

### Tautology:

A statement formula which is true always irrespective of the truth values of the individual variables is called a tautology.

Eg:

$P \vee \neg P$  is a tautology.

### Contradiction:

A statement formula which is always false is called a contradiction (or) absurdity.

Eg:

$P \wedge \neg P$  is a contradiction.

### Contingency:

A statement formula which is neither tautology nor contradiction is called contingency.

Eg:

$P \leftrightarrow Q$  is contingency.

1. Show that  $[P \wedge (P \wedge Q)] \rightarrow Q$  is a tautology.

P	Q	$\neg P$	$P \wedge Q$	$\neg P \wedge (P \wedge Q)$	$[P \wedge (P \wedge Q)] \rightarrow Q$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	F	F	T

Since all the entries in the resulting column is true, the given expression is a tautology.

2. Show that  $(P \wedge Q) \wedge \neg(P \vee Q)$  is a contradiction.

P	Q	$P \wedge Q$	$P \vee Q$	$\neg(P \vee Q)$	$(P \wedge Q) \wedge \neg(P \vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since all the entries in the resulting column are F, the given expression is a contradiction.

3. Identify the given expression

		$\neg(P \vee Q)$		$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$			
P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \vee \neg Q$	$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T

Since the entries in the resulting column are T as well as F, the given expression is a contingency.

Examine whether  $[(P \vee Q) \rightarrow \neg Q] \leftrightarrow [\neg P \rightarrow \neg(P \vee Q)]$  is a tautology?

Equivalence:

Two statement formulas P and Q are equivalent iff  $P \leftrightarrow Q$  is a tautology. It is denoted by  $P \Leftrightarrow Q$ .

Show that the propositions are logically equivalent for the following.

(i)  $P \rightarrow Q \Leftrightarrow \neg P \vee Q$

(ii)  $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

(i)  $P \rightarrow Q \Leftrightarrow \neg P \vee Q$

P	Q	$P \rightarrow Q$	$\neg P$	$\neg P \vee Q$	$P \rightarrow Q \Leftrightarrow \neg P \vee Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Since  $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$  is a tautology.

Hence  $(P \rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

$$(ii) P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$$

P	Q	R	$Q \vee R$	$P \rightarrow (Q \vee R)$	$P \rightarrow Q$	$P \rightarrow R$	$(P \rightarrow Q) \vee (P \rightarrow R)$	$P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Here  $[P \rightarrow (Q \vee R)] \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$  is a tautology.

$$\text{Hence } P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$$