Strong induction [Second principle of mathematical Induction]

In this form, we use the same basis step as before, but we use a different inductive step.

i). Assume that $P(j)$ is true for $j = 1, 2, \dots, k$

ii). we have to prove that $P(k+1)$ is true

well ordering property:

Every nonempty set of non-negative integers has a least element.

Pigeonhole principle

If $(n+1)$ pigeon occupies n holes then atleast one hole has more than one pigeon

Proof:

Assume that there are $(n+1)$ pigeons and n holes.

To prove atleast one hole has more than one pigeon.

we prove this by method of contradiction.

Suppose not, atleast one hole has not more than one pigeon.

From this each and every hole has exactly one pigeon.

Since there are n holes, which implies we have totally n pigeons which is a contradiction to our assumption.

Hence atleast one hole has more than one pigeon.

Generalized pigeonhole principle

If 'm' pigeon occupies 'n' holes then atleast one hole has more than $\left[\frac{m-1}{n}\right]+1$ pigeons.

Here $[x]$ denotes the greatest integer less than or equal to x, which is a real number.

1. Show that among 100 people, atleast 9 of them were born in the same month.

m = number of pigeons = No. of people = 100

n = No. of Holes = No. of month = 12

By Generalized PHP,

$$\left\lfloor \frac{m-1}{n} \right\rfloor + 1$$

$$= \left\lfloor \frac{100-1}{12} \right\rfloor + 1 = 9$$ were born in the same month.

2. Show that if 25 dictionaries in a library contain a total of 40,325 pages, then one of the dictionaries must have atleast 1614 pages.

No. of pigeons : m = No. of pages = 40,325

No. of holes : n = No. of dictionaries = 25

By Generalized PHP,

$$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = \left\lfloor \frac{40,325-1}{25} \right\rfloor + 1$$

= 1614 pages in the dictionaries

HW

1. Show that in a group of 6 people, atleast 3 must be mutual friends (or) atleast 3 must be strangers.

3. Show that if 7 colors are used to paint 50 bicycles, atleast 8 bicycles will be the same colour.

m = No. of bicycles = 50

n = No. of colour = 7

By Generalized PHP,

$$\left\lfloor \frac{m-1}{n} \right\rfloor + 1 = \left\lfloor \frac{50-1}{7} \right\rfloor + 1$$

$= 7 + 1$

$= 8$ bicycles will be the same colour.