Discrete Mathematics

Unit - I

Logics and Proofs

* Propositional logic
* Proportional Equivalences
* predicates and Quantifiers
* Nested Quantifiers
* Rules of inference
* Introduction to proofs
* Proof methods and Strategy

Logics and proofs

Proposition:

A proposition (or) Statement is a declarative sentence which is either true or false but not both.

Egs:

1. Newdelhi is the capital of India. [True]
2. Chennai is in England. [False]

Non proposition:

Questions, Exclamations and commands are non proposition.

Egs:

1. What is the height of himalaya. [Interrogative sentence]
2. Obey my orders. [Command]
3. $x+5=3$ [Neither true nor false]

Types of proposition:

Simple proposition:

A declarative sentence which cannot be further split up into simple sentences are called primary (or) atomic (or) simple statements.

Eg: Nandhini is a lawyer.

Compound proposition:

A statement which contain one or more primary statements and some connectives are called compound (or) molecular (or) composite statements.

Eg: Lotus is a flower and it is the national flower of India.

Connectives:

Connective is an operation which is used to connect two or more than two statements. ~~are known~~

There are five basic connectives.

S.No.	Logical connectives	Name	Symbols	Type of operator
1.	NOT	Negation (or) Denial	$\neg$ (or) $\sim$	Unary
	Eg: P: Chennai is a city $\neg$P: Chennai is not a city.			
2.	AND	Conjunction	$\wedge$	Binary
	Eg: P: It is raining Q: I am getting cold $P \wedge Q$: It is raining and I am getting cold.			
3.	OR	Disjunction	$\vee$	Binary
	Eg: P: $3+5=8$ Q: $5<3$ $P \vee Q$: $3+5=8$ or $5<3$			
4.	If ... then	Conditional	$\rightarrow$	Binary
	Eg: P: There is a flood Q: The crop will be destroyed. $P \rightarrow Q$: If there is a flood then the crop will be destroyed.			
5.	If and only if	Biconditional	$\leftrightarrow$ (or) $\rightleftarrows$	Binary
	Eg: P: Students will come to college Q: Friday is a working day $P \leftrightarrow Q$: Students will come to college if and only if Friday is a working day			

Truth table:

P	Q	¬P	P∧Q	P∨Q	P→Q	P↔Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Instead of T & F as 0 and 1, we can use

1. Using the statements

P : x is rich

Q : x is happy

write the following statements in symbolic form:

(a) x is poor

(b) x is poor but happy

(c) x is rich or unhappy

(d) x is neither rich nor happy

(e) x is poor or he is both rich and unhappy.

Soln.:

(a) $\neg A$

(b) $\neg A \wedge B$

(c) $\neg A \vee \neg B$

(d) $\neg A \wedge \neg B$

(e) $\neg A \vee (A \wedge \neg B)$

2. write the statements for the following symbolic form

P : It is hot day

Q : Temperature is 45°C.

(i) $\neg P$ (ii) $\neg(P \vee Q)$ (iii) $P \wedge Q$ (iv) $\neg(\neg P)$

(v) $\neg P \wedge \neg Q$ (vi) $\neg P \vee \neg Q$ (vii) $\neg(\neg P \vee \neg Q)$

Soln.:

(i) $\neg P \Rightarrow$ It is not hot day

(ii) $\neg(P \vee Q) \Rightarrow$ It is false that it is hot day or the temperature is 45°C.

(iii) $P \wedge Q \Rightarrow$ It is hot day and the temperature is 45°C

(iv) $\neg(\neg P) \Rightarrow$ It is hot day.

(v) $\neg P \wedge \neg Q \Rightarrow$ It is not hot day and the temperature is not 45°C. (or)

Neither it is hot day nor the temperature is 45°C.

(vi) $\neg P \vee \neg Q \Rightarrow$ It is not hot day or the temperature is not 45°C. (or)

~~Either~~ it is not hot day or the temp. is not 45°C

(vii) $\neg(\neg P \vee \neg Q) \Rightarrow$ It is false that it is not hot day or the temperature is not 45°C (or)

It is hot day ~~or~~ the temp. is 45°C.

3. Let P: Triangle ABC is an isosceles
Q: Triangle ABC is an equilateral
R: Triangle ABC is an equiangular.

Translate each of the following notation into a statement.

(i) $Q \to P$ (ii) $\neg P \to \neg Q$ (iii) $Q \leftrightarrow R$ (iv) $P \to \neg Q$

(v) $R \to P$ (vi) $(P \vee Q) \to R$ (vii) $(\neg P \wedge Q) \to \neg R$

Soln.:

(i) $Q \to P \Rightarrow$ If $\triangle ABC$ is an equilateral then $\triangle ABC$ is an isosceles.

(ii) $\neg P \to \neg Q$

$\Rightarrow$ If $\triangle ABC$ is not an isosceles then $\triangle ABC$ is not an equilateral.

(iii) $Q \leftrightarrow R \Rightarrow$ $\triangle ABC$ is an equilateral iff $\triangle ABC$ is an equiangular.

(iv) $P \to \neg Q \Rightarrow$ If $\triangle ABC$ is an isosceles then $\triangle ABC$ is not an equilateral.

(v) $R \to P \Rightarrow$ If $\triangle ABC$ is an equiangular then $\triangle ABC$ is an isosceles

(vi) $(P \vee Q) \to R \Rightarrow$ If either $\triangle ABC$ is an isosceles then $\triangle ABC$ is an equiangular.

(vii) $(\neg P \wedge Q) \to \neg R$

$\Rightarrow$ If $\triangle ABC$ is not an isosceles and equilateral then $\triangle ABC$ is not an equiangular.

4. Construct the truth table $\neg(P \vee Q) \vee (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \vee \neg Q$	$\neg(P \vee Q)$	$\neg(P \vee Q) \vee (\neg P \vee \neg Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	T	F	T
F	T	T	F	T	T	F	T
F	F	T	T	F	T	T	T

5]. Construct the truth table for the following:

(i). $(P \to Q) \wedge (Q \to P)$

(ii). $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$

(i) $(P \to Q) \wedge (Q \to P)$

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(ii) $\neg(P \wedge Q) \longleftrightarrow (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$	$\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

6]. How many rows are needed in the truth table of the given statement formula.

$$(P \to Q) \wedge (\neg R \vee S) \rightleftarrows T$$

Since the given statement formula consisting of P, Q, R, S, T.

Hence the truth table have 2^5 rows $= 32$ rows

7]. Negate the statement "for all real numbers x, if $x > 4$ then $x^2 > 16$"

For some x, if $x^2 \leq 16$, then $x \leq 4$.