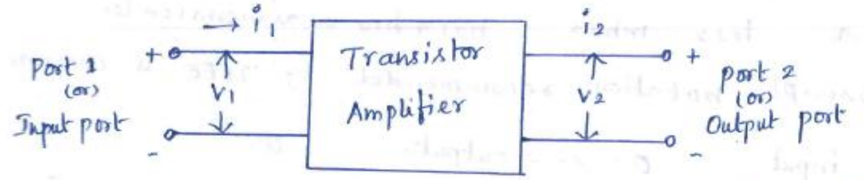
UNIT- II

BJT AMPLIFIERS

Method of drawing Small-Signal Equivalent Circuit

* A transistor can be treated as a two port Network.
* The terminal behaviour of any 2 port network can be specified by the terminal voltage v_1 & v_2 at port 1 & port 2 respectively & currents i_1 & i_2 entering port 1 & 2 respectively as shown in Fig:



* From 4 variables v_1, v_2 & i_1, i_2 2 can be selected as independent variables & remaining 2 can be expressed interms of the independent variables.
* The transistor can be analyzed using various 2 port parameters which of the following are more important.

1. Z - parameter (or) Impedance parameter
2. Y - parameter (or) Admittance parameter
3. H - parameter (or) Hybrid parameter.

H-parameter (or) Hybrid parameters: ·*·2Mark

* If the input current i_1 & the output voltage v_2 are taken as independent variables, the input voltage v_1 & output current i_2 can be expressed as

$$v_1 = h_{11} i_1 + h_{12} v_2 \quad — ①$$

$$i_2 = h_{21} i_1 + h_{22} v_2 \quad — ①$$

* The 4 h-parameters h_{11}, h_{12}, h_{21}, h_{22} are defined as

$h_{11} = \left[\frac{v_1}{i_1}\right]$ with $v_2 = 0$ in eqn ① → input impedance with output port short circuited..

$h_{22} = \left[\frac{i_2}{v_2} \right]$ with $i_1 = 0$ in eqn ② $\Rightarrow$ output admittance with input port short circuited.

$h_{12} = \left[\frac{v_1}{v_2} \right]$ with $i_1 = 0$ in eqn ① $\Rightarrow$ Reverse voltage transfer gain with input port open circuited.

$h_{21} = \left[\frac{i_2}{i_1} \right]$ with $V_2 = 0$ in eqn ② $\Rightarrow$ Forward current gain with output port short circuited.

* The dimensions of h-parameters are

$$h_{11} = \Omega \quad h_{22} = \text{mhos} \quad h_{21} \,\&\, h_{12} = \text{dimension less}$$

* Alternate subscript notation recommended by IEEE is commonly used

$$i = 11 = \text{input} \; ; \quad o = 22 = \text{output} \quad — ③$$

$$f = 21 = \text{Forward Transfer} \quad r = \text{reverse Transfer} \quad — ④$$

* According to eqn ③ & ④ for common Emitter Amplifier

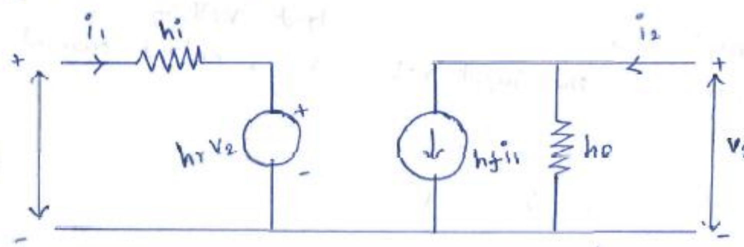
$$h_{11} = h_{ie} \; ; \; h_{22} = h_{oe} \quad — ⑤$$

$$h_{12} = h_{re} \; ; \; h_{21} = h_{fe} \quad — ⑥$$

$$① \Rightarrow \quad V_1 = h_i i_1 + h_r V_2 \quad — ⑦$$

$$② \Rightarrow \quad i_2 = h_f i_1 + h_o V_2 \quad — ⑧$$

* For this equation we want to draw equivalent circuit & it verify using KVL to input, KCL to output node.



H-parameter for all the 3 configurations

parameters	CE	CB	CC
Input resistance (h_{11})	hie	hib	hic
Reverse Voltage gain	hre	hrb	hrc
Forward Transfer current gain	hfe	hfb	hfc
Output admittance	hoe	hob	hoc

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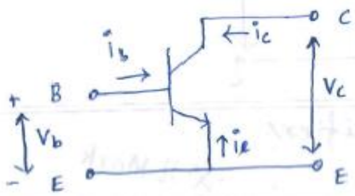
$$V_1 = h_{11} i_1 + h_{12} V_2 \quad -(1)$$

$$i_2 = h_{21} i_1 + h_{22} V_2 \quad -(2)$$

For CE configuration :

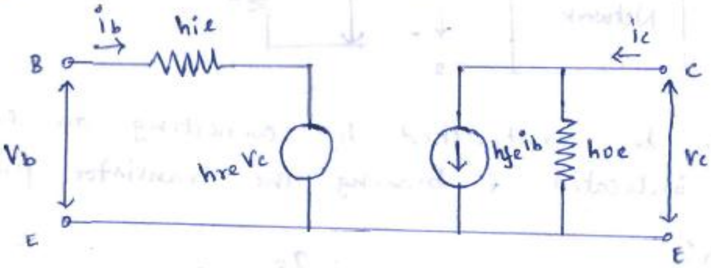


(1) ⇒ can be modified based on CE

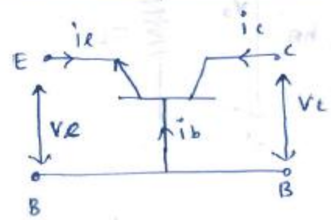
$$V_b = h_{ie} i_b + h_{re} V_c \quad -(3)$$

(2) $\quad i_c = h_{fe} i_b + h_{oe} V_c \quad -(4)$

For eqn (3) & (4) we have to draw the small signal Equivalent circuit.



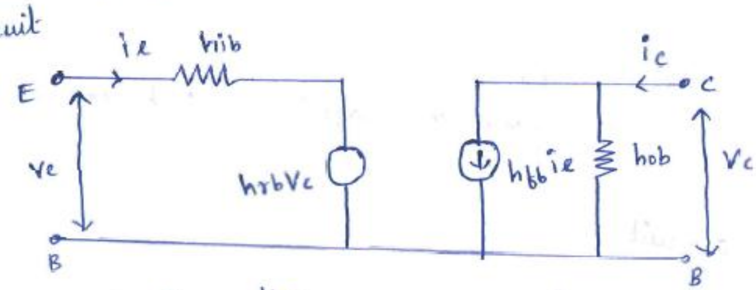
For CB configuration



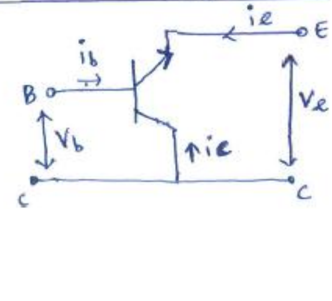
(1) ⇒ $\quad V_e = h_{ib} i_e + h_{rb} V_c \quad -(5)$

(2) ⇒ $\quad i_c = h_{fb} i_e + h_{ob} V_c \quad -(6)$

For eqn (5) & (6) we have to draw the small signal equivalent circuit



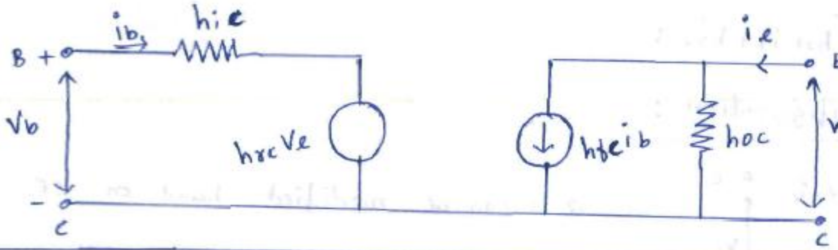
For CC configuration



(1) ⇒ $\quad V_b = h_{ic} i_b + h_{rc} V_e \quad -(7)$

(2) ⇒ $\quad i_e = h_{fc} i_b + h_{oc} V_e \quad -(8)$

For eqn ⑦ & ⑧ We have to draw small signal equivalent circuit



Midband Analysis of BJT single stage Amplifiers

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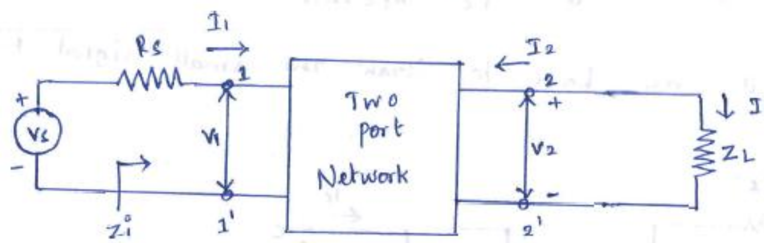
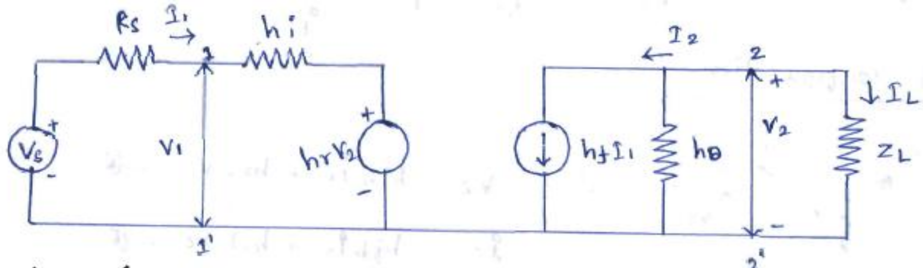


A Transistor can be constructed by connecting an external load & signal source as indicated & biasing the transistor properly.



1. current gain (or) Current Amplification (A_I)

It's the ratio of output current to input current.

$$\boxed{A_I = \frac{I_L}{I_1} = \frac{-I_2}{I_1}}$$

Here I_L & I_2 are equal in magnitude but opposite in sign

ie $I_L = -I_2$

From the circuit

$$I_2 = h_f I_1 + h_o V_2$$

Then

$$V_2 = I_L Z_L = -I_2 Z_L$$

Substitute V_2 in I_2

$$I_2 = h_f I_1 + h_o(-I_2 Z_L)$$

$$I_2 + h_o I_2 Z_L = h_f I_1$$

$$I_2(1 + h_o Z_L) = h_f I_1$$

$$\frac{I_2}{I_1} = \frac{h_f}{1 + h_o Z_L}$$

$$\boxed{\therefore\ A_I = \frac{-I_2}{I_1} = \frac{-h_f}{1 + h_o Z_L}}$$

2. Input Impedance (Z_i)

$$\boxed{Z_i = \frac{V_1}{I_1}}$$

From input circuit

$$V_1 = h_i I_1 + h_r V_2$$

Substitute V_1 in Z_i

$$Z_i = \frac{h_i I_1 + h_r V_2}{I_1} = h_i + h_r \frac{V_2}{I_1} \quad — ①$$

$$\because\ V_2 = -I_2 Z_L$$

$$= -(-A_I I_1) Z_L$$

$$V_2 = A_I I_1 Z_L \quad — ②$$

sub ② in ①

$$Z_i = h_i + h_r \frac{A_I I_1 Z_L}{I_1}$$

$$= h_i + h_r A_I Z_L$$

Sub A_I in Z_i

$$Z_i = h_i + h_r\left(\frac{-h_f}{1 + h_o Z_L}\right) Z_L$$

$$= h_i - \frac{h_r h_f}{1 + h_o Z_L} \cdot Z_L = h_i - \frac{(h_r h_f) Z_L}{Z_L(\frac{1}{Z_L} + h_o)}$$

$$= h_i - \frac{h_r h_f}{\frac{1}{Z_L} + h_o} \qquad \because Y_L = \frac{1}{Z_L}$$

$$\boxed{Z_i = h_i - \frac{h_r h_f}{Y_L + h_o}} \quad ⑤$$

→ The input impedance is the function of Load impedance.

3. Voltage gain (or) Voltage Amplification factor (A_v)

It's the ratio of output voltage to input voltage

$$\boxed{A_v = \frac{V_2}{V_1}}$$

$$V_2 = -I_2 Z_L = A_I I_1 Z_L \qquad \because A_I = \frac{-I_2}{I_1}$$

Sub V_2 in A_v

$$A_v = \frac{A_I I_1 Z_L}{V_1} \qquad \frac{I_1}{V_1} = \frac{1}{Z_i}$$

$$\boxed{A_v = \frac{A_I Z_L}{Z_i}}$$

4. Output Admittance (Y_o)

It's the ratio of output current (I_2) to the output voltage (V_2).

$$\boxed{Y_o = \frac{I_2}{V_2} \text{ with } V_s = 0}$$

$$I_2 = h_f I_1 + h_o V_2 \quad -① $$

divide eqn ① by V_2

$$Y_o \Leftarrow \frac{I_2}{V_2} = \frac{h_f I_1 + h_o V_2}{V_2} = \frac{h_f I_1}{V_2} + h_o \quad -②$$

With $V_s = 0$ by Apply KVL to the above circuit

$$R_s I_1 + h_i I_1 + h_r V_2 = 0$$

$$I_1 (R_s + h_i) + h_r V_2 = 0$$

$$I_1 (R_s + h_i) = -h_r V_2$$

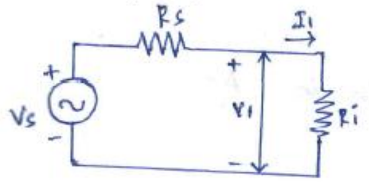
$$\frac{I_1}{V_2} = \frac{-h_r}{(R_s + h_i)}$$

$$Y_o = h_f \left(\frac{-h_r}{(R_s + h_i)} \right) + h_o$$

$$\boxed{Y_o = h_o - \frac{h_f h_r}{(R_s + h_i)}}$$

Output impedance is a function of source ⇒ resistance. If the source Impedance is resistive then Y_o is real.

5. Voltage gain (Avs)



$$A_{vs} = \frac{V_2}{V_s}$$

Multiply 4 divide by V_1

$$A_{vs} = \frac{V_2}{V_1} \cdot \frac{V_1}{V_s}$$

$$= A_v \cdot \frac{V_1}{V_s}$$

From the equivalent input circuit using thevenin's equivelance for the source

$$V_1 = \frac{Z_i}{Z_i + R_s} V_s$$

$$\therefore \frac{V_1}{V_s} = \frac{Z_i}{Z_i + R_s}$$ substitute this in A_{vs}

$$A_{vs} = A_v \cdot \frac{Z_i}{Z_i + R_s}$$

$$\because A_v = \frac{A_I Z_L}{Z_i}$$

$$= \frac{A_I Z_L}{\cancel{Z_i}} \cdot \frac{\cancel{Z_i}}{Z_i + R_s}$$

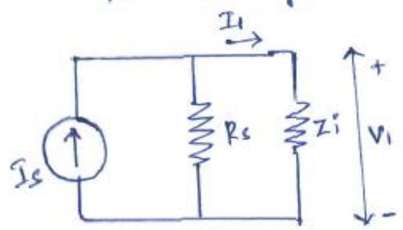
$$\boxed{A_{vs} = \frac{A_I Z_L}{Z_i + R_s}}$$

Note:

substitute $R_s = 0$: Then $\Rightarrow$ $\boxed{A_{vs} = \frac{A_I Z_L}{Z_i} = A_v}$

6. Current Gain (AIs)

The voltage source is replaced by current source



$$A_{Is} = \frac{-I_2}{I_s}$$ => Multiply 4 divide by I_1

$$= \frac{-I_2}{I_1} \cdot \frac{I_1}{I_s}$$

$$= A_I \cdot \frac{I_1}{I_s}$$

From The above figure

$$I_1 = I_S \cdot \frac{R_s}{R_s + Z_i}$$

$$\frac{I_1}{I_s} = \frac{R_s}{R_s + Z_i}$$ substitute this in A_{IS}

$$\boxed{A_{IS} = A_I \cdot \frac{R_s}{R_s + Z_i}}$$

Note:

Suppose $R_S = \infty$ Then $\Rightarrow$ $\boxed{A_{IS} = A_I}$

7. power gain (A_P)

It's The ratio of average power delivered to the load R_L to the input power. $\boxed{A_P = \frac{P_2}{P_1}}$

$$P_2 = |V_2| \, |I_L| \cos\theta$$

θ - is the phase angle between V_2 & I_L

Assume $Z_L = R_L$ Then the power delivered to the load is

$$P_2 = V_2 I_L = -V_2 I_2$$

The input power P_1 is

$$P_1 = V_1 I_1$$

$$A_P = \frac{P_2}{P_1} = \frac{-V_2 I_2}{V_1 I_1}$$

$\because A_I = -\frac{I_2}{I_1}$

$\because A_V = \frac{V_2}{V_1}$

$$A_P = A_V A_I \quad \text{--- (1)}$$

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$$A_V = \frac{A_I Z_L}{Z_i}$$ sub in A_P

$$A_P = \frac{A_I Z_L}{Z_i} A_I$$

Here $Z_L = R_L$; $Z_i = R_i$ so $A_P = A_I \cdot \frac{R_L}{R_i} \cdot A_I$

$$\boxed{A_P = A_I^2 \left(\frac{R_L}{R_i}\right)}$$

Relation between A_{vs} and A_{Is}

$$A_{vs} = \frac{A_I Z_L}{Z_i + R_s} \qquad A_{Is} = \frac{A_I R_s}{Z_i + R_s}$$

Taking the ratio of these 2 equations

$$\frac{A_{vs}}{A_{Is}} = \frac{\cancel{A_I} Z_L}{\cancel{Z_i + R_s}} \times \frac{\cancel{Z_i + R_s}}{\cancel{A_I} R_s} = \frac{Z_L}{R_s}$$

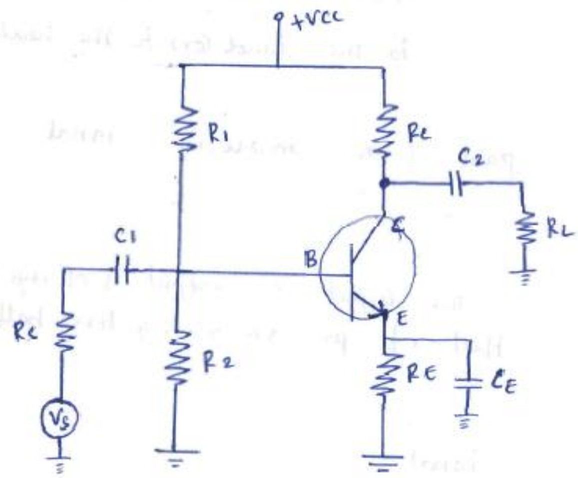
$$Z_L \simeq R_L$$

So

$$\frac{A_{vs}}{A_{Is}} = \frac{R_L}{R_s}$$

$$\boxed{A_{vs} = A_{Is}\left(\frac{R_L}{R_s}\right)}$$

Common Emitter Amplifier



* Amplifier is used to increase the signal level ie the amplifier is used to get a large signal output from a small signal input.

* For Ex: We give sin wave as input. We get the same sin wave output. The frequency is not change but Amplitude increased.

* To make the transistor as an amplifier, it's to be biased to operate in active region ie the Base-Emitter junction is forward bias and Base-collector junction is Reverse biased.

1. Biasing circuit

* Resistance R_1, R_2 & R_E forms the voltage divider bias circuit for the CE amplifier.

* It sets the proper operating point for the CE amplifier.

2. Input capacitor C_1

* This capacitor couples the signal to the base of the transistor.
* It blocks any dc component present in the signal & passes only ac signals for amplification.
* Because of this biasing conditions are maintained constant.

3. Emitter Bypass Capacitor C_E

* C_E is connected in parallel with the R_E to provide a low resistance path to the amplified ac signal.
* It's not inserted, the amplified ac signal passing through R_E will cause a voltage drop across it.
* This will reduce the output voltage, reducing the gain of the amplifier.

4. Output Coupling Capacitor C_2

* C_2 couples the output of the amplifier to the load (or) to the next stage of the amplifier.
* It blocks dc & passes only ac part of the amplifier signal.

5. Phase Reversal

* The phase relationship between the input & output voltages can be determined by considering the effect of positive & negative half cycle separately.

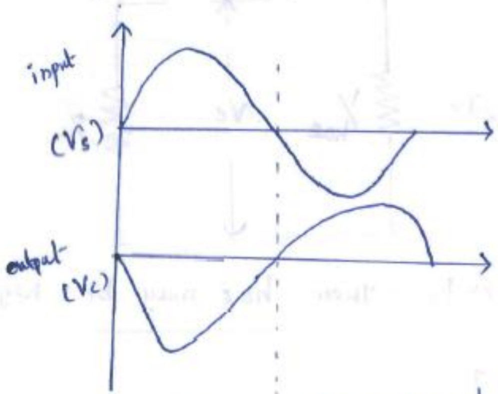
Consider positive half cycle of input signal :

* In which terminal 'A' is positive w.r.to 'B'.
* Due to this 2 voltages, ac & dc will be adding each other, increased forward bias on the Base - Emitter junction.

$\underset{V_s}{ac}$ & $\underset{V_{cc}}{dc}$

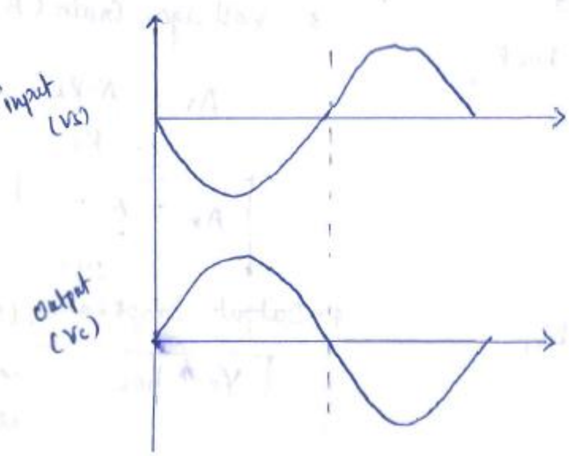
* This will increase the base current I_B.
* The I_C is β times the I_B ie $I_C = \beta I_B$ hence the I_C will also increases.
* This will increases the voltage drop across R_C.
* Since $V_C = V_{CC} - I_C R_C$, the increase in I_C results in a drop in V_C, as V_{CC} is constant.

* Thus, as V_i increases in a positive direction, V_o goes in a negative direction & we get negative half cycle of output voltage for positive half cycle of the input.



Consider negative half cycle of input signal:

* In which terminal 'A' is negative w.r.to 'B'
* Due to this 2 voltages V_c & dc will be opposing to each other, decreased forward biase on the Base-Emitter junction.
* This will decrease the base current I_B.
* The I_c decreases & the voltage drop across R_c decreases, increase the output voltage.
* Thus as we get positive half cycle at the output for the negative half cycle of the input.
* ∴ We can say that there is a phase shift of 180° between input & output voltages for a CE amplifier.



Small signal Analysis of common Emitter | Analysis of CE circuit Using Simplified Hybrid Model (single stage BJT Amplifiers)

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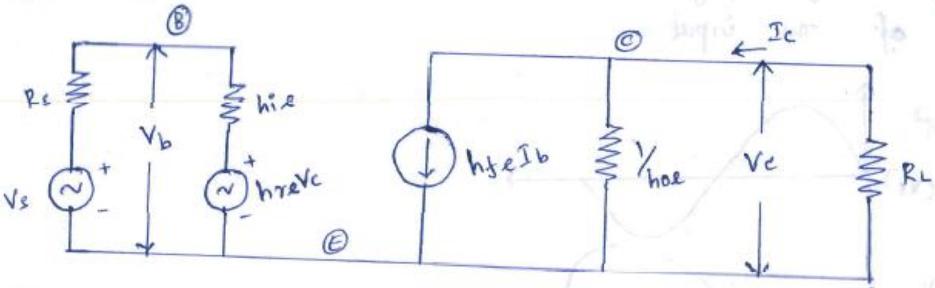


Here, $\frac{1}{h_{oe}} \parallel R_L$ & if $\frac{1}{h_{oe}} >> R_L$, then h_{oe} may be neglected.

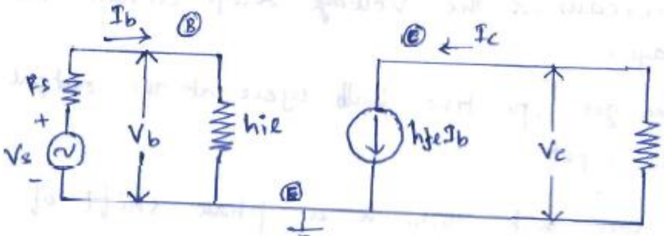
If h_{oe} neglected $I_c = h_{fe} I_b$

Magnitude of voltage generator in the Emitter $h_{re}|V_c| = h_{re} I_c R_L$

$= h_{fe} I_b h_{re} R_L$.

Since $h_{re} h_{fe} \simeq 0.01$ this voltage neglected in compasision with $h_{ie} I_b$ drop across h_{ie} provided that R_L is too large.

R_L small means h_{re}, h_{oe} neglected so,



1. current Gain (A_I)

$$\boxed{A_I = \frac{-I_c}{I_b} = \frac{-h_{fe}}{1 + h_{oe} R_L}}$$

neglecting h_{oe}

$$\boxed{A_I = -h_{fe}}$$

2. Input Impedance (R_i)

$$\boxed{R_i = h_{ie} + h_{re} A_I R_L}$$

neglecting h_{re}

$$\boxed{R_i = h_{ie}}$$

3. Voltage Gain (A_V)

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$$A_V = \frac{A_i R_L}{R_i}$$

$$\boxed{A_V = \frac{A_i R_L}{h_{ie}}}$$

4. Output Impedance (Y_o)

$$\boxed{Y_o = h_{oe} - \frac{h_{fe} h_{re}}{h_{ie} + R_s}}$$

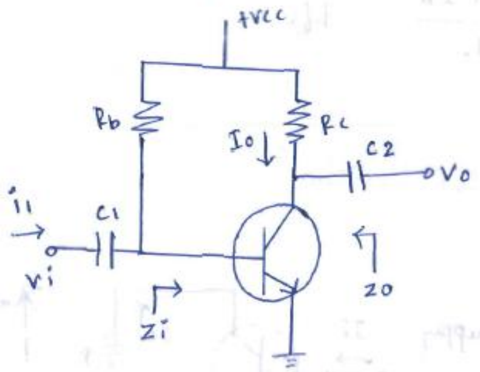
by neglecting h_{oe} & h_{re}

$$\boxed{Y_o = 0}$$

$$R_o = \frac{1}{Y_o} = \frac{1}{0} = \infty$$

$$\boxed{R_o = \infty}$$

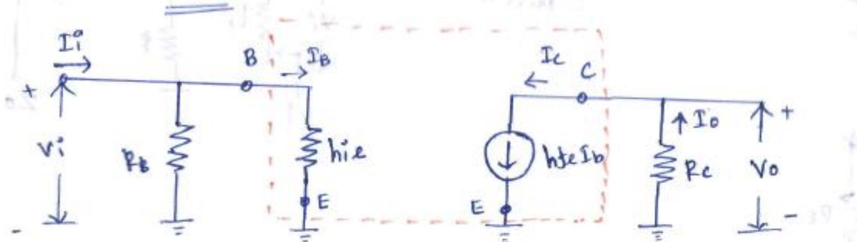
1. CE Amplifier with Fixed Bias



* Remove the dc effect of power supply by grounding them.

* Replace the capacitors (C_1 & C_2) by short circuits.

Small signal Model:



1. Input Impedance (Z_i)

$Z_i = R_B \parallel h_{ie}$; $R_B \gg h_{ie}$

$$\boxed{Z_i \simeq h_{ie}}$$

2. Output Impedance (Z_o)

It's the impedance determined with $V_i = 0$ with $V_i = 0$, $I_b = 0$. $h_{fe} I_b = 0$ indicating an open circuit equivalent for the current source.

$$\boxed{Z_o = R_L \simeq R_c}$$

3. Voltage gain (A_v)

$A_v = \dfrac{V_o}{V_i}$

$V_o = -I_o R_c$ & $I_o = h_{fe} I_b$ substitute this in V_o we get

$V_o = -h_{fe} I_b R_c$

Assume that $R_B \gg h_{ie}$

$I_i \simeq I_b$ & $V_i = I_b h_{ie}$

$\therefore A_v = \dfrac{-h_{fe} \cancel{I_b} R_c}{\cancel{I_b} h_{ie}} \Rightarrow \boxed{A_v = \dfrac{-h_{fe} R_c}{h_{ie}}}$

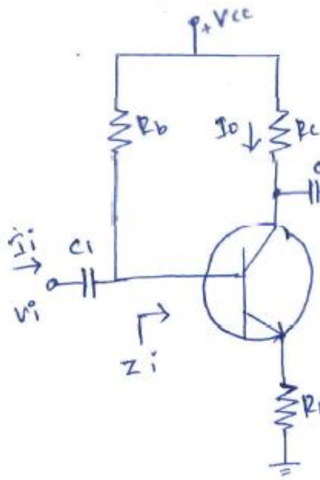
* As h_{fe} & h_{ie} are positive, A_v is negative. The negative sign indicate a 180 phase shift between input & output signals.

4. Current gain (A_I)

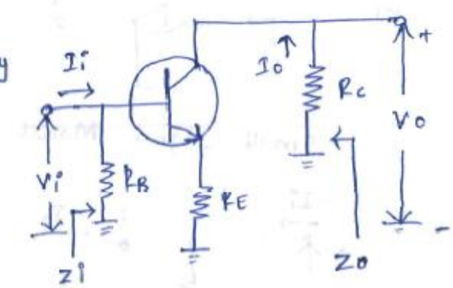
$$A_I = \frac{I_L}{I_i} = \frac{-I_o}{I_i} = \frac{-h_{fe} I_b}{I_b} = -h_{fe}$$

$$\boxed{A_I = -h_{fe}}$$

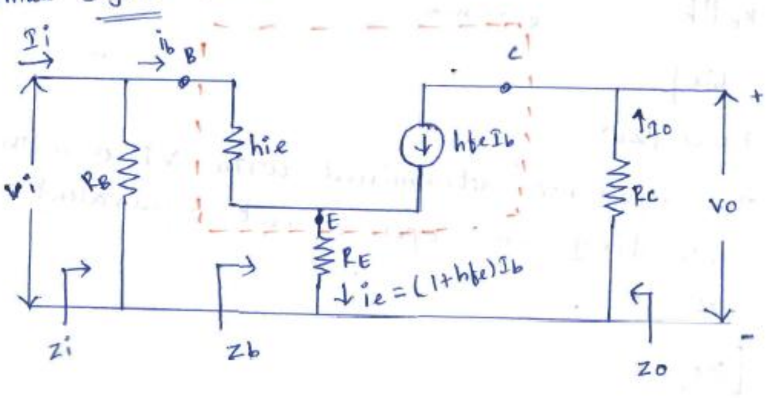
CE Amplifier with Unbypassed Emitter Resistor



⟹ Remove dc supply & capacitors



Small signal Model



1. Input Impedance (Z_i)

$$I_e = I_b + h_{fe} I_b = I_b(1 + h_{fe})$$

$$V_i = I_b * h_{ie} + (1 + h_{fe}) I_b R_E$$

$$Z_b = \frac{V_i}{I_b} = \frac{I_b * h_{ie} + (1 + h_{fe}) I_b R_E}{I_b} = h_{ie} + (1 + h_{fe}) R_E$$

$\because h_{fe} >> 1$

$$\boxed{Z_b = h_{ie} + h_{fe} R_E}$$

$\because h_{fe} R_E >> h_{fe}$

$$\boxed{Z_i = R_B \parallel Z_b}$$

2. Output Impedance (Z_o)

$V_i = 0$, $I_b = 0$, $h_{fe} I_b = 0$ indicating open circuit for current source.

$$\boxed{Z_o = R_C}$$

3. Voltage gain (A_V)

$$A_V = \frac{V_o}{V_i}$$

$$V_o = -I_o R_C = (-h_{fe} I_b) R_C \qquad \because I_b = \frac{V_i}{Z_b}$$

$$= -h_{fe} \left(\frac{V_i}{Z_b}\right) R_C$$

$$A_V = -h_{fe} \left(\frac{\cancel{V_i}}{Z_b}\right) R_C \times \frac{1}{\cancel{V_i}}$$

$$A_V = \frac{-h_{fe} R_C}{Z_b}$$

$$\because Z_b \approx h_{fe} R_E$$

$$A_V = \frac{-\cancel{h_{fe}} R_C}{\cancel{h_{fe}} R_E}$$

$$\boxed{A_V = -\frac{R_C}{R_E}}$$

4. Current Gain (A_I)

$$A_I = -I_o / I_i$$

$$I_o = h_{fe} I_b$$

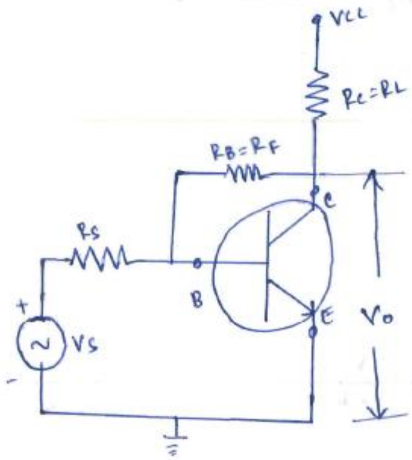
$$I_b = I_i \times \frac{R_B}{R_B + Z_b}$$

$$I_o = h_{fe} I_i \times \frac{R_B}{R_B + Z_b}$$

$$A_I = -\frac{I_o}{I_i} = \frac{-h_{fe} \cancel{I_i} \times R_B}{(R_B + Z_b) \cancel{I_i}}$$

$$\boxed{A_I = \frac{-h_{fe} R_B}{R_B + Z_b}}$$

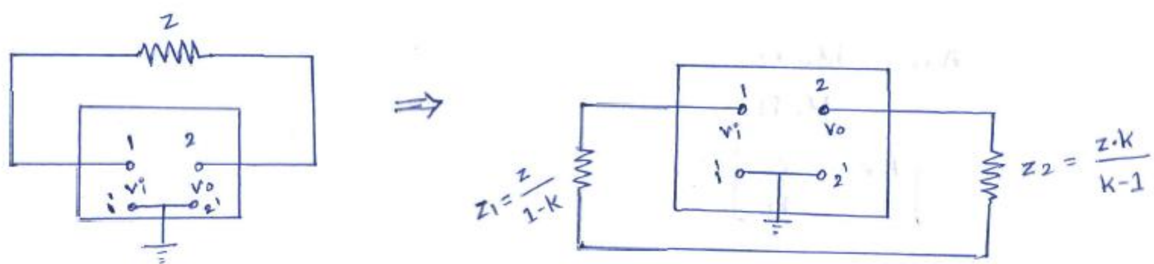
2. CE Amplifier with Collector to Base Bias



* The resistance R_F in this configuration is connected between input & Output.
* For analysis of this circuit it's necessary to split this resistance for input & output.
* This can be achieved by Millers Theorem.

Millers Theorem ※ 2 Marks

* It's used for converting one circuit configuration to another circuit configuration.

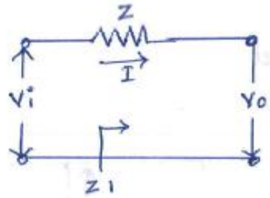


* If Z is the impedance connected between 2 nodes, node 1 & node 2 it can be replaced by 2 separate impedances Z_1 & Z_2; where Z_1 is connected b/w node 2 & ground.

* The V_i & V_o are the voltages at the node 1 & 2.
* The value of Z_1 & Z_2 can be derived from the ratio of V_o & V_i.
* The V_o/V_i is denoted as K.
* It's not necessary to know the values of V_i & V_o to calculate the value of Z_1 & Z_2.
* The values of impedance Z_1 & Z_2 are given as

$$Z_1 = \frac{Z}{1-K} \quad \& \quad Z_2 = \frac{Z \cdot K}{K-1}$$

Proof of Millers Theorem · ẋ · 4 (or) 6 Mark



* Millers Theorem states that, the effect of resistance Z on the input circuit is a ratio of input voltage V_i to the current I which flows from input to the output.

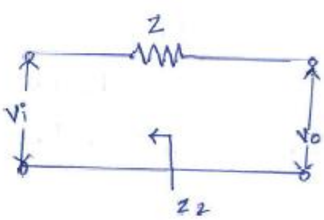
$$\therefore Z_1 = \frac{V_i}{I}$$

Where

$$I = \frac{V_i - V_o}{Z} = \frac{V_i\left[1 - \frac{V_o}{V_i}\right]}{Z} = \frac{V_i[1 - A_v]}{Z} \qquad \because A_v = \frac{V_o}{V_i} = 1$$

$$Z_1 = \frac{V_i}{\frac{V_i[1-A_v]}{Z}} = \frac{Z}{1-A_v}$$

$$\boxed{Z_1 = \frac{Z}{1-K}}$$



* Millers Theorem states that, the effect of resistance Z on the output circuit is the ratio of output voltage V_o to the current I which flows from the output to the input.

$$\therefore Z_2 = \frac{V_o}{I}$$

Where

$$I = \frac{V_o - V_i}{Z} = \frac{V_o\left[1 - \frac{V_i}{V_o}\right]}{Z} = \frac{V_o\left[1 - \frac{1}{A_v}\right]}{Z}$$

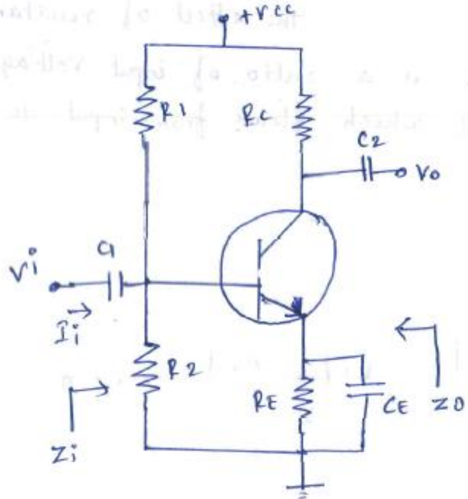
$$= \frac{V_o\left[\frac{A_v - 1}{A_v}\right]}{Z}$$

$$\therefore Z_2 = \frac{V_o}{\frac{V_o\left[\frac{A_v-1}{A_v}\right]}{Z}} = \frac{Z}{\frac{A_v-1}{A_v}}$$

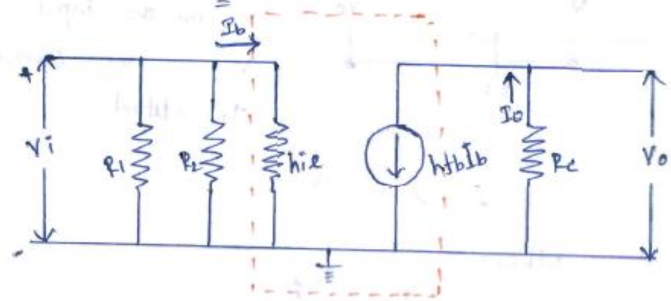
$$\therefore Z_2 = \frac{Z \cdot A_v}{A_v - 1} \qquad \because A_v = \frac{V_o}{V_i} = k$$

$$\therefore \boxed{Z_2 = \frac{Z \cdot K}{K-1}}$$

3. CE Amplifier with Voltage Divider Bias



Small signal Model



1. Input Impedance (Zi)

$Z_i = R_B \| h_{ie}$

$\because R_B >> h_{ie}$

$\boxed{Z_i \simeq h_{ie}}$

2. Output Impedance (Zo)

$V_i = 0;\ I_b = 0;\ h_{fe} I_b = 0$

$\boxed{Z_o = R_c}$

3. Voltage gain (Av)

$A_v = \frac{V_o}{V_i}$

$V_o = -I_o R_c$

$I_o = h_{fe} I_b$

$V_o = -h_{fe} I_b R_c$

$R_B >> h_{ie}$

$I_i \simeq I_b$

$V_i = I_b h_{ie}$

$A_v = -\frac{h_{fe} \cancel{I_b} R_c}{\cancel{I_b} h_{ie}}$

$\boxed{A_v = -\frac{h_{fe} R_c}{h_{ie}}}$

4) Current gain (AI)

$A_I = -I_o / I_i$

$I_o = -h_{fe} I_b$

$I_b = I_i \times \frac{R_B}{R_B + h_{ie}}$

$I_o = -h_{fe} I_i \times \frac{R_B}{R_B + h_{ie}}$

$A_I = I_o / I_i$

$= \frac{h_{fe} \cancel{I_i} R_B}{(R_B + h_{ie}) * \cancel{I_i}}$

$\boxed{A_I = \frac{h_{fe} R_B}{(R_B + h_{ie})}}$