Even and Odd Signals

(1) Find even + odd components of

$$x(t) = \cos t + \sin t$$

$$x_e(t) = \frac{x(t)+x(-t)}{2} \qquad x_o(t) = \frac{x(t)-x(-t)}{2}$$

$$x(-t) = \cos(-t) + \sin(-t)$$

$$= \cos t - \sin t$$

$$x_o(t) = \frac{\cos t + \sin t - \cos t + \sin t}{2} = \frac{2\sin t}{2}$$

$$x_o(t) = \sin t$$

$$x_e(t) = \frac{\cos t + \cancel{\sin t} + \cos t - \cancel{\sin t}}{2}$$

$$= \frac{\cancel{2}\cos t}{\cancel{2}}$$

$$= \cos t$$

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(2) $x(t) = \cos t + \sin t + \cos t \cdot \sin t$

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\cdot\sin(-t)$$

$$= \cos t - \sin t - \cos t \sin t$$

$$x_e(t) = \frac{\cos t + \cancel{\sin t} + \cancel{\cos t \sin t} + \cos t - \cancel{\sin t} - \cancel{\cos t \sin t}}{2}$$

$$= \frac{\cancel{2}\cos t}{\cancel{2}}$$

$$x_e(t) = \cos t$$

$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

$$= \frac{\cancel{\cos t} + \sin t + \cos t \sin t - \cancel{\cos t} + \sin t + \cos t \sin t}{2}$$

$$= \frac{\cancel{2}(\sin t + \cos t \sin t)}{\cancel{2}}$$

$$= \sin t + \cos t \sin t \quad = \sin t$$

(B) $x(n) = \{3, 2, \underset{\uparrow}{1}, 4, 5\}$

index value

The value pointout by this index should be in 0^{th} position.

$x(-2) = 3,\ x(-1) = 2,\ x(0) = 1,\ x(1) = 4,\ x(2) = 5$

Even: $x_e(n) = \frac{x(n) + x(-n)}{2}$

$n = -2$

$$x_e(-2) = \frac{x(-2) + x(2)}{2} = \frac{3+5}{2} = \frac{8}{2} = 4$$

$n = -1$

$$x_e(-1) = \frac{x(-1) + x(1)}{2} = \frac{2+4}{2} = \frac{6}{2} = 3$$

$n = 0$

$$x_e(0) = \frac{x(0)}{2} + \frac{x(0)}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$$

$n = 1$

$$x_e(1) = \frac{x(1) + x(-1)}{2} = \frac{4+2}{2} = \frac{6}{2} = 3$$

$n = 2$

$$x_e(2) = \frac{x(2) + x(-2)}{2} = \frac{5+3}{2} = \frac{8}{2} = 4$$

$$x_e(n) = \{4, 3, \underset{\uparrow}{1}, 3, 4\}$$

Odd: $x_o(n) = \frac{x(n) - x(-n)}{2}$

$n = -2$

$$x_o(-2) = \frac{x(-2) - x(2)}{2} = \frac{3-5}{2} = \frac{-2}{2} = -1$$

$n = -1$

$$x_o(1) = \frac{x(-1) - x(1)}{2} = \frac{2-4}{2} = \frac{-2}{2} = -1$$

$n = 0$

$$x_o(0) = \frac{x(0) - x(0)}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$$

$n = 1$

$$x_o(1) = \frac{x(1) - x(-1)}{2} = \frac{4-2}{2} = \frac{2}{2} = 1$$

$n = 2$

$$x_o(2) = \frac{x(2) - x(-2)}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$$

$$x_o(n) = \{-1, -1, \underset{\uparrow}{0}, 1, 1\}$$

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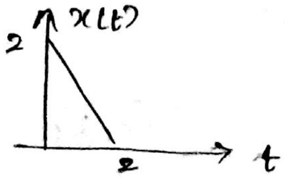
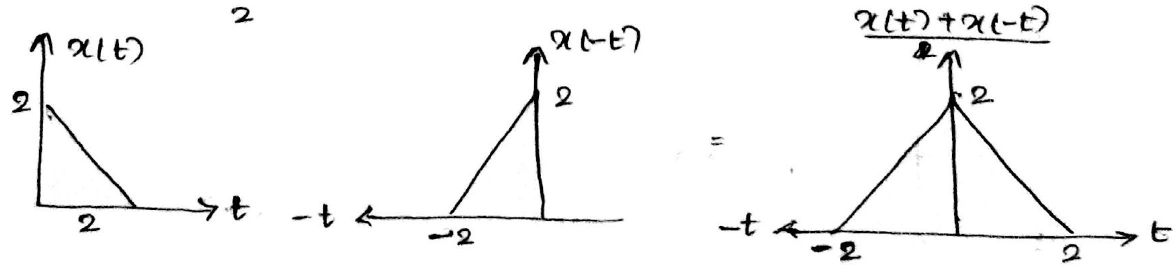
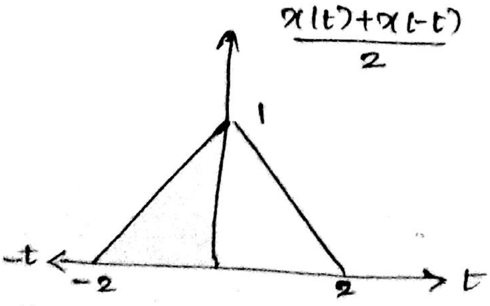
(4) Find the even & odd component of

$x(t) =$

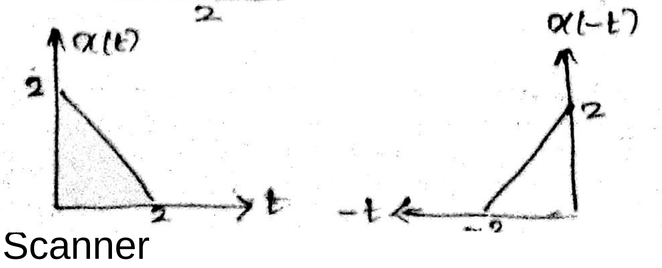


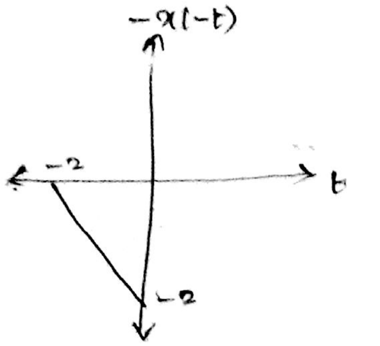
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

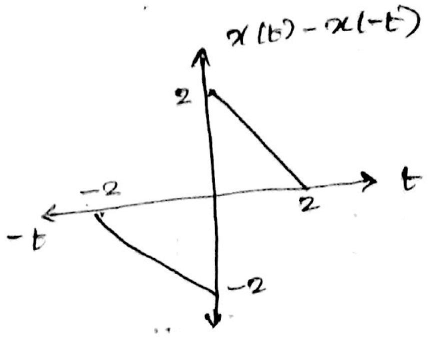


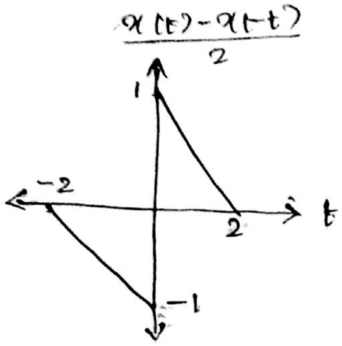


$$x_o(t) = \frac{x(t) - x(-t)}{2}$$









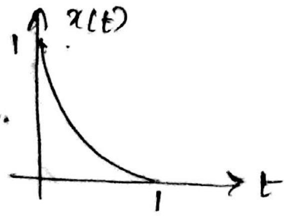
(5) find the even + odd components of

(1) $x(n) = \{-2, 1, \underset{\uparrow}{2}, -1, 3\}$

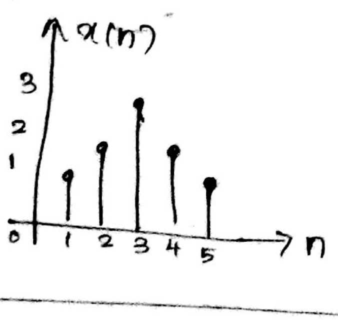
(2) $x(t) = 3\sin t + 2\sin t + 2\sin^2 t\cos t$

(3) $x(n) = \{1, 0, -1, 2, 3\}$

(4)



(5)



(i) $x(n) = \{-2, 1, \underset{\uparrow}{2}, -1, 3\}$

$x(-2) = -2$, $x(-1) = 1$, $x(0) = 2$, $x(1) = -1$, $x(2) = 3$

Even: $x_e(n) = \frac{x(n) + x(-n)}{2}$

$n=-2$

$$x_e(t) = \frac{x(-2)+x(2)}{2} = \frac{-2+3}{2} = \frac{1}{2}$$

$n=-1$

$$x_e(-1) = \frac{x(-1)+x(1)}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$$

$n=0$

$$x_e(0) = \frac{x(0)+x(0)}{2} = \frac{2+2}{2} = \frac{4}{2} = 2$$

$n=1$

$$x_e(1) = \frac{x(1)+x(-1)}{2} = \frac{-1+1}{2} = \frac{0}{2} = 0$$

$n=2$

$$x_e(2) = \frac{x(2)+x(-2)}{2} = \frac{3-2}{2} = \frac{1}{2}$$

$$x_e(n) = \{ \tfrac{1}{2}, 0, \underset{\uparrow}{2}, 0, \tfrac{1}{2} \}$$

Odd : $x_o(t) = \dfrac{x(n) - x(-n)}{2}$

$n=-2$, $x_o(-2) = \dfrac{x(+2) - x(-2)}{2} = \dfrac{-2-3}{2} = \dfrac{-5}{2}$

$n=-1$, $x_o(-1) = \dfrac{x(+1) - x(1)}{2} = \dfrac{1+1}{2} = \dfrac{2}{2} = 1$

$n=0$, $x_o(0) = \dfrac{x(0) - x(0)}{2} = \dfrac{2-2}{2} = \dfrac{0}{2} = 0$

$n=1$, $x_o(1) = \dfrac{x(1) - x(-1)}{2} = \dfrac{-1-1}{2} = \dfrac{-2}{2} = -1$

$n=2$, $x_o(2) = \dfrac{x(2) - x(-2)}{2} = \dfrac{3+2}{2} = \dfrac{5}{2}$

$$x_o(n) = \{ -\tfrac{5}{2}, 1, \underset{\uparrow}{0}, -1, \tfrac{5}{2} \}$$

$\cos 2t = 1 - 2\sin^2 t$

$\sin^2 t = \frac{\cos 2t}{2}$ $\frac{1+\cos 2t}{2}$

(ii) $x(t) = \sin t + 2\sin t + 2\sin^2 t \cos t$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x(-t) = \sin(-t) + 2\sin(-t) + 2\sin^2(-t)\cos(-t)$$

$$= -\sin t - 2\sin t + 2\sin^2 t \cos t$$

$$x_e(t) = \frac{\sin t + 2\sin t + 2\sin^2 t\cos t - \sin t - 2\sin t + 2\sin^2 t \cos t}{2}$$

$$= \frac{\cancel{4}^{2}\sin^2 t\cos t}{\cancel{2}}$$

$$= \cancel{2}\left(\frac{1+\cos 2t}{\cancel{2}}\right)\cos t$$

$$x_e(t) = \cos t\,(1+\cos 2t) = 2\sin^2 t\cos t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{\sin t + 2\sin t + 2\sin^2 t\cos t + \sin t + 2\sin t - 2\sin^2 t\cos t}{2}$$

$$= \frac{2\sin t + 4\sin t}{2}$$

$$= \frac{\cancel{2}(\sin t + 2\sin t)}{\cancel{2}}$$

$$x_o(t) = \sin t + 2\sin t = 3\sin t$$

(iii) $x(n) = \{1, 0, -1, 2, 3\}$

$x(0) = 1,\ x(1) = 0,\ x(2) = -1,\ x(3) = 2,\ x(4) = 3$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$n = 0,\ x_e(0) = \frac{x(0) + x(0)}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$$

$$n = 1,\ x_e(1) = \frac{x(1) + x(-1)}{2} = \frac{0+0}{2} = \frac{0}{2} = 0$$

$n=2,\ x_e(2) = \frac{x(2)-x(-2)}{2} = \frac{-1+0}{2} = -1/2$

$n=3,\ x_e(3) = \frac{x(3)-x(-3)}{2} = \frac{2+0}{2} = \frac{2}{2} = 1$

$n=4,\ x_e(4) = \frac{x(4)-x(-4)}{2} = \frac{3+0}{2} = 3/2$

$x_e(n) = \{\underset{\uparrow}{1}, 0, -1/2, 1, 3/2\}$

Odd : $x_o(n) = \frac{x(n)-x(-n)}{2}$

$n=0,\ x_o(0) = \frac{x(0)-x(0)}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$

$n=1,\ x_o(1) = \frac{x(1)-x(-1)}{2} = \frac{0-0}{2} = \frac{0}{2} = 0$

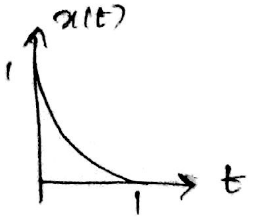
$n=2,\ x_o(2) = \frac{x(2)-x(-2)}{2} = \frac{-1-0}{2} = \frac{-1}{2} = -1/2$

$n=3,\ x_o(3) = \frac{x(3)-x(-3)}{2} = \frac{2-0}{2} = \frac{2}{2} = 1$

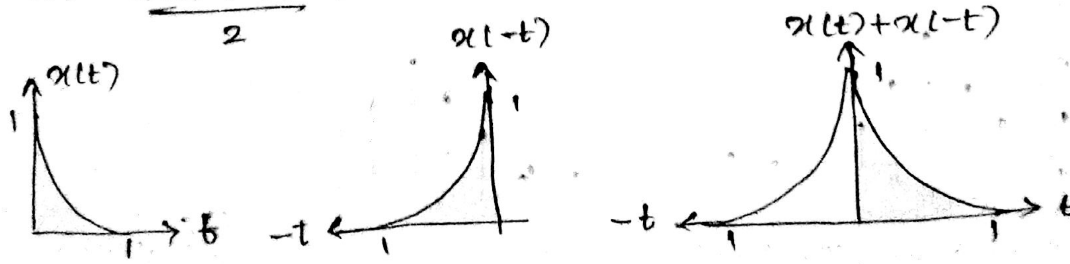
$n=4,\ x_o(4) = \frac{x(4)-x(-4)}{2} = \frac{3-0}{2} = 3/2$

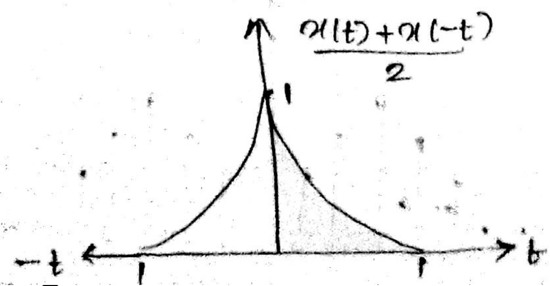
$x_o(n) = \{\underset{\uparrow}{0}, 0, -1/2, 1, 3/2\}$

(iv)

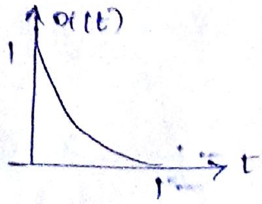


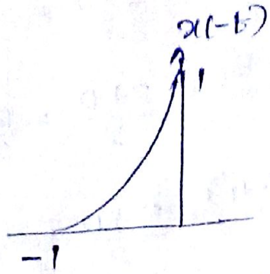
$x(t) = \frac{x(t)+x(-t)}{2}$

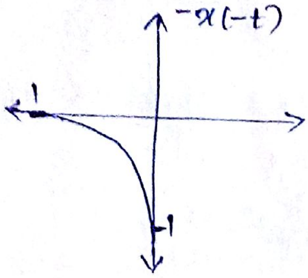


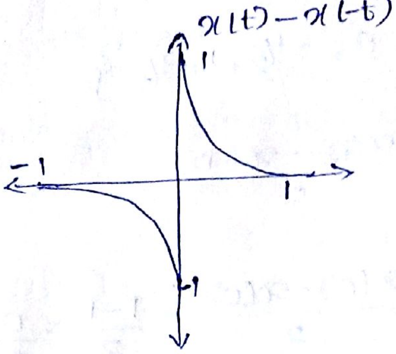


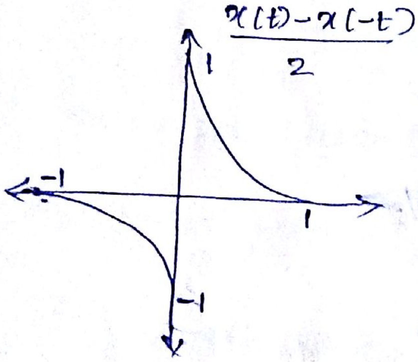
$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



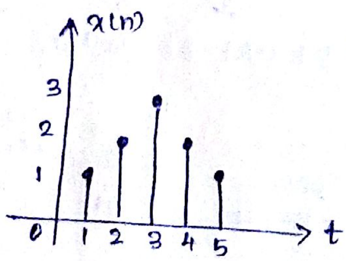




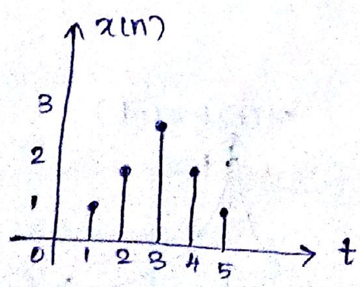


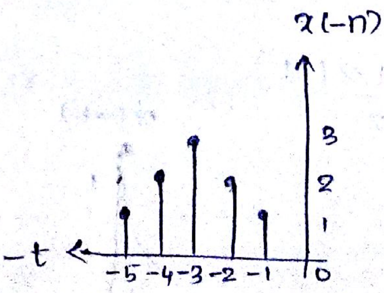


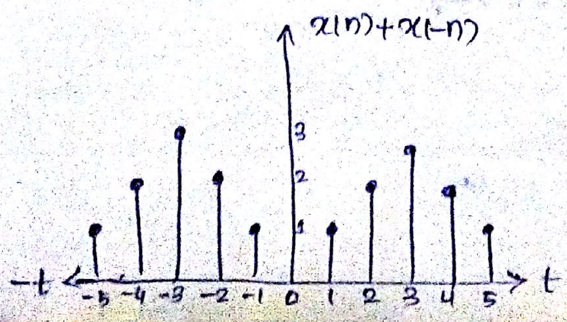
(v)

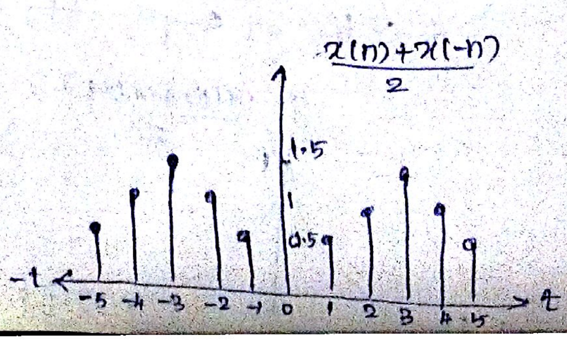


$$x_e(n) = \frac{x(n) + x(-n)}{2}$$









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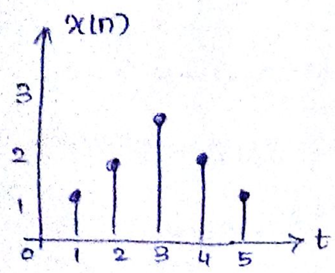
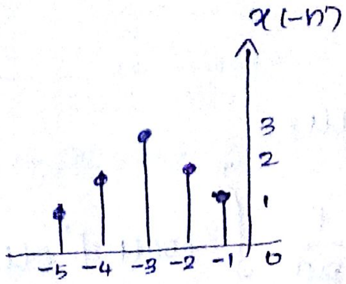
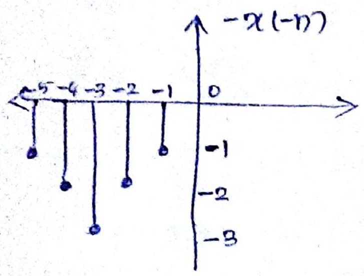
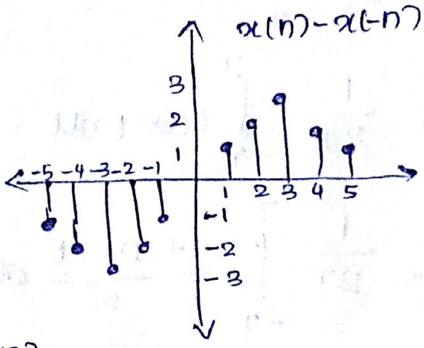
$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

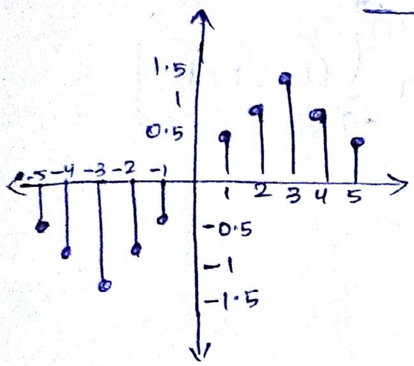








$\frac{x(n) - x(-n)}{2}$



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Energy and Power signal :

Integration of cosine wave over full cycle $= 0$

(1) $x(t) = \cos t$

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} \cos^2 t \, dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} \frac{1 + \cos 2t}{2} dt$$

$$= \frac{1}{2} \left[\lim_{T \to \infty} \int_{-T}^{T} 1 \, dt + \lim_{T \to \infty} \underbrace{\int_{-T}^{T} \cancel{\cos 2t} \, dt}_{} \right]$$

$$= \frac{1}{2} \lim_{T \to \infty} [T + T]$$