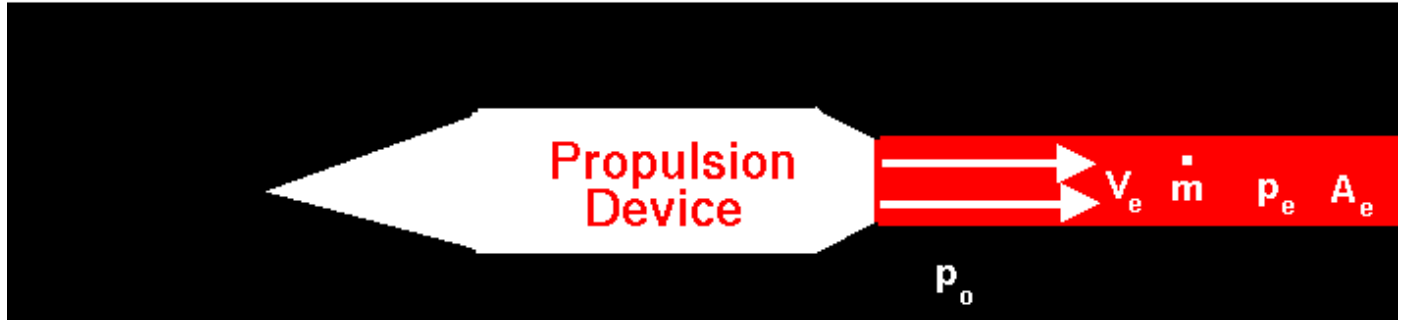




Specific Impulse

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Rocket Thrust Equation $F = \dot{m} V_e + (p_e - p_o) A_e$

where p = pressure, V = velocity, A = area, \dot{m} = mass flow rate, F = thrust

Define: Equivalent Velocity: $V_{eq} = V_e + \frac{(p_e - p_o) A_e}{\dot{m}}$ $F = \dot{m} V_{eq}$

Define: Total Impulse: $I = F \Delta t = \int F dt = \int \dot{m} V_{eq} dt = m V_{eq}$

Define: Specific Impulse: $I_{sp} = \frac{\text{Total Impulse}}{\text{Weight}} = \frac{I}{m g_o} = \frac{V_{eq}}{g_o}$ **units = sec**

$$I_{sp} = \frac{F}{\dot{m} g_o}$$

Thrust is the [force](#) which moves a rocket through the air. Thrust is generated by the rocket engine through the [reaction](#) of accelerating a mass of gas. The gas is accelerated to the rear and the rocket is accelerated in the opposite direction. To accelerate the gas, we need some kind of [propulsion system](#). We will discuss the details of various propulsion systems on some other pages. For right now, let us just think of the propulsion system as some machine which accelerates a gas.

From Newton's [second law](#) of motion, we can define a force to be the change in momentum of an object with a change in time. **Momentum** is the object's mass times the velocity. When dealing with a gas, the basic [thrust equation](#) is given as:

$$F = \dot{m} V_e - \dot{m}_0 V_0 + (p_e - p_0) A_e$$

Thrust F is equal to the exit [mass flow rate](#) \dot{m} times the exit velocity V_e minus the free stream mass flow rate \dot{m}_0 times the free stream velocity V_0 plus the pressure difference across the engine $p_e - p_0$ times the engine area A_e .

For [liquid](#) or [solid](#) rocket engines, the propellants, fuel and oxidizer, are carried on board. There is no free stream air brought into the propulsion system, so the [thrust equation](#) simplifies to:

$$F = \dot{m} V_e + (p_e - p_0) A_e$$

where we have dropped the exit designation on the mass flow rate.

Using algebra, let us divide by \dot{m} :

$$F / \dot{m} = V_e + (p_e - p_0) A_e / \dot{m}$$

We define a new velocity called the **equivalent velocity V_{eq}** to be the velocity on the right hand side of the above equation:

$$V_{eq} = V_e + (p_e - p_0) * A_e / \dot{m}$$

Then the rocket thrust equation becomes:

$$F = \dot{m} * V_{eq}$$

The **total impulse (I)** of a rocket is defined as the average thrust times the total time of firing. On the slide we show the total time as " Δt ". (Δ is the Greek symbol that looks like a triangle):

$$I = F * \Delta t$$

Since the thrust may change with time, we can also define an integral equation for the total impulse. Using the symbol (\int) for the integral, we have:

$$I = \int F dt$$

Substituting the equation for thrust given above:

$$I = \int (\dot{m} * V_{eq}) dt$$

Remember that **\dot{m}** is the mass flow rate; it is the amount of exhaust mass per time that comes out of the rocket. Assuming the equivalent velocity remains constant with time, we can integrate the equation to get:

$$I = m * V_{eq}$$

where **m** is the total mass of the propellant. We can divide this equation by the weight of the propellants to define the **specific impulse**. The word "[specific](#)" just means "divided by weight". The specific impulse **Isp** is given by:

$$I_{sp} = V_{eq} / g_0$$

where g_0 is the gravitational acceleration constant (32.2 ft/sec² in English units, 9.8 m/sec² in metric units). Now, if we substitute for the equivalent velocity in terms of the thrust:

$$I_{sp} = F / (\dot{m} * g_0)$$

Mathematically, the Isp is a [ratio](#) of the thrust produced to the weight flow of the propellants. A quick check of the units for Isp shows that:

$$I_{sp} = \text{m/sec} / \text{m/sec}^2 = \text{sec}$$

Why are we interested in specific impulse? First, it gives us a quick way to determine the thrust of a rocket, if we know the weight flow rate through the nozzle. Second, it is an indication of engine efficiency. Two different rocket engines have different values of specific impulse. The engine with the higher value of specific impulse is more efficient because it produces more thrust for the same amount of propellant. Third, it simplifies our mathematical analysis of rocket thermodynamics. The units of specific impulse are the same whether we use English units or metric units. Fourth, it gives us an easy way to "size" an engine during preliminary analysis. The result of our thermodynamic analysis is a certain value of specific impulse. The rocket weight will define the required value of thrust. Dividing the thrust required by the specific impulse will tell us how much weight flow of propellants our engine must produce. This information determines the physical size of the engine.

There is a similar efficiency parameter called the [specific thrust](#) which is used to characterize turbine engine performance.
