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CHAPTER 3

FLUID FLOW THEORY (cont'd)

FLUID DYNAMICS

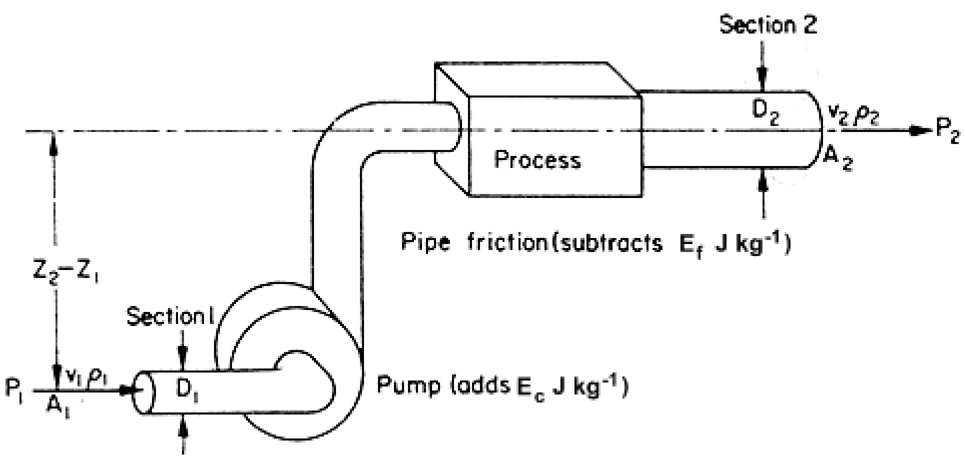
- Mass balance
- Energy balance
- Potential energy
- Kinetic energy
- Pressure energy
- Friction loss
- Mechanical energy
- Other effects
- Bernouilli's equation

In most processes fluids have to be moved so that the study of fluids in motion is important. Problems on the flow of fluids are solved by applying the principles of conservation of mass and energy. In any system, or in any part of any system, it must always be possible to write a mass balance and an energy balance. The motion of fluids can be described by writing appropriate mass and energy balances and these are the bases for the design of fluid handling equipment.

Mass Balance

Consider part of a flow system, such for example as that shown in **Fig. 3.3**.

This consists of a continuous pipe that changes its diameter, passing into and out of a unit of processing plant, which is represented by a tank. The processing equipment might be, for example, a pasteurizing heat exchanger. Also in the system is a pump to provide the energy to move the fluid.



Fiigure 3.3. Mass and energy balance in fluid flow

In the flow system of **Fig. 3.3** we can apply the law of conservation of mass to obtain a mass balance. Once the system is working steadily, and if there is no accumulation of fluid in any part the system, the quantity of fluid that goes in at section 1 must come out at section 2. If the area of the pipe at section 1 is A_1 , the velocity at this section, v_1 and the fluid density ρ_1, and if the corresponding values at section 2 are A_2, v_2, ρ_2, the mass balance can be expressed as

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad (3.4)$$

If the fluid is incompressible $\rho_1 = \rho_2$ so in this case

$$A_1 v_1 = A_2 v_2 \qquad (3.5)$$

Equation (3.5) is known as the **continuity equation** for liquids and is frequently used in solving flow problems. It can also be used in many cases of gas flow in which the change in pressure is very small compared with the system pressure, such as in many air-ducting systems, without any serious error.

EXAMPLE 3.4. Velocities of flow

Whole milk is flowing into a centrifuge through a full 5 cm diameter pipe at a velocity of 0.22 m s^{-1}, and in the centrifuge it is separated into cream of specific gravity 1.01 and skim milk of specific gravity 1.04. Calculate the velocities of flow of milk and of the cream if they are discharged through 2 cm diameter pipes. The specific gravity of whole milk of 1.035.

From eqn. (3.4):

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 + \rho_3 A_3 v_3$$

where suffixes 1, 2, 3 denote respectively raw milk, skim milk and cream. Also, since volumes will be conserved, the total leaving volumes will equal the total entering volume and so
$A_1 v_1 = A_2 v_2 + A_3 v_3$ and from this equation

$$v_2 = (A_1 v_1 - A_3 v_3)/A_2 \qquad (a)$$

This expression can be substituted for v_2 in the mass balance equation to give:

$$\rho_1 A_1 v_1 = \rho_2 A_2 (A_1 v_1 - A_3 v_3)/A_2 + \rho_3 A_3 v_3$$
$$\rho_1 A_1 v_1 = \rho_2 A_1 v_1 - \rho_2 A_3 v_3 + \rho_3 A_3 v_3.$$

$$\text{So } A_1 v_1 (\rho_1 - \rho_2) = A_3 v_3 (\rho_3 - \rho_2) \qquad (b)$$

From the known facts of the problem we have:

$$A_1 = (\pi/4) \times (0.05)^2 = 1.96 \times 10^{-3}\ \text{m}^2$$

$$A_2 = A_3 = (\pi/4) \times (0.02)^2 = 3.14 \times 10^{-4}\ \text{m}^2$$
$$v_1 = 0.22\ \text{m s}^{-1}$$
$$\rho_1 = 1.035 \times \rho_w,\ \rho_2 = 1.04 \times \rho_w,\ \rho_3 = 1.01 \times \rho_w$$

where ρ_w is the density of water.

Substituting these values in eqn. (b) above we obtain:

$$-1.96 \times 10^{-3} \times 0.22\ (0.005) = -3.14 \times 10^{-4} \times v_3 \times (0.03)$$

so $\qquad v_3 = 0.23\ \text{m s}^{-1}$

Also from eqn. (a) we then have, substituting 0.23 m s^{-1} for v_3,

$$v_2 = [(1.96 \times 10^{-3} \times 0.22) - (3.14 \times 10^{-4} \times 0.23)] / 3.14 \times 10^{-4}$$

$$= \underline{1.1\text{m s}^{-1}}$$

Energy Balance

In addition to the mass balance, the other important quantity we must consider in the analysis of fluid flow, is the energy balance. Referring again to **Fig. 3.3**, we shall consider the changes in the total energy of unit mass of fluid, one kilogram, between Section 1 and Section 2.

Firstly, there are the changes in the intrinsic energy of the fluid itself which include changes in:
(1) Potential energy.
(2) Kinetic energy.
(3) Pressure energy.

Secondly, there may be energy interchange with the surroundings including:
(4) Energy lost to the surroundings due to friction.
(5) Mechanical energy added by pumps.
(6) Heat energy in heating or cooling the fluid.

In the analysis of the energy balance, it must be remembered that energies are normally measured from a datum or reference level. Datum levels may be selected arbitrarily, but in most cases the choice of a convenient datum can be made readily with regard to the circumstances.

Potential energy

Fluid maintained above the datum level can perform work in returning to the datum level. The quantity of work it can perform is calculated from the product of the distance moved and the force resisting movement; in this case the force of gravity. This quantity of work is called the potential energy of the fluid.
Thus the potential energy of one kilogram of fluid at a height of Z (m) above its datum is given by E_p, where

$$E_p = Zg \quad \text{(J)}$$

Kinetic energy

Fluid that is in motion can perform work in coming to rest. This is equal to the work required to bring a body from rest up to the same velocity, which can be calculated from the basic equation

$$v^2 = 2as, \text{ therefore } s = v^2/2a,$$

where v (m s^{-1}) is the final velocity of the body, a (m s^{-2}) is the acceleration and s (m) is the distance the body has moved.

Also work done = $W = F \times s$, and from Newton's Second Law, for m kg of fluid

$$F = ma$$

and so $E_k = W = mas = mav^2/2a = mv^2/2$

The energy of motion, or kinetic energy, for 1 kg of fluid is therefore given by E_k where

$$E_k = v^2/2 \quad \text{(J)}.$$

Pressure energy

Fluids exert a pressure on their surroundings. If the volume of a fluid is decreased, the pressure exerts a force that must be overcome and so work must be done in compressing the fluid. Conversely, fluids under pressure can do work as the pressure is released. If the fluid is considered as being in a cylinder of cross-sectional area A (m^2) and a piston is moved a distance L (m) by the fluid against the pressure P (Pa) the work done is PAL joules. The quantity of the fluid performing this work is $AL\rho$ (kg). Therefore the pressure energy that can be obtained from one kg of fluid (that is the work that can be done by this kg of fluid) is given by E_r where

$$E_r = PAL / AL\rho$$
$$= P/\rho \quad \text{(J)}$$

Friction loss

When a fluid moves through a pipe or through fittings, it encounters frictional resistance and energy can only come from energy contained in the fluid and so frictional losses provide a drain on the energy resources of the fluid. The actual magnitude of the losses depends upon the nature of the flow and of the system through which the flow takes place. In the system of **Fig. 3.3**, let the energy lost by 1 kg fluid between section 1 and section 2, due to friction, be equal to E_f (J).

Mechanical energy

If there is a machine putting energy into the fluid stream, such as a pump as in the system of **Fig. 3.3**, the mechanical energy added by the pump per kg of fluid must be taken into account. Let the pump energy added to 1 kg fluid be E_c (J). In some cases a machine may extract energy from the fluid, such as in the case of a water turbine.

Other effects

Heat might be added or subtracted in heating or cooling processes, in which case the mechanical equivalent of this heat would require to be included in the balance. Compressibility terms might also occur, particularly with gases, but when dealing with low pressures only they can usually be ignored.

For the present let us assume that the only energy terms to be considered are E_p, E_k, E_r, E_f, E_c.

Bernouilli's Equation

We are now in a position to write the energy balance for the fluid between section 1 and section 2 of **Fig. 3.3**.

The total energy of one kg of fluid entering at section 1 is equal to the total energy of one kg of fluid leaving at section 2, less the energy added by the pump, plus friction energy lost in travelling between the two sections. Using the subscripts 1 and 2 to denote conditions at section 1 or section 2, respectively, we can write

$$E_{p1} + E_{k1} + E_{r1} = E_{p2} + E_{k2} + E_{r2} + E_f - E_c. \qquad (3.6.)$$

Therefore $$Z_1g + v_1^2/2 + P_1/\rho_1 = Z_2g + v_2^2/2 + P_2/\rho_2 + E_f - E_c. \qquad (3.7)$$

In the special case where no mechanical energy is added and for a frictionless fluid,
$E_c = E_f = 0$, and we have

$$Z_1g + v_1^2/2 + P_1/\rho_1 = Z_2g + v_2^2/2 + P_2/\rho_2 \qquad (3.8)$$

and since this is true for any sections of the pipe the equation can also be written

$$Zg + v^2/2 + P/\rho = k \qquad (3.9)$$

where k is a constant.

Equation (3.9) is known as **Bernouilli's equation**. First discovered by the Swiss mathematician Bernouilli in 1738, it is one of the foundations of fluid mechanics. It is a mathematical expression, for fluid flow, of the principle of conservation of energy and it covers many situations of practical importance.

Application of the equations of continuity, eqn. (3.4) or eqn. (3.5), which represent the mass balance, and eqn. (3.7) or eqn. (3.9), which represent the energy balance, are the basis for the solution of many flow problems for fluids. In fact much of the remainder of this chapter will be concerned with applying one or another aspect of these equations.

The Bernouilli equation is of sufficient importance to deserve some further discussion. In the form in which it has been written in eqn. (3.9) it will be noticed that the various quantities are in terms of energies per unit mass of the fluid flowing. If the density of the fluid flowing multiplies both sides of the equation, then we have pressure terms and the equation becomes:

$$\rho Zg + \rho v^2/2 + P = k' \qquad (3.10)$$

and the respective terms are known as the potential head pressure, the velocity pressure and the static pressure.

On the other hand, if the equation is divided by the acceleration due to gravity, g, then we have an expression in terms of the head of the fluid flowing and the equation becomes:

$$Z + v^2/2g + P/\rho g = k'' \qquad (3.11)$$

and the respective terms are known as the potential head, the velocity head and the pressure head.
The most convenient form for the equation is chosen for each particular case, but it is important to be consistent having made a choice.

If there is a constriction in a pipe and the static pressures are measured upstream or downstream of the constriction and in the constriction itself, then the Bernouilli equation can be used to calculate the rate of flow of the fluid in the pipe. This assumes that the flow areas of the pipe and in the constriction are known. Consider the case in which a fluid is flowing through a horizontal pipe of cross-sectional area A_1 and then it passes to a section of the pipe in which the area is reduced to A_2. From the continuity equation [eqn. (3.5)] assuming that the fluid is incompressible:

$$A_1 v_1 = A_2 v_2$$

and so

$$v_2 = v_1 A_1 / A_2$$

Since the pipe is horizontal

$$Z_1 = Z_2$$

Substituting in eqn. (3.8)

$$v_1^2/2 + P_1/\rho_1 = v_1^2 A_1^2/(2 A_2^2) + P_2/\rho_2$$

and since $\rho_1 = \rho_2$ as it is the same fluid throughout and it is incompressible,

$$P_1 - P_2 = \rho_1 v_1^2((A_1^2/A_2^2) - 1)/2. \qquad (3.12)$$

From eqn. (3.12), knowing P_1, P_2, A_1, A_2, ρ_1, the unknown velocity in the pipe, v_1, can be calculated.

Another application of the Bernouilli equation is to calculate the rate of flow from a **nozzle** with a known pressure differential. Consider a nozzle placed in the side of a tank in which the surface of the fluid in the tank is Z ft above the centre line of the nozzle as illustrated in **Fig. 3.4.**

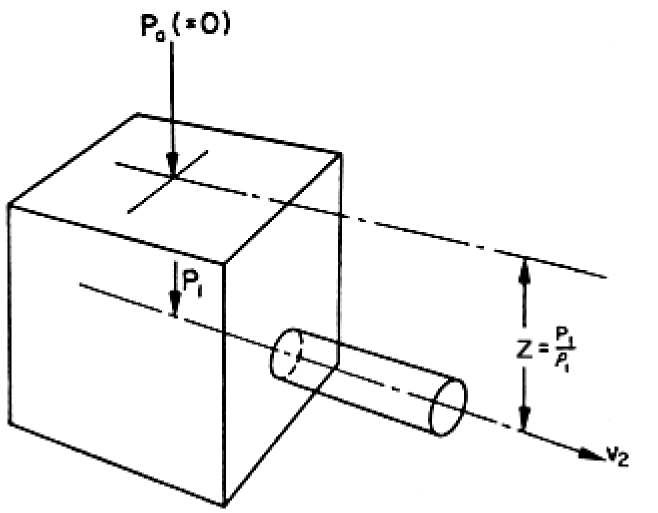


FIG. 3.4. Flow from a nozzle.

Take the datum as the centre of the nozzle. The velocity of the fluid entering the nozzle is approximately zero, as the tank is large compared with the nozzle. The pressure of the fluid entering the nozzle is P_1 and the density of the fluid ρ_1. The velocity of the fluid flowing from the nozzle is v_2 and the pressure at the nozzle exit is 0 as the nozzle is discharging into air at the datum pressure. There is no change in potential energy as the fluid enters and leaves the nozzle at the same level. Writing the Bernouilli equation for fluid passing through the nozzle:

$$0 + 0 + P_1/\rho_1 = 0 + v_2^2/2 + 0$$

$$v_2^2 = 2P_1/\rho_1$$

whence

$$v_2 = \sqrt{(2P_1/\rho_1)}$$

but $P_1/\rho_1 = gZ$

(where Z is the head of fluid above the nozzle) therefore

$$v_2 = \sqrt{(2gZ)} \qquad (3.13)$$

EXAMPLE 3.5. Pressure in a pipe

Water flows at the rate of 0.4 $m^3\ min^{-1}$ in a 7.5 cm diameter pipe at a pressure of 70 kPa. If the pipe reduces to 5 cm diameter calculate the new pressure in the pipe.

Density of water is 1000 kg m^{-3}.

Flow rate of water = 0.4 $m^3\ min^{-1}$ = 0.4/60 $m^3\ s^{-1}$.

Area of 7.5 cm diameter pipe $= (\pi/4)D^2$

$= (\pi/4)(0.075)^2$

$= 4.42 \times 10^{-3}\ m^2$.

So velocity of flow in 7.5 cm diameter pipe,

$$v_1 = (0.4/60)/(4.42 \times 10^{-3}) = 1.51\ m\ s^{-1}$$

Area of 5 cm diameter pipe $= (\pi/4)(0.05)^2$

$= 1.96 \times 10^{-3}\ m^2$

and so velocity of flow in 5 cm diameter pipe,

$$v_2 = (0.4/60)/(1.96 \times 10^{-3}) = 3.4\ m\ s^{-1}$$

Now

$$Z_1 g + v_1^2/2 + P_1/\rho_1 = Z_2 g + v_2^2/2 + P_2/\rho_2$$

and so

$$0 + (1.51)^2/2 + 70 \times 10^3/1000 = 0 + (3.4)^2/2 + P_2/1000$$

$$0 + 1.1 + 70 = 0 + 5.8 + P_2/1000$$

$$P_2/1000 = (71.1 - 5.8) = 65.3$$

$$\underline{P_2 = 65.3k\ Pa}.$$

EXAMPLE 3.6. Flow rate of olive oil

Olive oil of specific gravity 0.92 is flowing in a pipe of 2 cm diameter. Calculate the flow rate of the olive oil, if an orifice is placed in the pipe so that the diameter of the pipe in the constriction is reduced to 1.2 cm, and if the measured pressure difference between the clear pipe and the most constricted part of the pipe is 8 cm of water.

Diameter of pipe, in clear section, equals 2 cm and at constriction equals 1.2 cm.

$$A_1/A_2 = (D_1/D_2)^2 = (2/1.2)^2$$

Differential head = 8 cm water.

Differential pressure $= Z\rho g$

$= 0.08 \times 1000 \times 9.81 = 785$ Pa.

substituting in eqn. (3.12)

$$785 = 0.92 \times 1000 \times v^2\ [(2/1.2)^4 - 1]/2$$

$$v^2 = 785/3091$$

$$\underline{v = 0.5\ m\ s^{-1}}$$

EXAMPLE 3.7. Mass flow rate from a tank

The level of water in a storage tank is 4.7 m above the exit pipe. The tank is at atmospheric pressure and the exit pipe discharges into the air. If the diameter of the exit pipe is 1.2 cm what is the mass rate of flow through this pipe?

From eqn. (3.13)

$v = \sqrt{(2\, gZ)}$
$v = \sqrt{(2 \times 9.81 \times 4.7)}$
$= 9.6\ m\ s^{-1}$.

Now area of pipe, *A*

$= (\pi/4)D^2$
$= (\pi/4) \times (0.012)^2$
$= 1.13 \times 10^{-4}\ m^2$

Volumetric flow rate, *Av*

$= 1.13 \times 10^{-4}\ m^2 \times 9.6\ m\ s^{-1}$
$= 1.13 \times 10^{-4} \times 9.6\ m^3\ s^{-1}$
$= 1.08 \times 10^{-3}\ m^3\ s^{-1}$

Mass flow rate, ρAv

$= 1000\ kg\ m^{-3} \times 1.08 \times 10^{-3}\ m^3\ s^{-1}$
$= \underline{1.08\ kg\ s^{-1}}$

EXAMPLE 3.8. Pump horsepower

Water is raised from a reservoir up 35 m to a storage tank through a 7.5 cm diameter pipe. If it is required to raise 1.6 cubic metres of water per minute, calculate the horsepower input to a pump assuming that the pump is 100% efficient and that there is no friction loss in the pipe.
1 Horsepower = 0.746 kW.

Volume of flow, *V*

$= 1.6\ m^3\ min^{-1} = 1.6/60\ m^3\ s^{-1} = 2.7 \times 10^{-2}\ m^3\ s^{-1}$

Area of pipe, *A*

$= (\pi/4) \times (0.075)^2 = 4.42 \times 10^{-3}\ m^2$,

Velocity in pipe, *v*

$= 2.7 \times 10^{-2}/(4.42 \times 10^{-3}) = 6\ m\ s^{-1}$,

And so applying eqn. (3.7)

$E_c = Z_g + v^2/2$

$E_c = 35 \times 9.81 + 6^2/2$
$= 343.4 + 18$
$= 361.4\ J$

Therefore total power required

$= E_c \times$ mass rate of flow
$= E_c V\rho$
$= 361.4 \times 2.7 \times 10^{-2} \times 1000\ J\ s^{-1}$
= 9758 J s-1

and, since 1 h.p. $= 7.46 \times 10^2\ J\ s^{-1}$,

<u>required power = 13 h.p</u>.

Fluid-flow theory > VISCOSITY

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