WAVES BETWEEN PARALLEL PLANES

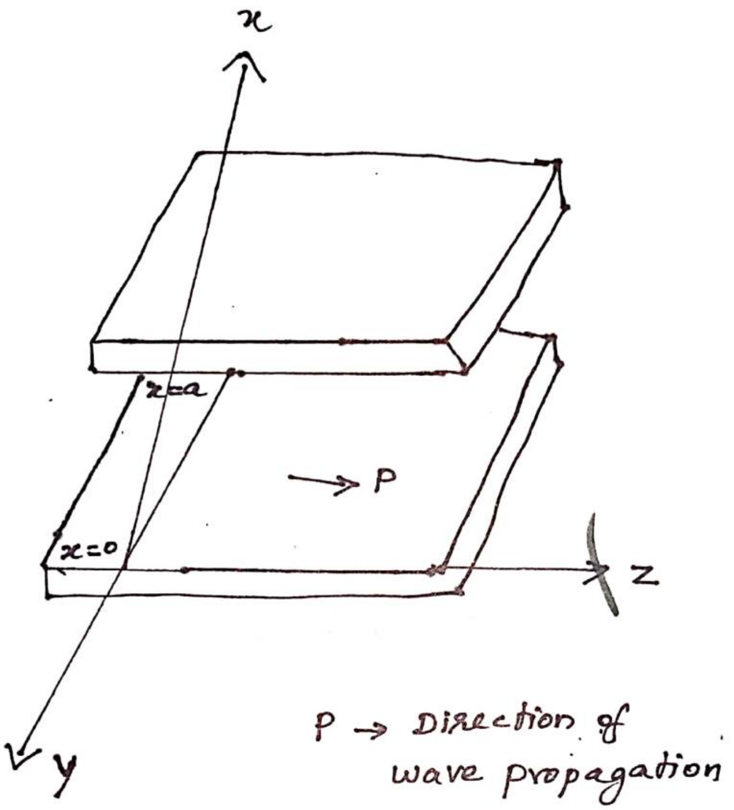

Fig (i) parallel conducting planes

In order to determine the EM field configuration in the region between the planes

→ Maxwell's equations will be solved

→ subject to appropriate boundary conditions.

Maxwell's Equations

$\nabla \times H = \sigma + j\omega\varepsilon E$

($\sigma = 0$, since medium between the plane is air)

$\therefore \nabla \times H = j\omega\varepsilon E$

$\nabla \times E = -j\omega\mu H$

Boundary conditions for perfectly conducting planes

$E_{tan} = 0$

$H_{nor} = 0$

Wave equations

$$\nabla^2 E = \gamma^2 E$$
$$\nabla^2 H = \gamma^2 H$$

$$\gamma = \sqrt{(\sigma + j\omega\varepsilon)(j\omega\mu)}$$
$$\sigma = 0$$
$$\therefore \gamma = \sqrt{j\omega\varepsilon \times j\omega\mu} = j\omega\sqrt{\mu\varepsilon}$$

In rectangular co-ordinates & for non-conducting region $\boxed{\nabla \times H = -j\omega\varepsilon E}$.

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = -j\omega\varepsilon\left[E_x \vec{a_x} + E_y \vec{a_y} + E_z \vec{a_z}\right]$$

By equating, we get three equations.

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \rightarrow (1)$$
$$-\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_x}{\partial z}\right] = j\omega\varepsilon E_y \rightarrow (2)$$
$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (3)$$

III^ly for $\boxed{\nabla \times E = -j\omega\mu H}$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu E_x \rightarrow (4)$$
$$-\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] = -j\omega\mu E_y \rightarrow (5)$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu E_z \rightarrow (6)$$

Propagation constant

$$\boxed{\bar{\gamma} = \bar{\alpha} + j\bar{\beta}}$$

* If $\bar{\gamma} \rightarrow$ real, α have value, $\beta = 0$

$\rightarrow$ represents there is no wave motion, but only an exponential decrease in Amplitude.

* If $\bar{\gamma} \rightarrow$ imaginary, $\bar{\alpha} = 0$, $\bar{\beta}$ have value

$\rightarrow$ represents a wave propagation, but no attenuation.

Important Assumptions

* In y-direction $\rightarrow$ the field is uniform and constant.

$$\therefore \boxed{\frac{\partial}{\partial y} = 0}$$

* In x-direction $\rightarrow$ Certain boundary must met.

$$\text{so } \boxed{\frac{\partial}{\partial x} \rightarrow \text{no change}}$$

* In z-direction $\rightarrow$ The wave is assumed to propagate

$$\therefore \boxed{\begin{aligned} \frac{\partial}{\partial z} &= -\bar{\gamma} \\ \frac{\partial^2}{\partial z^2} &= \bar{\gamma}^2 \end{aligned}}$$

After substituting assumptions $\rightarrow$ eqns (1) to (6) becomes,

$$\bar{\gamma} H_y = j\omega\varepsilon E_x \rightarrow (7)$$

$$-\bar{\gamma} H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \rightarrow (8)$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z \rightarrow (9)$$

$$\bar{\gamma} E_y = -j\omega\mu H_x \rightarrow (10)$$

$$-\bar{\gamma} E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \rightarrow (11)$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_x \rightarrow (12)$$

Solving Eqns. (7), (8), (10) & (11) simultaneously, we get

$$\boxed{\begin{aligned} E_x &= \frac{\bar{\gamma}}{h^2}\frac{\partial E_z}{\partial x} && \rightarrow (12) \\ H_y &= \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} && \rightarrow (13) \\ E_y &= \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} && \rightarrow (14) \\ H_x &= -\frac{\bar{\gamma}}{h^2}\frac{\partial H_z}{\partial x} && \rightarrow (15) \end{aligned}}$$

where $\boxed{h^2 = \bar{\gamma}^2 + \omega^2\mu\varepsilon}$