FLUID MECHANICS AND MACHINERY

UNIT-I

FLUID PROPERTIES AND FLOW CHARACTERISTICS

Part B Qn (6 to 16 marks) University Questions and Answers.

(1). Find the height through which water rises by
(a) Capillary action in a 2mm bore, if Surface tension at the Prevailing temperature is 0.075g/cm. (April 2003)

G.D Surface tension $\sigma = 0.075\ g/cm$

Diameter of tube $d = 2mm = 0.002m$

Solution:

$$\sigma = \frac{0.075 \times 9.81 \times 100}{1000} = 0.0736\ N/m$$

For water $\beta = 0$

Capillary rise $h = \frac{4\sigma \cos\beta}{wd} = \frac{4 \times 0.0736 \times 1}{9810 \times 0.002}$

$$h = 0.015m = 15mm$$

(b) An oil film of thickness 10mm is used for lubrication between the two Square parallel plates of Size 0.9x0.9 m each, in which the upper plate moves at 2m/s required a force of 100N to maintain this Speed. Determine (i) dynamic viscosity of the oil
(ii) Kinematic viscosity of oil if the specific gravity of the oil is 0.95.

Oil film thickness $t = 0.01m$ 1-2

Plate Size $= 0.9 \times 0.9 m$

Area of plate $= 0.9 \times 0.9 = 0.81 m^2$

Velocity of the upper plate $= u = 2 m/s$.

Force on the upper plate $F = 100N$

Specific gravity of oil $S = 0.95$

Shear Stress $\tau = \frac{F}{A} = \frac{100}{0.81} = 123.45 \ N/m^2$

We know that $\tau = \mu \frac{du}{dy} = \mu \frac{u}{t}$

($\because$ Assuming Linear relationship).

$$123.45 = \mu \frac{2}{10 \times 10^{-3}}$$

$\therefore$ dynamic viscosity $\mu = 0.6172 \ N\text{-}s/m^2$

density of oil $\rho =$ sp. gravity $\times$ density of water

$= 0.95 \times 1000 = 950 \ kg/m^3$

Kinematic viscosity $\nu = \frac{\mu}{\rho} = \frac{0.6172}{950} = \cancel{6.4}$

$\nu = 6.498 \times 10^{-4} \ m^2/s$.

(2) A drainage pipe is tapered in a section running with full of water. The pipe diameters at the inlet and exit are 1000mm and 500mm respectively.
The water surface is 2m above the centre of the inlet and exit is 3m above the free surface of the water.
The pressure at the exit is 250mm of Hg vacuum. The friction loss between the inlet and exit of the pipe is 1/10 of the velocity head at the exit. Determine the discharge through the pipe.
(April 2010)

Solution

Given Data

$D_1 = 1000mm = 1m$ | $D_2 = 500mm = 0.5m$

$Z_1 = 2m$, | $Z_2 = -3m$

$P_1 = P_{atm} = 101.3 \text{ KN/m}^2$

$P_2 = P_{atm} - P_v = 101.3 - 33.32$

$P_2 = 67.98 \text{ KN/m}^2$

$P_v = 250mm$ of Hg Vacuum

$P_v = \frac{250}{760} \times 101.3$

$P_v = 33.32 \text{ KN/m}^2$

$h_f = \frac{1}{10}$ of velocity head at exit $= 0.1 \left(\frac{V_2^2}{2g}\right) \Rightarrow$

$$= 0.1 \frac{\left[\frac{Q}{A_2}\right]^2}{2g}$$

Specific weight of water $w = 9.81 \text{ KN/m}^3$

From Continuity equation

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4} D_1^2} = \frac{Q}{\frac{\pi}{4} \times 1^2} = 1.273\,Q$$

Similarly

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi}{4} D_2^2} = \frac{Q}{\frac{\pi}{4} \times 0.5^2} = 5.09\,Q$$

$$h_f = 0.1 \frac{\left(\frac{Q}{A_2}\right)^2}{2g} = 0.1 \frac{\left[\frac{Q}{\frac{\pi}{4} \times 0.5^2}\right]^2}{2 \times 9.81}$$

$$h_f = 0.132 Q^2$$

According to Bernoullis equation

$$\frac{V_1^2}{2g} + \frac{P_1}{W} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{W} + Z_2 + h_f$$

$$\frac{(1.273 Q)^2}{2 \times 9.81} + \frac{101.3}{9.81} + 2 = \frac{(5.09 Q)^2}{2 \times 9.81} + \frac{67.98}{9.81} - 3 + 0.132 Q^2$$

$$1.37 Q^2 = 8.396$$

$$\boxed{Q = 2.476 \ m^3/s}$$

(3) A pipe of 300mm diameter inclined at 30° to the horizontal is carrying gasoline (sp. gravity = 0.82). A Venturimeter is fitted in the pipe to find out the flow rate whose throat diameter is 150mm. The throat is 1.2m from the entrance along its length. The Pressure gauges fitted to the Venturimeter read 140 kN/m2 and 80 kN/m2 respectively. Find out the Co-efficient of discharge of venturimeter if the flow is 0.20 m3/s (April 2010)

Given data

d_1 diameter of the pipe = 300mm = 0.3m

Vertical inclination $\theta = 30°$

Specific gravity $S = 0.82$

Throat diameter $d_2 = 150mm = 0.15m$

Dist of throat from entrance $= 1.2m$

$P_1 = 140\ KN/m^2 = 140 \times 10^3 N/m^2$

$P_2 = 80\ KN/m^2 = 80 \times 10^3\ N/m^2$

Rate of flow $Q = 0.2m^3/s$

We know that

$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 0.3^2 = 0.070m^2$

$a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times 0.15^2 = 0.0176m^2$

Pr. head at entrance $\frac{P_1}{W}$

$$\frac{P_1}{sg\,\cancel{g}} = \frac{140 \times 10^3}{5 \times 1000} = \frac{140 \times 10^3}{9810 \times 0.82} = 17.40 \text{ m of oil}$$

Pressure head at throat $\frac{P_2}{W} = \frac{80 \times 10^3}{9810 \times 0.82}$

$= 9.945$ m of oil

$Z_1 = 0$

$Z_2 = 1.2 \sin 30° = 0.6$

We know that

$$h = \left[\frac{P_1}{W} + Z_1\right] - \left[\frac{P_2}{W} + Z_2\right]$$

$$h = (17.4 + 0) - (9.945 + 0.6)$$

$$\boxed{h = 6.855m}$$

The discharge through the Venturimeter is

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$0.2 = C_d \times \frac{(0.070 \times 0.0176)}{\sqrt{0.07^2 - 0.0176^2}} \times \sqrt{2 \times 9.81 \times 6.855}$$

$$0.2 = C_d \times 0.018 \times 11.59$$

$$\boxed{C_d = 0.948}$$

(4) A U-tube is made of two Capillaries of diameter 1mm and 1.5mm respectively. The tube is kept vertically and partially filled with water of surface tension 0.0736 N/m and zero contact angle. Calculate the difference in the levels of the menisci caused by the Capillary (Nov 2010)

Dia of tubes $d_1 = 1\,mm = 0.001\,m$, $d_2 = 1.5\,mm = 0.0015\,m$

Surface tension $\sigma = 0.0736$ N/m

Contact angle $\beta = 0$

To find:

Diff in water levels in the two limbs caused by the surface tension effect

We know that

Capillary rise $h = \frac{4\sigma \cos\beta}{wd}$

1. For $d_1 = 0.001m$ $h_1 = \frac{4 \times 0.0736 \times \cos 0}{9810 \times 0.001} = 0.030m$

2. For $d_2 = 0.0015m$ $h_2 = \frac{4 \times 0.0736 \times \cos 0}{9810 \times 0.0015} = 0.02$

$h_1 - h_2 = 0.03 - 0.02 = 0.01m$

For practice

(5) Lateral stability of a long shaft 150mm in diameter is obtained by means of a 250mm stationary bearing having an internal diameter of 150.25 mm. If the space between bearing and shaft is filled with a lubricant having viscosity 0.245 NS/m². what power will be required to overcome the viscous resistance when the shaft is rotated at a constant rate of 180rpm. Dec 2010.

(5) A pipe line 60cm in diameter bifurcated at a Y Junction into two branches 40cm and 30cm in diameter. If the rate of flow in the main pipe is 1.5 m3/s, and the mean velocity of flow in the 30cm pipe is 7.5 m/s determine the rate of flow in the 40cm pipe (Dec 2010)

Solution:

Given Data

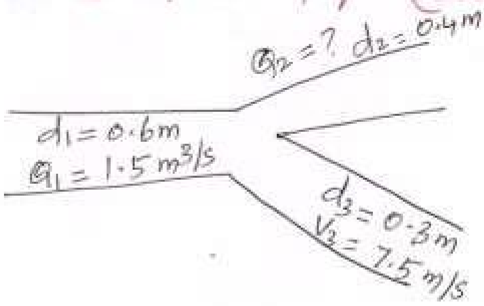
$d_1 = 0.6m$

$Q_1 = 1.5 m^3/s$

$d_2 = 0.4m$

$d_3 = 0.3m$

$V_3 = 7.5m/s$



To find

$Q_2 = ?$

We know that $Q_1 = Q_2 + Q_3$

$Q_1 = A_1 V_1 \Rightarrow 1.5 = Q_2 + Q_3 \Rightarrow 1.5 = Q_2 + [A_3 V_3]$

$$1.5 = Q_2 + \left[\frac{\pi}{4} \times 0.3^2\right] \times (7.5)$$

$$1.5 = Q_2 + 0.530$$

$$\boxed{Q_2 = 0.969 \ m^3/s}$$

(6) Desire an expression for the Capillary rise at a liquid in a Capillary tube of radius r having Surface tension S and Contact angle θ. If the plates are of glass, what will be the Capillary rise of water having

$\sigma = 0.073$ N/m $\theta = 0$ Take $r = 1$ mm

(Nov Dec 2011)

The Surface tension force acting around the circumference of the tube

$$S = \sigma \times \pi d$$

The vertical Component of this force $= \sigma \pi d \cos\theta$ —— (1)

Rise of the water in the tube $= h$

The weight of the liquid Column of height h in the tube

= Area of the tube × h × Specific weight

$$= \frac{\pi d^2}{4} \times h \times W \quad \text{—— (2)}$$

1-c

Equating ① & ②

$$\sigma \times \pi d \times \cos\theta = \frac{\pi d^2}{4} \times h \times w$$

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi d^2}{4} \times w}$$

$$h = \frac{4\sigma \cos\theta}{wd}$$

For water and glass tube interface $\theta = 0$

So, the capillary rise of water in the glass tube

$$h = \frac{4\sigma}{wd}$$

If $\sigma = 0.073$ N/m $\theta = 0$ $r = 1mm = 0.001m$

$$h = \frac{4 \times 0.073 \times \cos 0}{9810 \times 0.002}$$

$$\boxed{h = 0.014\,m = 14.88\,mm}$$

⑦ A venturimeter having inlet & throat diameter 30 cm and 15 cm is fitted in a horizontal diesel pipe line (sp. gravity = 0.92) to measure the discharge through the pipe. The venturimeter is connected to a mercury manometer. It was found that the discharge is 8 litre/sec. Find the reading of mercury manometer head in cm. Take Cd = 0.96

(Nov 2011)

Solution:

Given data: $d_1 = 30\,cm = 0.3\,m$

Throat dia $d_2 = 15\,cm = 0.15\,m$

sp. gravity of mercury $S_m = 13.6$

sp. gravity of oil $S_0 = 0.92$

$Q = 8\ lit/s$

$= 8 \times 10^{-3}\ m^3/s$

$Cd = 0.96$

Area at inlet section $a_1 = \frac{\pi}{4} d_1^2 = 0.0707\,m^2$

Area at throat $a_2 = \frac{\pi}{4} d_2^2 = 0.0177\,m^2$

$$Q = Cd \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\boxed{h = 0.011\ m\ of\ oil}$$

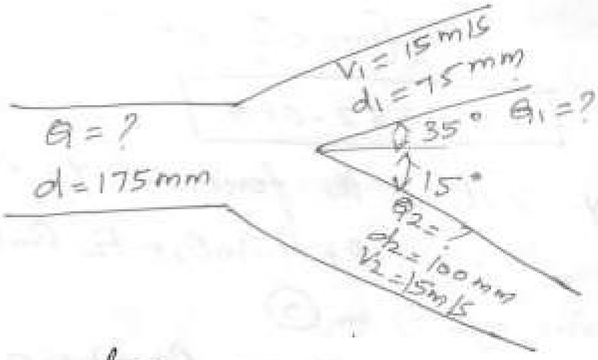
When a differential manometer is connected between throat and pipe section then the difference in pressure head.

$$h = x\left[\frac{S_m}{S_0} - 1\right]$$

$$0.011 = x\left[\frac{13.6}{0.92} - 1\right]$$

$$\boxed{x = 0.798\ mm}$$

(8) A pipe line of 175 mm diameter branches into two pipes which delivers the water at atmosphere Pressure. The diameter of the branch 1 which is at 35° counter clockwise to the pipe axis is 75mm and the velocity is at outlet 15 m/s. The branch 2 is at 15°c with the pipe centre line in the clockwise direction has a diameter 100mm. The outlet velocity is 15 m/s The pipes lie in a horizontal plane. Determine the magnitude and direction of the forces (on) the pipes.



Solution

Given data:

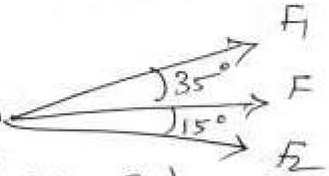
By Continuity equation

$$AV = A_1V_1 + A_2V_2$$

$$\frac{\pi}{4} \times 0.175^2 \times V = \left[\frac{\pi}{4} \times 0.075^2 \times 15\right] + \left[\frac{\pi}{4} \times 0.1^2 \times 15\right]$$

$$V = 7.65 \text{ m/s}$$

By resolving forces in x-direction



$$F_x = F\cos\theta + F_1\cos\theta_1 + F_2\cos(360-\theta_2) \quad (1)$$

We know that

Force $F = mV$

Mass of water $m = \rho A V$ —— (2)

Where $\rho = 1000\ kg/m^3$

A = Area of pipe

V = velocity of water

Sub eqn (2) in (1)

$$F_x = \rho A V^2 \cos\theta + \rho A_1 V_1^2 \cos\theta_1 + \rho A_2 V_2^2 \cos(360-\theta_2)$$

$$F_x = \left[1000 \times \frac{\pi}{4} \times 0.175^2 \times 7.65 \cos 0^\circ\right] + \left[1000 \times \frac{\pi}{4} \times 0.075^2\ 15 \cos 35^\circ\right] + \left[1000 \times \frac{\pi}{4} \times 0.1^2 \times 15 \cos(360-15)\right]$$

$$\boxed{F_x = 352.08 N}$$

By resolving the forces in Y-direction.

$$F_y = F \sin\theta + F_1 \sin\theta_1 + F_2 \sin(360-\theta_2) \quad —— (3)$$

Sub eqn (2) in (3)

$$F_y = \rho A V^2 \sin\theta + \rho A_1 V_1^2 \sin\theta_1 + \rho A_2 V_2^2 \sin(360-\theta_2)$$

(or)

$$F_y = \rho A V^2 \sin\theta + \rho A_1 V_1^2 \sin\theta_1 - \rho A_2 V_2^2 \sin\theta_2$$

$$F_y = \left[1000 \times \frac{\pi}{4} \times 0.175^2 \times 7.65 \sin 0^\circ\right] + \left[1000 \times \frac{\pi}{4} \times 0.075^2 \times 15 \sin 35\right] + \left[1000 \times \frac{\pi}{4} \times 0.1^2 \left(-15 \sin 15^\circ\right)\right]$$

$$F_y = 7.52 N$$

Resultant force $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{352.08^2 + 7.52^2}$$

$$F_R = 352.16\,N$$

The direction of resultant force with x-axis is

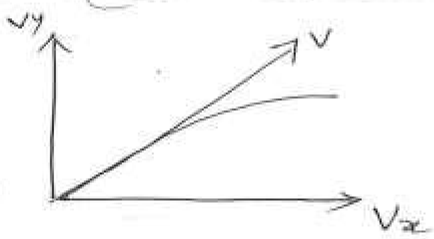
$$\tan\theta = \frac{F_y}{F_x} = \frac{7.52}{352.08} = 0.0214$$

$$\boxed{\theta = 1.23^\circ}$$

(9) A Jet issuing at a velocity of 25 m/s is directed at 35° to the horizontal. Calculate the height cleared by the Jet at 28m from the discharge location. Also determine the maximum height the Jet will clear and the corresponding horizontal location (Dec 2011)

Solution.

Given Data:



Jet velocity $V = 25\,m/s$

Angle $\theta = 35^\circ$

Discharge location $x = 28m$

We know that

$V_x = V\cos\theta = 25\cos 35^\circ = 20.48\ m/s$

$V_y = V\sin\theta = 25\sin 35^\circ = 14.34\ m/s$

According to Newton's Law

$$Z = V_y t - \tfrac{1}{2} g t^2 \quad \text{——} \quad (1)$$

Displacement $x = V_x t$

$$t = \frac{x}{V_x}$$

Sub t value in eqn ①

$$Z = V_y \frac{x}{V_x} - \frac{1}{2} g \left[\frac{x}{V_x}\right]^2$$

Hight cleared at 28m

$$Z = 14.34 \times \frac{28}{20.48} - \frac{1}{2} \times 9.81 \times \left[\frac{28}{20.48}\right]^2$$

$$= 10.44\,m$$

Maximum height of the trajectory $= \dfrac{V_y^2}{2g}$

$$= \frac{14.34^2}{2 \times 9.81}$$

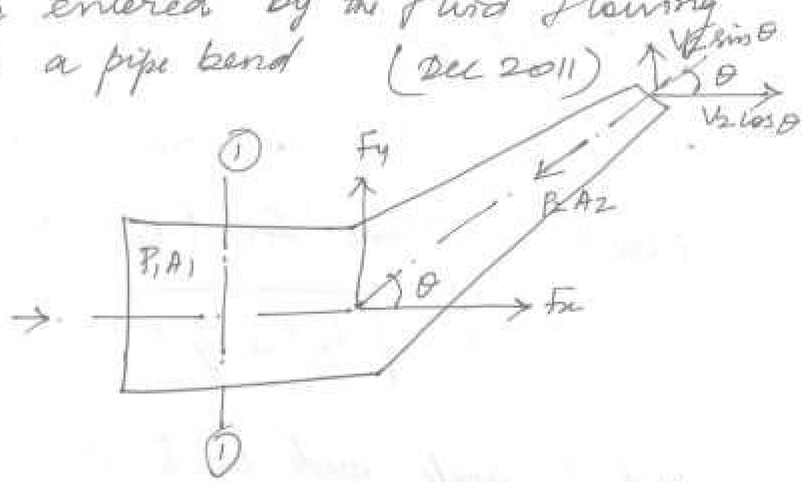
$$= 10.48\,m$$

Horizontal distance $= \dfrac{V_y V_x}{g}$

$$= \frac{14.34 \times 20.48}{9.81}$$

$$= 29.94\,m$$

(16) Derive the Linear momentum equation using the Control volume approach and determine the force entered by the fluid flowing through a pipe bend (Dec 2011)



Let P_1 A_1 V_1 are density area & velocity at Sec ①
P_2 A_2 V_2 are density, area & velocity at Sec ②

The momentum equation in x-direction is given by
net force acting in x direction = Rate of change of momentum in x-direction

$P_1 A_1 - P_2 A_2 \cos\theta - F_x = (\text{mass/sec})$ change of velocity

$= \rho Q$ final velocity in x direction − Initial velocity in x direction

$= \rho Q (V_2 \cos\theta - V_1)$

$F_x = \rho Q (V_1 - V_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta$

lly The momentum eqn in y-direction $\sin\theta$

$$0 - P_2 A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0)$$

$$F_y = \rho Q \cdot (-V_2 \sin\theta - P_2 A_2 \sin\theta)$$

Now the resultant force (F_R) acting on the bend

$$F_R = \sqrt{F_x^2 + F_y^2}$$

and the angle made by the resultant force with horizontal direction is given by

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

(11) The space between two square flat parallel plate is filled with oil. Each of the plate is 600mm. The thickness of the oil film is 12.5mm. The upper plate, which moves at 2.5 m/s requires a force of 98.1N to maintain the speed.

Determine.

(i) The dynamic viscosity of the oil in poise

(ii) The kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95

(Dec 2012)

Solution:

Given Data:

oil film thickness $dy = 12.5 mm = 0.0125$

Area of the plate $= 600 \times 600 = 360000\ mm^2$

$= 0.36 m^2$

Velocity of the upper plate du $= 2.5 m/s$

Force on " " $= 98.1$

To find

1. dynamic viscosity μ
2. Kinematic viscosity ν

We know that

Shear stress $\tau = \frac{F}{A} = \frac{98.1}{0.36} = 272.5\ N/m^2$

and also

$$\tau = \mu \times \frac{du}{dy}$$

$$272.5 = \mu \times \frac{2.5}{0.0125}$$

$$272.5 = \mu \times 200$$

$$\mu = 1.3625\ N\text{-}s/m^2$$

$$\boxed{\mu = 13.625 \text{ poise}}$$

$\therefore$ 1 poise $= \frac{1}{10}$ N-s/m²

2. Kinematic viscosity

$$\nu = \frac{\mu}{\rho} = \frac{1.3625}{\rho}$$

$$= \frac{1.3625}{0.95 \times 1000}$$

$$\boxed{\nu = 1.434 \times 10^{-3} \, m^2/s.}$$