

Minterms: [product consisting of all variables]

- (i) $P\bar{A}Q, \bar{T}P\bar{A}Q, P\bar{A}TQ, \bar{T}P\bar{A}TQ$ are minterms in P, Q .
- (ii) $P\bar{A}Q\bar{A}R, P\bar{A}TQ\bar{A}R, \bar{T}P\bar{A}Q\bar{A}R, P\bar{A}Q\bar{A}T\bar{R}, \bar{T}P\bar{A}Q\bar{A}T\bar{R}$ are minterms in P, Q and R .

Maxterms: [sum consisting of all variables]

- (i) $P\bar{V}Q, \bar{T}P\bar{V}Q, P\bar{V}TQ, \bar{T}P\bar{V}TQ$ are maxterms in P, Q .
- (ii) $P\bar{V}Q\bar{V}R, \bar{V}TQ\bar{V}R, \bar{T}P\bar{V}Q\bar{V}R, P\bar{V}Q\bar{V}T\bar{R}, \bar{T}P\bar{V}Q\bar{V}T\bar{R}$ are maxterms in P, Q, R .

PDNF principal disjunctive normal form
 The PDNF of a given formula is an equivalent formula which consisting of disjunction of minterms only. i.e., $PDNF = (minterms) \vee (minterms) \vee \dots \vee (minterms)$

PCNF principal conjunctive normal form
 The PCNF of a given formula is an equivalent formula which consisting of conjunction of maxterms only. i.e., $PCNF = (maxterms) \wedge (maxterms) \wedge \dots \wedge (maxterms)$

Obtain the PDNF of

- i) $\bar{T}P\bar{V}Q$
- ii) $(P\bar{A}Q) \vee (\bar{T}P\bar{A}R) \vee (Q\bar{A}R)$
- vi) $\bar{T}P\bar{V}Q \Leftrightarrow (\bar{T}P\bar{A}T) \vee (Q\bar{A}T)$
 $\Leftrightarrow (\bar{T}P\bar{A}(Q\bar{V}TQ)) \vee (Q\bar{A}(P\bar{V}T\bar{P}))$
 $\Leftrightarrow (\bar{T}P\bar{A}Q) \vee (\bar{T}P\bar{A}TQ) \vee (Q\bar{A}P) \vee (Q\bar{A}T\bar{P})$ Distributive laws
 $\Leftrightarrow (\bar{T}P\bar{A}Q) \vee (\bar{T}P\bar{A}TQ) \vee (P\bar{A}Q) \vee (Q\bar{A}T\bar{P})$ Commutative law
- ii) $(P\bar{A}Q) \vee (\bar{T}P\bar{A}R) \vee (Q\bar{A}R)$
 $\Leftrightarrow (P\bar{A}Q\bar{A}T) \vee (\bar{T}P\bar{A}R\bar{A}T) \vee (Q\bar{A}R\bar{A}T)$
 $\Leftrightarrow (P\bar{A}Q\bar{A}R\bar{V}T\bar{R}) \vee (\bar{T}P\bar{A}R\bar{A}(Q\bar{V}TQ)) \vee (Q\bar{A}R\bar{A}(P\bar{V}T\bar{P}))$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \\ \vee (\neg P \wedge R \wedge \neg Q) \vee (Q \wedge R \wedge P) \vee (Q \wedge R \wedge \neg P)$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \\ \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge \neg R)$$

iii) obtain the PDNF of $P \rightarrow ((P \rightarrow Q) \wedge \neg (\neg Q \vee \neg P))$

$$P \rightarrow ((P \rightarrow Q) \wedge \neg (\neg Q \vee \neg P))$$

$$\Leftrightarrow \neg P \vee ((P \rightarrow Q) \wedge \neg (\neg Q \vee \neg P))$$

$$\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge \neg (\neg Q \vee \neg P))$$

$$\Leftrightarrow \neg P \vee ((\neg P \vee Q) \wedge (Q \wedge P))$$

$$\Leftrightarrow \neg P \vee [(P \wedge (Q \wedge P)) \vee (Q \wedge (\neg P \wedge P))]$$

$$\Leftrightarrow \neg P \vee [(P \wedge (P \wedge Q)) \vee (Q \wedge (\neg P \wedge P))]$$

$$\Leftrightarrow \neg P \vee [(P \wedge P \wedge Q) \vee (Q \wedge \neg P \wedge P)]$$

$$\Leftrightarrow \neg P \vee [P \wedge Q \vee (Q \wedge \neg P)]$$

$$\Leftrightarrow \neg P \vee [P \wedge Q]$$

$$\Leftrightarrow \neg P \vee (P \wedge Q)$$

$$\Leftrightarrow (\neg P \wedge \neg Q) \vee (P \wedge Q)$$

$$\Leftrightarrow (\neg P \wedge (Q \vee \neg Q)) \vee (P \wedge Q)$$

$$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$$

obtain the PCNF of ^{PDNF} i). $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ and

PCNF of ii). $P \rightarrow (\neg P \wedge (Q \rightarrow P))$

$$i). (\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$$

$$\text{PCNF} \Leftrightarrow (P \vee R) \wedge [(Q \rightarrow P) \wedge (P \rightarrow Q)]$$

$$\Leftrightarrow (P \vee R) \wedge [(\neg Q \vee P) \wedge (P \vee \neg Q)]$$

$$\Leftrightarrow (P \vee R \vee (Q \wedge \neg Q)) \wedge (\neg Q \vee P \vee (R \wedge \neg R)) \wedge (P \vee \neg Q \vee (R \wedge \neg R))$$

$$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee R) \wedge (P \vee \neg Q \vee R)$$

$$\wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R)$$

$$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee T \vee R) \wedge (P \vee T \vee Q \vee R) \wedge \\ (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee T \vee R)$$

PDFN :

TS: conjunction of the remaining maxterms.

$$TS: (P \vee Q \vee T \vee R) \wedge (\neg P \vee T \vee Q \vee R) \wedge (\neg P \vee T \vee Q \vee T \vee R)$$

$$T(TS) : \neg(P \vee Q \vee T \vee R) \vee \neg(\neg P \vee T \vee Q \vee R) \vee \neg(\neg P \vee T \vee Q \vee T \vee R)$$

$$\Leftrightarrow (\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R) \text{ which} \\ \text{is the required PDFN.}$$

ii). $P \rightarrow (\neg P \wedge (Q \rightarrow P))$

PCNF :

$$\Leftrightarrow \neg P \vee [\neg P \wedge (Q \rightarrow P)] \text{ material implication Rule}$$

$$\Leftrightarrow \neg P \vee [\neg P \wedge (\neg Q \vee P)] \text{ material implication Rule}$$

$$\Leftrightarrow (\neg P \vee \neg P) \wedge (\neg P \vee (\neg Q \vee P)) \text{ Distributive law}$$

$$\Leftrightarrow \neg P \wedge (\neg P \vee (P \vee \neg Q)) \text{ Idempotent law}$$

$$\Leftrightarrow \neg P \wedge [(P \vee P) \vee \neg Q] \text{ Commutative law}$$

$$\Leftrightarrow \neg P \wedge [(P \vee P) \vee \neg Q] \text{ Associative law}$$

$$\Leftrightarrow \neg P \wedge (T \vee \neg Q)$$

$$\Leftrightarrow \neg P \wedge T$$

$$\Leftrightarrow \neg P$$

$$\Leftrightarrow \neg P \vee F$$

$$\Leftrightarrow [\neg P \vee (Q \wedge \neg Q)]$$

$$\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee \neg Q) \text{ which is the required} \\ \text{PCNF.}$$

iii). $(Q \rightarrow P) \wedge (\neg P \wedge Q)$

$$\Leftrightarrow (\neg Q \vee P) \wedge (\neg P \wedge Q)$$

$$\Leftrightarrow (\neg Q \vee P) \wedge [(\neg P \wedge Q) \vee F]$$

$$\Leftrightarrow (\neg Q \vee P) \wedge [(\neg P \wedge Q) \vee (P \wedge \neg P)]$$

$$\Leftrightarrow (\neg Q \vee P) \wedge [(\neg P \wedge Q) \vee P] \wedge [(\neg P \wedge Q) \vee \neg P]$$

$$\Leftrightarrow (TQVP) \wedge (TPVP) \wedge (QVP) \wedge (TPVTP) \wedge (QVTP)$$

$$\Leftrightarrow (TQVP) \wedge T \wedge (QVP) \wedge (TPV(QVTP))$$

$$\wedge \neg P$$

$$\Leftrightarrow (PV\bar{T}Q) \wedge (PVQ) \wedge (\bar{T}PV(TPVQ))$$

$$\Leftrightarrow (PV\bar{T}Q) \wedge (PVQ) \wedge ((\bar{T}PVTP) \vee Q)$$

$$\Leftrightarrow (PV\bar{T}Q) \wedge (PVQ) \wedge (TPVQ)$$

min \rightarrow few
max \rightarrow con tra

Truth Table (PDNF & PCNF)

~~PDNF~~:

J. Obtain PDNF & PCNF for the following using truth table

i). $P \wedge (Q \rightarrow R)$

ii). $P \rightarrow [(P \rightarrow Q) \wedge (TQVTP)]$

i). $P \wedge (Q \rightarrow R)$

P	Q	R	$Q \rightarrow R$	$P \wedge (Q \rightarrow R)$	min terms	max terms
T	T	T	T	<u>T</u>	$P \wedge Q \wedge R$	$\bar{P} \vee \bar{Q} \vee \bar{R}$
T	T	F	F	F*		
T	F	T	T	<u>T</u>	$P \wedge \bar{Q} \wedge R$	
T	F	F	T	<u>T</u>	$P \wedge \bar{Q} \wedge \bar{R}$	
F	T	T	T	F*		$P \vee \bar{Q} \vee \bar{R}$
F	T	F	F	F*		$P \vee Q \vee \bar{R}$
F	F	T	T	F*		$\bar{P} \vee Q \vee R$
F	F	F	T	F*		

PDNF: Disjunction of min terms

$$= (P \wedge Q \wedge R) \vee (P \wedge \bar{Q} \wedge R) \vee (P \wedge \bar{Q} \wedge \bar{R})$$

PCNF: Conjunction of max terms

$$= (\bar{P} \vee \bar{Q} \vee R) \wedge (\bar{P} \vee Q \vee \bar{R}) \wedge (P \vee \bar{Q} \vee \bar{R}) \wedge (P \vee Q \vee R)$$

$S \Leftrightarrow$

ii). $P \rightarrow [(P \rightarrow Q) \wedge (\neg Q \wedge TP)]$

P	Q	$P \rightarrow Q$	$\neg Q$	TP	$\neg Q \wedge TP$	$[(P \rightarrow Q) \wedge (\neg Q \wedge TP)]$	S	min terms	max terms
T	T	T	F	F	F	F	F		$TP\neg Q$
T	F	F	T	F	F	F	F		$TP\neg Q$
F	T	T	F	T	F	F	T	$TP\neg Q$	
F	F	T	T	T	T	T	T	$TP\neg Q$	

PDNF : Disjunction of minterms

$$= (TP \wedge Q) \vee (TP \wedge \neg Q)$$

PCNF : Conjunction of maxterms

$$= (TP \vee \neg Q) \wedge (TP \vee Q)$$

HW 11. $(TP \rightarrow R) \wedge (Q \leftrightarrow P)$ using truth table.