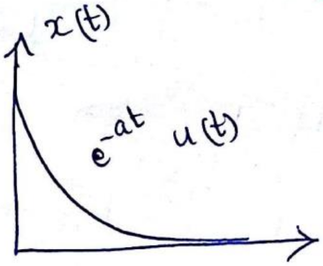
PROBLEMS :-

1) Find the Fourier transform of decaying Exponential function




$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

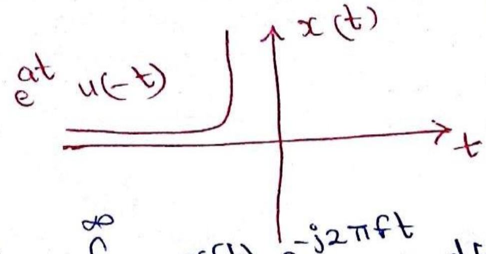
$$= \int_{-\infty}^{\infty} e^{-at}\, u(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-j2\pi ft}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j2\pi f)t}\, dt \Rightarrow \left[\frac{e^{-(a+j2\pi f)}}{-(a+j2\pi f)}\right]_{0}^{\infty}$$

$$X(F) = \frac{1}{a+j2\pi f}$$

2) Find the Fourier transform of rising Exponential function (7)




$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{0} e^{+at}\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{0} e^{t(a-j2\pi f)}\, dt \Rightarrow \left[\frac{e^{(a-j2\pi f)t}}{a-j2\pi f}\right]_{-\infty}^{0}$$

$$X(F) = \frac{1}{a-j2\pi f}$$

HW

3) Find the Fourier transform of $x(t) = e^{-0.5t}\, u(t)$

$$X(F) = \frac{1}{0.5+j2\pi f}$$

4) Find the Fourier transform of $u(t)$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} u(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-j2\pi f t}\, dt$$

$$= \left[\frac{e^{-j2\pi f}}{-j2\pi f}\right]_{0}^{\infty}$$

$$X(F) = \frac{1}{j2\pi f}.$$

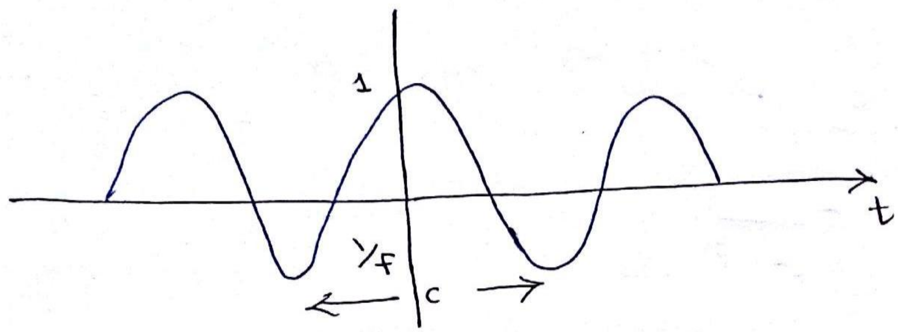
Signum Function :-

$$\text{Signum}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$$

Sinc Function :-

$$\text{sinc}\, x = \frac{\sin \pi x}{\pi x}$$

5) Find the fourier transform of cosine wave as shown in fig




$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}.$$

$$x(t) = \cos 2\pi f_c t$$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \cos 2\pi f_c t \; e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\, e^{-j2\pi ft}\, dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left[ e^{-j2\pi t(f-f_c)} + e^{-j2\pi t(f+f_c)} \right] dt$$

$$= \frac{1}{2} \left[ \int_{-\infty}^{\infty} e^{-j2\pi t(f-f_c)}\, dt + \int_{-\infty}^{\infty} e^{-j2\pi t(f+f_c)}\, dt \right]$$

Apply the result :- $\int_{-\infty}^{\infty} e^{-j2\pi (f-f_0)t} = \delta(f-f_0)$

$$X(F) = \frac{1}{2} \left[ \delta(f-f_c) + \delta(f+f_c) \right]$$

6) Find the Fourier transform of sine wave :- (8)

HW.

$$x(t) = \sin 2\pi f_c t \qquad \sin 2\pi f_c t = \frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}$$

$$X(F) = \frac{1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

7) Find the Fourier transform of $x(t) = A\cos(2\pi f_c t)\, u(t)$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$= \int_{-\infty}^{\infty} A\cos(2\pi f_c t)\, u(t)\, e^{-j2\pi f t}\, dt$$

$$= A\int_{0}^{\infty} \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\, e^{-j2\pi f t}\, dt$$

$$= A/2 \int_{0}^{\infty} \left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right) e^{-j2\pi f t}\, dt$$

$$= A/2\left[\int_{0}^{\infty} e^{-j2\pi(f-f_c)t}\, dt + \int_{0}^{\infty} e^{-j2\pi(f+f_c)t}\, dt\right]$$

$$= A/2\left[\frac{e^{-j2\pi(f-f_c)t}}{-j2\pi(f-f_c)} + \frac{e^{-j2\pi(f+f_c)t}}{j2\pi(f+f_c)}\right]_0^{\infty}$$

$$= A/2\left[\frac{1}{j2\pi(f-f_c)} + \frac{1}{j2\pi(f+f_c)}\right]$$

$$= A/2 \cdot \frac{1}{j2\pi}\left[\frac{1}{f-f_c} + \frac{1}{f+f_c}\right]$$

$$= \frac{A}{j4\pi}\left[\frac{f+\cancel{f_c}+f-\cancel{f_c}}{f^2-f_c^2}\right]$$

$$= \frac{A}{j\cancel{4}_2\pi}\left[\frac{\cancel{2}f}{f^2-f_c^2}\right] = \frac{A}{j2\pi}\left[\frac{f}{f^2-f_c^2}\right]$$

⑧ Find the Fourier transform of $x(t) = A\sin(2\pi f_c t)\,u(t)$

HW $X(F) = \frac{-A}{2\pi}\left[\frac{f_c}{f^2 - f_c^2}\right]$

⑨ Find the Fourier transform of $\cos^2(2\pi f_o t)$.

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} \cos^2(2\pi f_o t)\, e^{-j2\pi f t}\, dt$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \int_{-\infty}^{\infty} \left[\frac{1+\cos 4\pi f_o t}{2}\right] e^{-j2\pi f t}\, dt$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-j2\pi f t}\, dt + \int_{-\infty}^{\infty} \cos 4\pi f_o t\; e^{-j2\pi f t}\, dt\right]$$

$$= \frac{1}{2}\left[\delta(F) + \int_{-\infty}^{\infty} \cos 4\pi f_o t\; e^{-j2\pi f t}\, dt\right]$$

$$= \frac{1}{2}\left[\delta(F) + \int_{-\infty}^{\infty} \frac{e^{j4\pi f_o t} + e^{-j4\pi f_o t}}{2}\, e^{-j2\pi f t}\, dt\right]$$

$$= \frac{1}{2}\left[\delta(F) + \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{j4\pi f_o t} + e^{-j4\pi f_o t}\; e^{-j2\pi f t}\, dt\right]\right]$$

$$= \frac{1}{2}\left[\delta(F) + \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-j2\pi(f-2f_o)t} + e^{-j2\pi(f+2f_o)t}\right]\right]$$

$$= \frac{1}{2}\left[\delta(F) + \frac{1}{2}\left[\delta(f-2f_o) + \delta(f+2f_o)\right]\right]$$

$$X(F) = \frac{1}{2}\delta(F) + \frac{1}{4}\left[\delta(f-2f_o) + \delta(f+2f_o)\right]$$

Find the Fourier transform of Sine wave :-

$$x(t) = \sin 2\pi f_c t$$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} \sin 2\pi f_c t\; e^{-j2\pi f t}\, dt$$

$$\sin 2\pi f_c t = \frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}$$

$$= \int_{-\infty}^{\infty} \left[\frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}\right] e^{-j2\pi f t}\, dt$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty} e^{j2\pi f_c t}\, e^{-j2\pi f t}\, dt - \int_{-\infty}^{\infty} e^{-j2\pi f_c t}\, e^{-j2\pi f t}\, dt\right]$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty} e^{-j2\pi t(f - f_c)}\, dt - \int_{-\infty}^{\infty} e^{-j2\pi t(f + f_c)}\, dt\right]$$

Applying the result

$$= \int_{-\infty}^{\infty} e^{-j2\pi (f - f_0) t}\, dt = \delta(f - f_0)$$

$$X(F) = \frac{1}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]$$

Find the Fourier transform of $x(t) = A \sin(2\pi f_c t)\, u(t)$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} A \sin(2\pi f_c t)\, u(t)\, e^{-j2\pi f t}\, dt$$

$$\sin 2\pi f_c t = \frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}$$

$$= A \int_{0}^{\infty} \left[\frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2j}\right] e^{-j2\pi f t}\, dt$$

$$= \frac{A}{2j}\left[\int_0^{\infty} e^{j2\pi f_c t}\, e^{-j2\pi f t}\, dt - \int_0^{\infty} e^{-j2\pi f_c t}\, e^{-j2\pi f t}\, dt\right]$$

$$= \frac{A}{2j}\left[\int_0^{\infty} e^{-j2\pi (f-f_c)t}\, dt - \int_0^{\infty} e^{-j2\pi (f+f_c)t}\, dt\right]$$

$$= \frac{A}{2j}\left[\frac{e^{-j2\pi(f-f_c)t}}{-j2\pi(f-f_c)} - \frac{e^{-j2\pi(f+f_c)t}}{-j2\pi(f+f_c)}\right]_0^{\infty}$$

$$= \frac{A}{2j}\left[\frac{1}{j2\pi(f-f_c)} - \frac{1}{j2\pi(f+f_c)}\right]$$

$$= \frac{A}{j^2 4\pi}\left[\frac{1}{f-f_c} - \frac{1}{f+f_c}\right]$$

$$= \frac{-A}{4\pi}\left[\frac{\cancel{f} + f_c - \cancel{f} + f_c}{f^2 - f_c^2}\right] \Rightarrow -\frac{A}{\cancel{4}_{2}\pi}\left[\frac{\cancel{2} f_c}{f^2 - f_c^2}\right]$$

$$X(F) = -\frac{A}{2\pi}\left[\frac{f_c}{f^2 - f_c^2}\right]$$