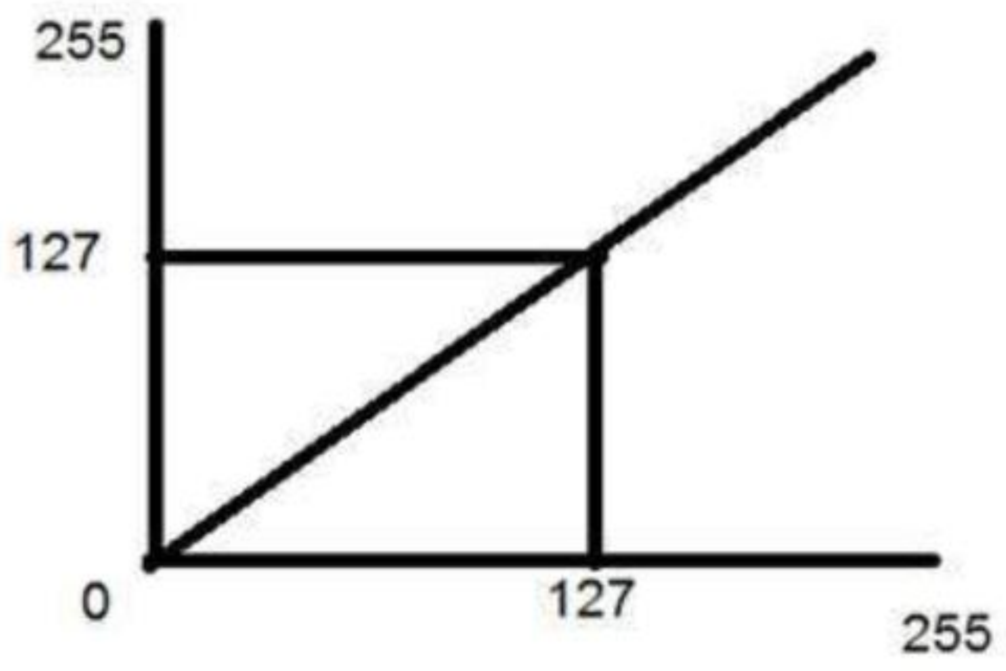
IMAGE TRANSFORMS

Mathematically this transformation function can be denoted as:

$$g(x,y) = \begin{cases} 0 & f(x,y) < 127 \\ 1 & f(x,y) > 127 \end{cases}$$

IMAGE TRANSFORMS

Consider another transformation like this



Now if you will look at this particular graph, you will see a straight transition line between input image and output image.

It shows that for each pixel or intensity value of input image, there is a same intensity value of output image. That means the output image is exact replica of the input image.

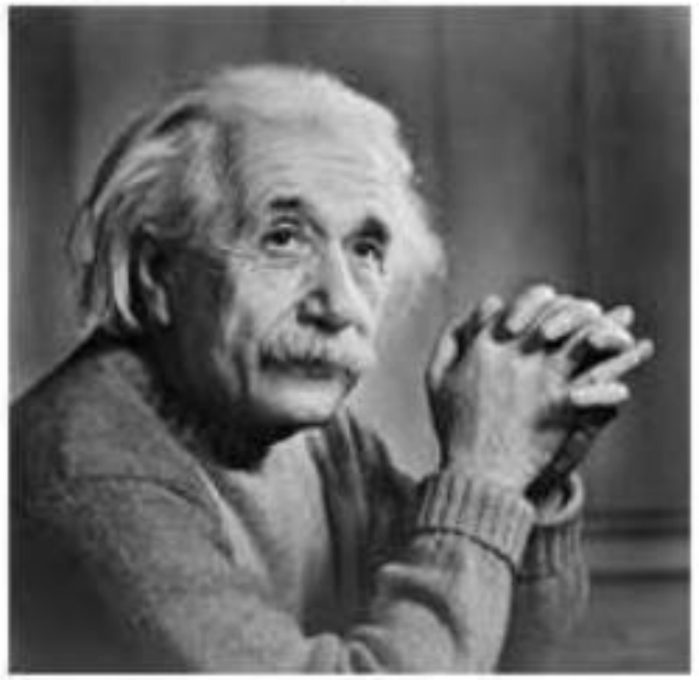
IMAGE TRANSFORMS

It can be mathematically represented as:

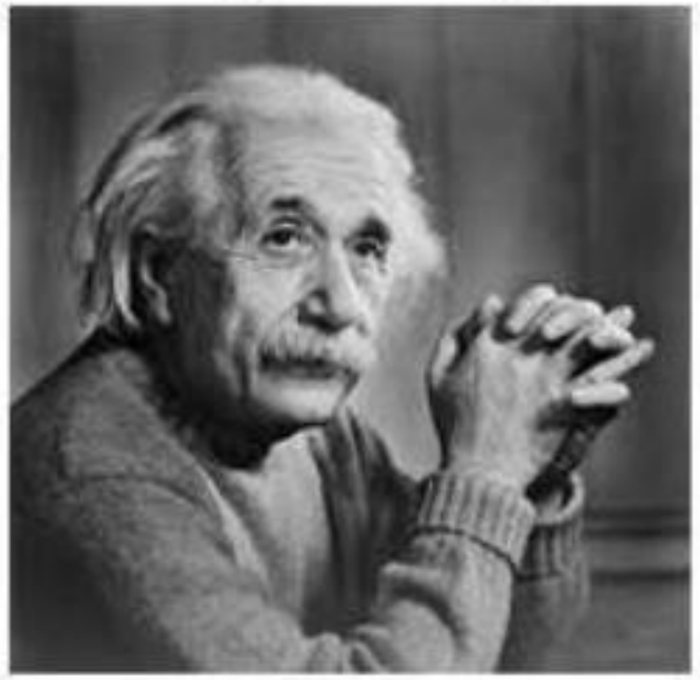
$g(x,y) = f(x,y)$

the input and output image would be in this case are shown below.

Input image



Output image



2D DFT

Forward 2D discrete Fourier Transformation:

Let we have an Image of size MxN then F(u,v) is the F T of image f(x,y)

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Where variable u = 0, 1, 2, ….., M-1 and v = 0, 1, 2, ….., N-1

Inverse (Backward) Fourier Transformation :

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Where variable x = 0, 1, 2, ….., M-1 and y = 0, 1, 2, ….., N-1

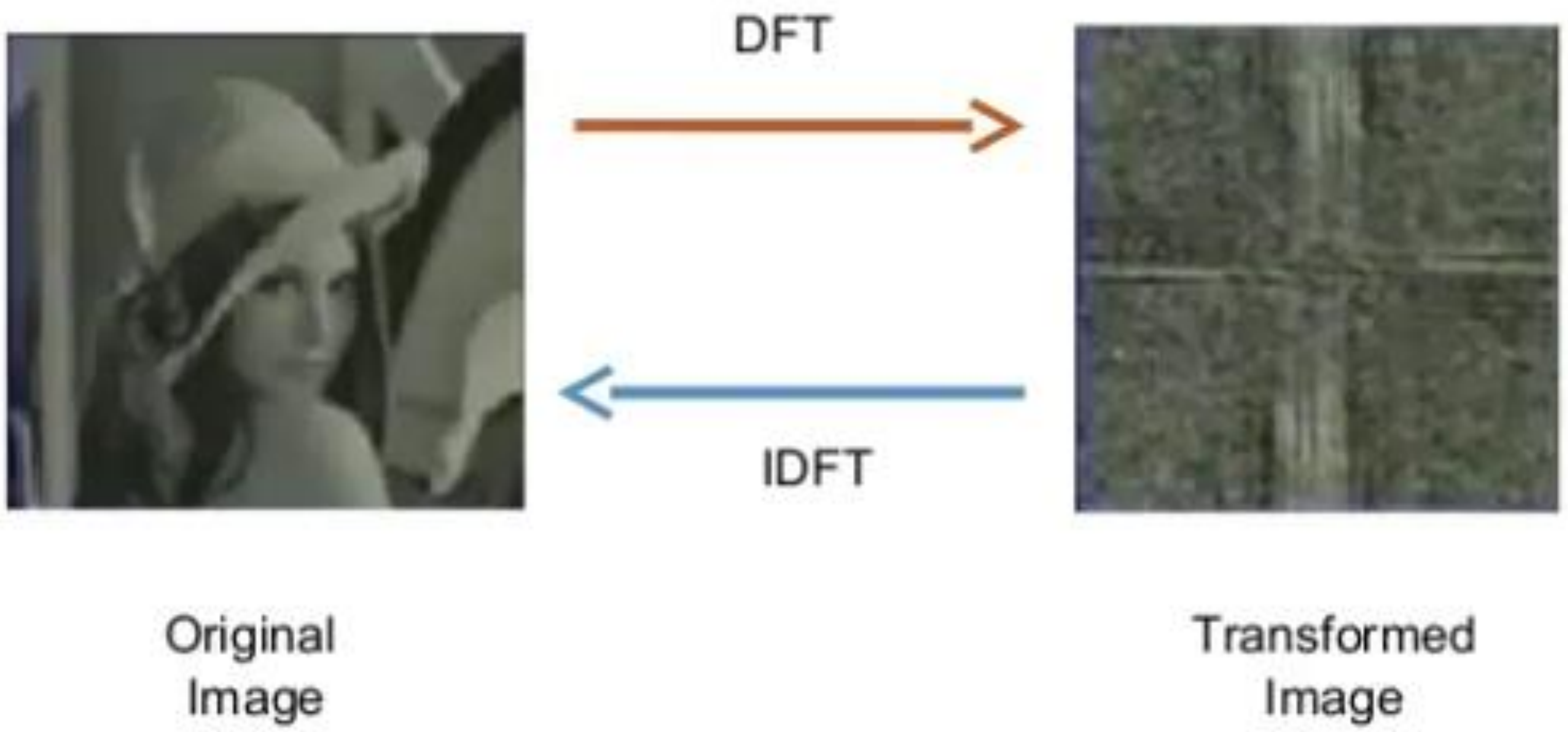
2D DFT

- For a square image i.e. M = N and the Fourier Transformation Pair is as follows

$$F(u,v) = \frac{1}{N} \sum_{x,y=0}^{N-1} \sum f(x,y) e^{-\frac{j2\pi}{N}(ux+vy)}$$

$$f(x,y) = \frac{1}{N} \sum_{u,v=0}^{N-1} \sum F(u,v) e^{\frac{j2\pi}{N}(ux+vy)}$$

DISCRETE FT RESULT



PROPERTIES

- Seperability
- Translation
- Periodicity
- Conjugate
- Rotation
- Distributive
- Scaling
- Convolution
- Corelation

PROPERTIES

Seperability

The separbility property says that we can do 2D Fourier transformation as two 1 D Fourier Transformation

$$F(u,v) = \frac{1}{N} \sum_{x,y=0}^{N-1} \sum f(x,y) e^{-\frac{j2\pi}{N}(ux+vy)}$$

Inverse Fourier Transform

$$F(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} e^{-j\frac{2\pi}{N}ux} \; N.\frac{1}{N} \sum_{y=0}^{N-1} f(x,y) e^{-\frac{j2\pi}{N}vy}$$

X represent row of image so x is fixed

$$F(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} e^{-j\frac{2\pi}{N}ux} \; N.F(x,v)$$

Fourier Transformation along row

$$F(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} F(x,v) \, e^{-j\frac{2\pi}{N}ux}$$

PROPERTIES

Seperability Cont…

2D Inverse Fourier transformation can also be viewed as two 1 D Inverse Fourier Transformation

$$f(x,y) = \frac{1}{N}\sum_{u,v=0}^{N-1}\sum F(u,v)e^{\frac{j2\pi}{N}(ux+vy)}$$

$$f(x,y) = \frac{1}{N}\sum_{u=0}^{N-1} e^{j\frac{2\pi}{N}ux} \; .N.\frac{1}{N}\sum_{v=0}^{N-1} F(u,v)e^{\frac{j2\pi}{N}vy}$$

IDFT along rows

$$f(x,y) = \frac{1}{N}\sum_{u=0}^{N-1} N\;.f(u,y)e^{j\frac{2\pi}{N}ux}$$

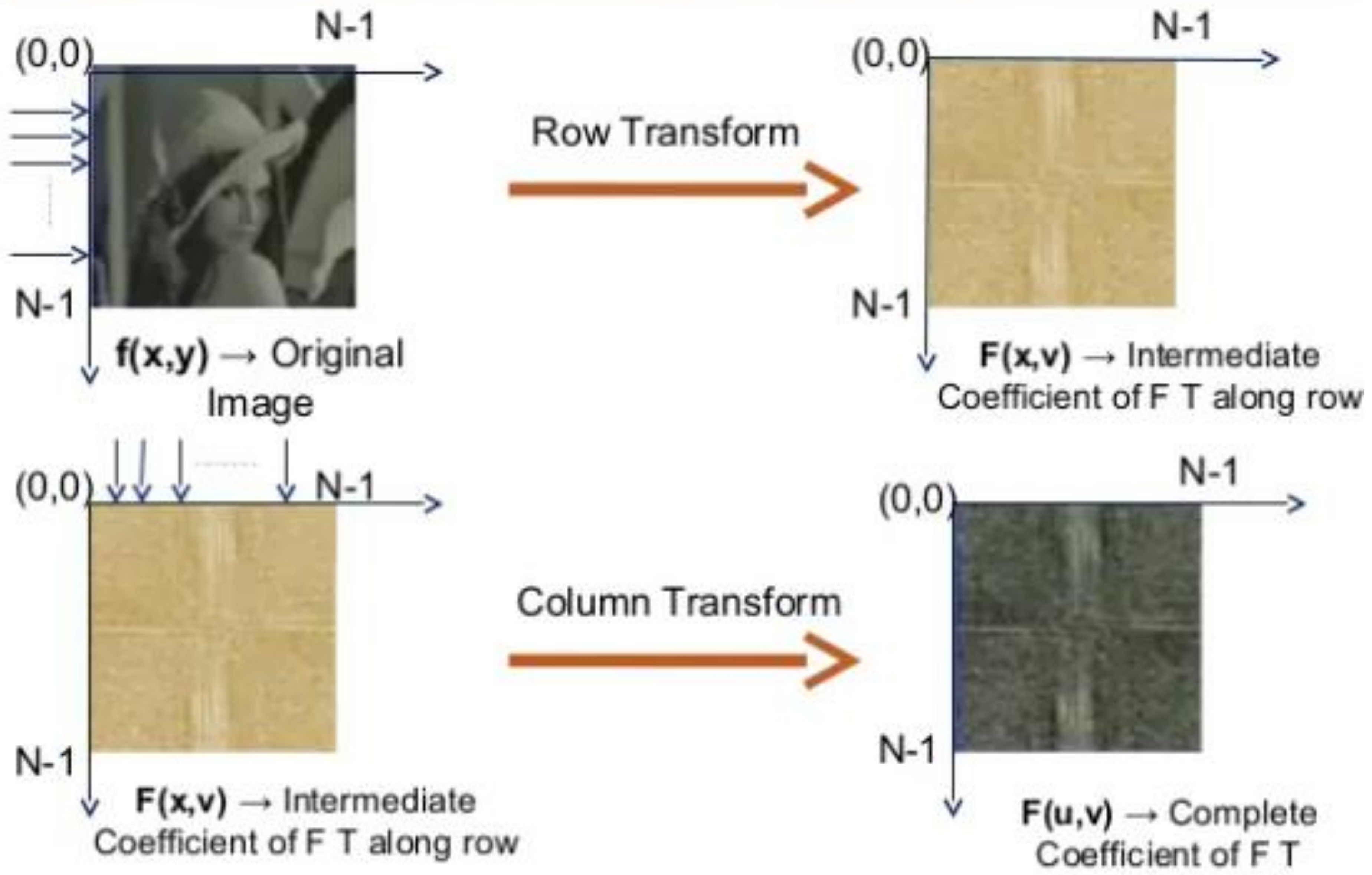
IDFT along columns

Advantage of Seperability:

Operation become much simpler and less time complexity

PROPERTIES

Seperability Concept



PROPERTIES

Translation

Translation of x and y by x_0 and y_0 respectively.

$$f(x,y) \xrightarrow{(x_0,\,y_0)} f(x-x_0, y-y_0)$$

Fourier Transform

$$F_t(u,v) = \frac{1}{N}\sum\sum f(x-x_0, y-y_0)e^{-\frac{j2\pi}{N}(u(x-x_0)+v(y-y_0))}$$

$$F_t(u,v) = \frac{1}{N}\sum\sum f(x-x_0, y-y_0)e^{-\frac{j2\pi}{N}(ux+vy)} . e^{-\frac{j2\pi}{N}(ux_0+vy_0)}$$

$$F_t(u,v) = F(u,v).e^{-j\frac{2\pi}{N}(ux_0+vy_0)}$$

Magnitude of FT remains same

Additional Phase

PROPERTIES

Translation Cont..

Inverse Fourier Transform

$$F(u-u_0,v-v_0)=f(x,y).e^{j\frac{2\pi}{N}(u_0x+v_0y)}$$

Here sift x_0, y_0 does not change Fourier spectrum but it add some phase sift diff

$$f(x,y).e^{j\frac{2\pi}{N}(u_0x+v_0y)} \Rightarrow F(u-u_0,v-v_0)$$

$$f(x-x_0,y-y_0) \Rightarrow F(u,v).e^{-j\frac{2\pi}{N}(ux_0+vy_0)}$$

Periodicity

Periodicity property says that the Discrete Fourier Transform and Inverse Discrete Fourier Transform are periodic with a period N

$$F(u,v) = F(u+N,v) = F(u,v+N) = F(u+N,v+N)$$

Proof:

$$F(u,v) = \frac{1}{N}\sum_{x,y=0}^{N-1}\sum f(x,y)e^{-\frac{j2\pi}{N}(ux+vy)}$$

$$F(u+N,v+N) = \frac{1}{N}\sum_{x,y=0}^{N-1}\sum f(x,y)e^{-\frac{j2\pi}{N}(ux+vy+Nx+Ny)}$$

$$F(u+N,v+N) = \frac{1}{N}\sum_{x,y=0}^{N-1}\sum f(x,y)e^{-\frac{j2\pi}{N}(ux+vy)}\,.\,e^{-j2\pi(x+y)}$$

$$F(u+N,v+N) = \frac{1}{N}\sum_{x,y=0}^{N-1}\sum f(x,y)e^{-\frac{j2\pi}{N}(ux+vy)}$$

$$F(u+N,v+N) = F(u,v)$$

So we can say that Discrete Fourier Transform is periodic with N

Conjugate

- If f(x,y) is a real valued function then

$$F(u,v) = F^* (-u, -v)$$

- Where F* indicate it complex conjugate
- Now Fourier Spectrum

$$|F(u,v)| = |F(-u,-v)|$$

- This property help to visualize Fourier Spectrum

Rotation

- Let $x = r\cos\theta$ and $y = \sin\theta$
- $u = w\cos\phi$ and $v = \sin\phi$
- Then we have

$$f(x,y) = f(r,\theta) \quad \text{in Spatial Domain}$$

$$F(u,v) = F(w, \phi) \quad \text{in Frequency Domain}$$

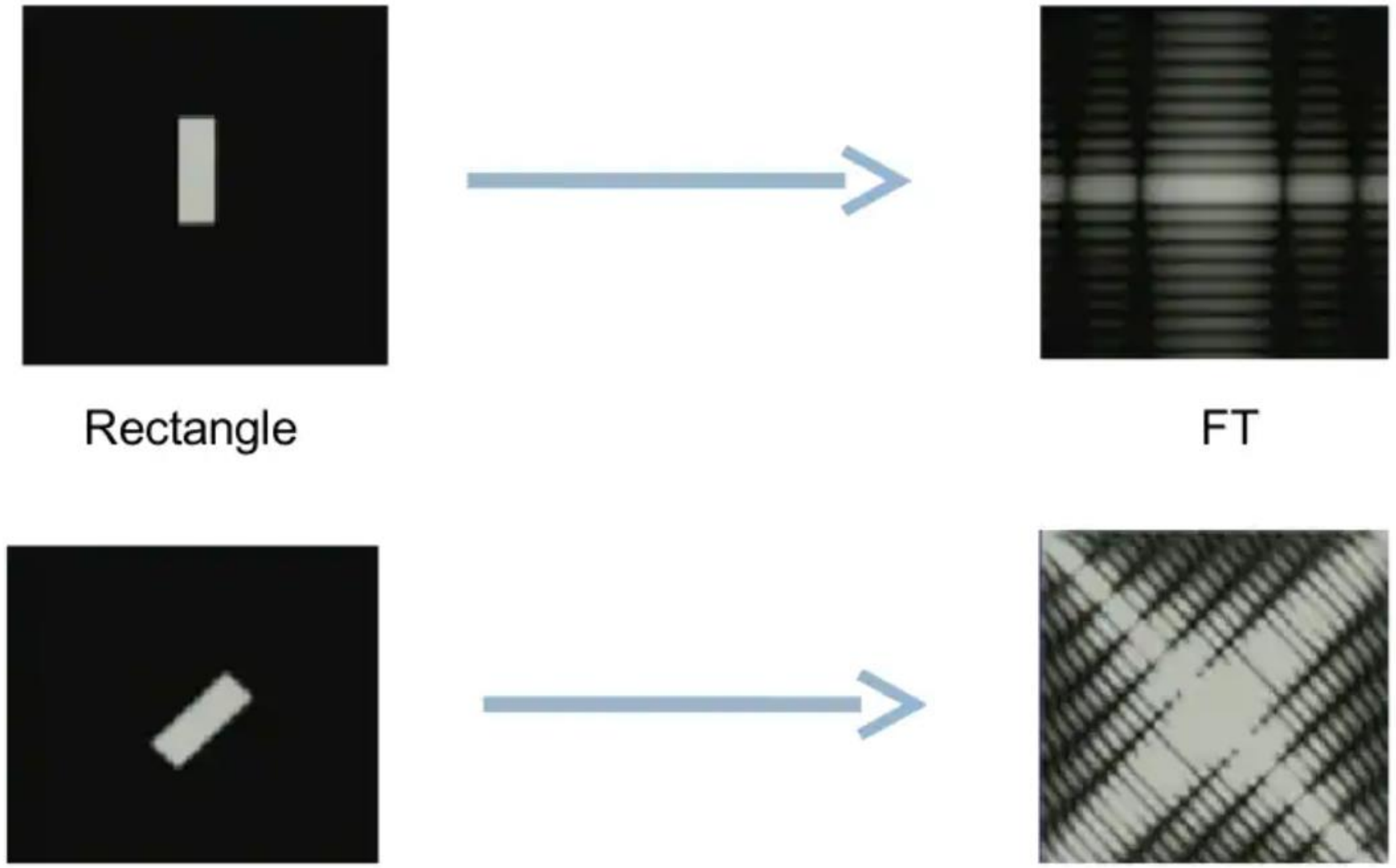
- Now Rotated Image is $f(r, \theta + \theta_0)$ and

$$f(r, \theta + \theta_0) \leftrightarrow F(w, \phi + \phi_0)$$

- $F(w, \phi + \phi_0)$ is F T of Rotated image

PROPERTIES

Rotation Concept



PROPERTIES

Distributivity

- DFT is distributive over addition but not on multiplication

$$\Im\{f_1(x,y) + f_2(x,y)\} = \Im\{f_1(x,y)\} + \Im\{f_2(x,y)\}$$

$$\Im\{f_1(x,y) \cdot f_2(x,y)\} \neq \Im\{f_1(x,y)\} \cdot \Im\{f_2(x,y)\}$$

PROPERTIES

Scaling

- If a and b are two scaling quantity then

$$a\, f(x,y) \leftrightarrow a\, F(u,v)$$

- If f(x,y) is multiplied by scalar quantity a then its F T is also multiplied by same scalar quantity
- Scaling Individual dimension

$$f(ax, by) \Leftrightarrow \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

PROPERTIES

Correlation & Correlation

- **Convolution:**

$$f(x).g(x) \Leftrightarrow F(u) * G(u)$$

$$F(u).G(u) \Leftrightarrow f(x) * g(x)$$

- Convolution in spatial domain is equivalent to multiplication in frequency domain and vice versa
- **Correlatio** $f(x,y) \odot g(x,y) \Leftrightarrow F^*(u,v).G(u,v)$

$$f^*(x,y).g(x,y) \Leftrightarrow F(u,v) \odot G(u,v)$$

- Where f* and F* indicate conjugates of f and F

Thank you!