Unit - 2

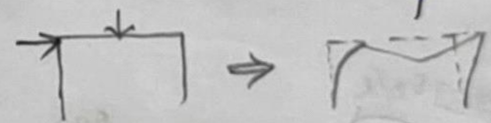
Slope Deflection Method

Introduction :

The slope deflection method is a structural analysis method for beams and frames. If the slopes at the ends and displacements are known then the end moment can be found interms of slopes, deflection, stiffness and length of the members.

Assumptions :

1. All the joints of the frame are rigid. The angle between the members at the joints do not change, when the members of the frame are loaded.



2. Distortion due to axial and shear stresses being very small, are neglected.

Degree of Freedom :

The no of joints rotation and independent joint translation in a structure is called the degree of freedom.

1. Rotation :

$Dr = j - f$

Dr = Degree of freedom
j = No of joints including supports
F = No. of fixed supports

2. Translation :

In a frame each joint has two translation ($2j$). But fixed and pinned supports resists 2 translation ($2f$), ($2h$), roller resists 1 translation and members ~~connect~~ resist 1 translation.

$$D_t = 2j - (2f + 2h + r + m)$$

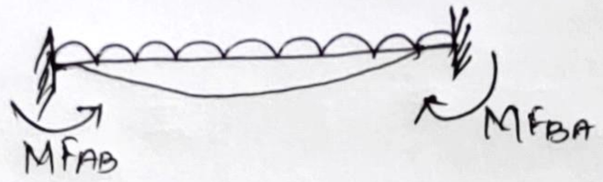
The combined Degree of freedom

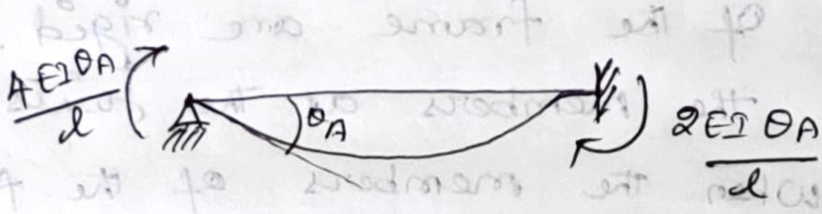
$$D = D_r + D_t$$

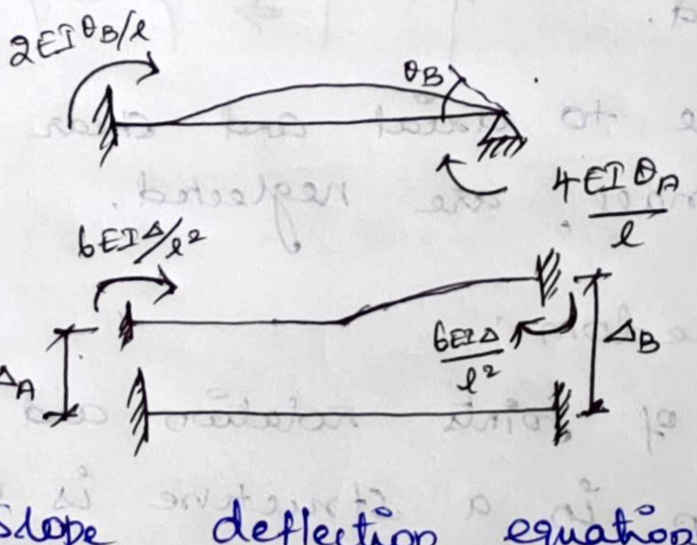
$$= (j - f) + 2j - (2f + 2h + r + m)$$

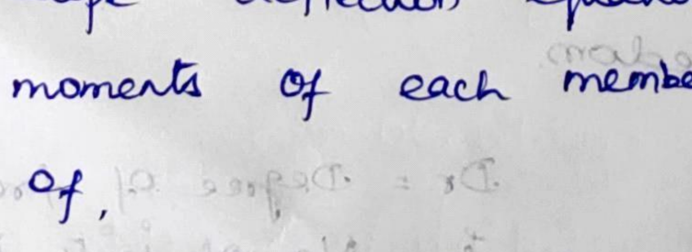
$$D = 3j - 3f - 2h - r - m$$

Slope Deflection Equations :









$$\Delta = \Delta_B - \Delta_A$$

Slope deflection equations express the final end moments of each member in a frame in terms of,

(i) Fixed end moments due to external loads.

(ii) Moments due to rotation at A

(iii) Moments due to rotation at B.

(iv) Moments due to differential transverse displacement of B above A.

Slope deflection equations are,

$$M_{AB} = M_{FAB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} + \frac{6EI\Delta}{l^2}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$M_{BA} = M_{FBA} + \frac{2EI\theta_A}{l} + \frac{4EI\theta_B}{l} + \frac{6EI\Delta}{l^2}$$

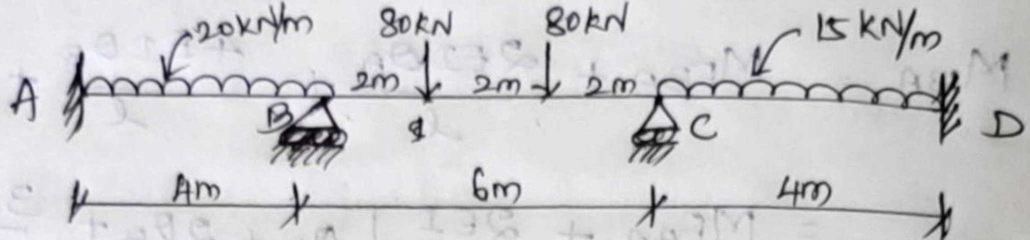
$$= M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B + \frac{3\Delta}{l}\right]$$

Step by step procedure:

1. For all the members write down the fixed end moments due to given loading.
2. For each member write down the slope deflection equation in terms of joint rotation and displacements ($\theta_A, \theta_B, \ldots \Delta_A, \Delta_B \ldots$)
3. At every joint Several members join. Each member has a moment acting on it. Write down the joint Equillibrium equations. (Ex ⇒. At Joint C members: AC, BC, BC Eq. Equation $M_{CA} + M_{CB} + M_{CD} = 0$)
4. Solve the equillibrium equations to get the unknown displacements.
5. Use this unknown values in slope deflection equations to obtain final joint moments.
6. Combine the joint moments of each member with the free bending moments to get the final bending moment diagram in each member.

1. Analyse the continuous beam loaded as shown in figure by the slope deflection method and sketch the bending moment diagram.

$$\left[2I_{AB} = I_{BC} = 2I_{CD}\right] \quad \frac{2I}{l}$$



Sol:

$$I_{AB} = I_{CD} = I, \quad I_{BC} = 2I$$

$$\theta_A = \theta_D = 0 \quad (A \text{ and } D \text{ are fixed})$$

Fixed End Moments:

Span AB, $MF_{AB} = \frac{-wl^2}{12} = -\frac{20 \times 4^2}{12} = -26.67\ kNm$

$$MF_{BA} = \frac{wl^2}{12} = \frac{20 \times 4^2}{12} = 26.67\ kNm$$

Span BC, $MF_{BC} = -\left(\frac{wa_1 b_1^2}{l^2} + \frac{wa_2 b_2^2}{l^2}\right)$

$$= -\left(\frac{80 \times 2 \times 4^2}{6^2} + \frac{80 \times 4 \times 2^2}{6^2}\right)$$

$$= -106.67\ kNm$$

$$MF_{CB} = \left(\frac{wa_1^2 b_1}{l^2} + \frac{wa_2^2 b_2}{l^2}\right)$$

$$= \left(\frac{80 \times 2^2 \times 4}{6^2} + \frac{80 \times 4^2 \times 2}{6^2}\right)$$

$$= 106.67\ kNm$$

Span CD, $MF_{CD} = -\frac{wl^2}{12} = -\frac{15 \times 4^2}{12} = -20\ kNm$

$$MF_{DC} = \frac{wl^2}{12} = \frac{15 \times 4^2}{12} = 20\ kNm$$

Slope Deflection Equation :

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + \frac{3\Delta}{l}\right]$$

$$= -26.67 + \frac{2EI}{4}(0 + \theta_B + 0)$$ $\Delta = 0$ no settlement

$$= -26.67 + 0.5\,EI\theta_B \longrightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A + \frac{3\Delta}{l}\right)$$

$$= 26.67 + \frac{2EI}{4}(2\theta_B + 0 + 0)$$

$$M_{BA} = 26.67 + EI\theta_B \longrightarrow (2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{3\Delta}{l}\right)$$

$$= -106.67 + \frac{2E \times 2I}{6}(2\theta_B + \theta_C + 0)$$

$$= -106.67 + EI(1.333\,\theta_B + 0.666\,\theta_C) \longrightarrow (3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{3\Delta}{l}\right)$$

$$= 106.67 + \frac{2E \times 2I}{6}(2\theta_C + \theta_B + 0)$$

$$= 106.67 + EI(1.333\,\theta_C + 0.666\,\theta_B) \longrightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D + \frac{3\Delta}{l}\right)$$

$$= -20 + \frac{2EI}{4}(2\theta_C + 0 + 0)$$

$$= -20 + EI\theta_C \longrightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C + \frac{3\Delta}{l}\right)$$

$$= 20 + \frac{2EI}{4}(0 + \theta_C + 0)$$

$$= 20 + EI(0.5\,\theta_C) \longrightarrow (6)$$

Equilibrium Equations:

$$M_{BA} + M_{BC} = 0 \rightarrow (7)$$

$$M_{CB} + M_{CD} = 0 \rightarrow (8)$$

Sub (2), (3) in (7) $\Rightarrow$

$$26.67 + EI\theta_B - 106.67 + (1.33\theta_B + 0.666\theta_C)EI = 0$$

$$2.333\theta_B + 0.666\theta_C = \frac{80}{EI} \rightarrow (9)$$

Sub (4), (5) in (8) $\Rightarrow$

$$106.67 + EI(0.666\theta_B + 1.333\theta_C) - 20 + EI\theta_C = 0$$

$$0.666\theta_B + 2.333\theta_C = -\frac{86.67}{EI} \rightarrow (10)$$

(9) & (10) can be written as,

$$\begin{pmatrix} 2.333 & 0.666 \\ 0.666 & 2.333 \end{pmatrix}\begin{pmatrix} \theta_B \\ \theta_C \end{pmatrix} = \frac{1}{EI}\begin{pmatrix} 80 \\ -86.67 \end{pmatrix}$$

$$\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \frac{1}{EI}\begin{Bmatrix} 80 \\ -86.67 \end{Bmatrix}\begin{pmatrix} 2.333 & 0.666 \\ 0.666 & 2.333 \end{pmatrix}^{-1}$$

$$= \frac{1}{EI}\begin{Bmatrix} 80 \\ -86.67 \end{Bmatrix}\frac{1}{5}\begin{pmatrix} 2.333 & -0.666 \\ -0.666 & 2.333 \end{pmatrix}$$

$$\boxed{A^{-1} = \frac{1}{|A|}\,\text{adj } A}$$

$$= \frac{1}{EI}\begin{Bmatrix} 16 \\ -17.34 \end{Bmatrix}\begin{Bmatrix} 2.333 & -0.666 \\ -0.666 & 2.333 \end{Bmatrix}$$

$$\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \frac{1}{EI}\begin{Bmatrix} 48.88 \\ -51.11 \end{Bmatrix}$$

$$\theta_B = \frac{48.88}{EI}, \quad \theta_C = \frac{-51.11}{\theta_C}$$

Final Moments:

$$M_{AB} = -26.67 + 0.5EI\left(\frac{48.88}{EI}\right) = -2.23\,kNm$$

$$M_{BA} = 26.67 + EI\left(\frac{48.88}{EI}\right) = 75.55\,kNm$$

$$M_{BC} = -106.67 + EI\left(\left(1.333 \times \frac{48.88}{EI}\right) + \left(0.666 \times \frac{-51.11}{EI}\right)\right)$$

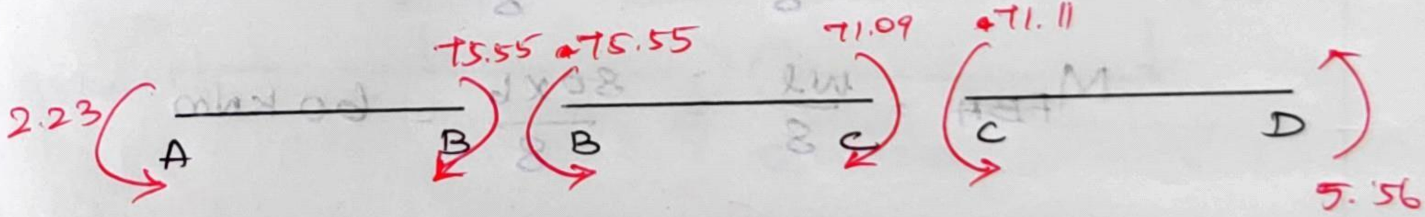
$$= -75.55\,kNm$$

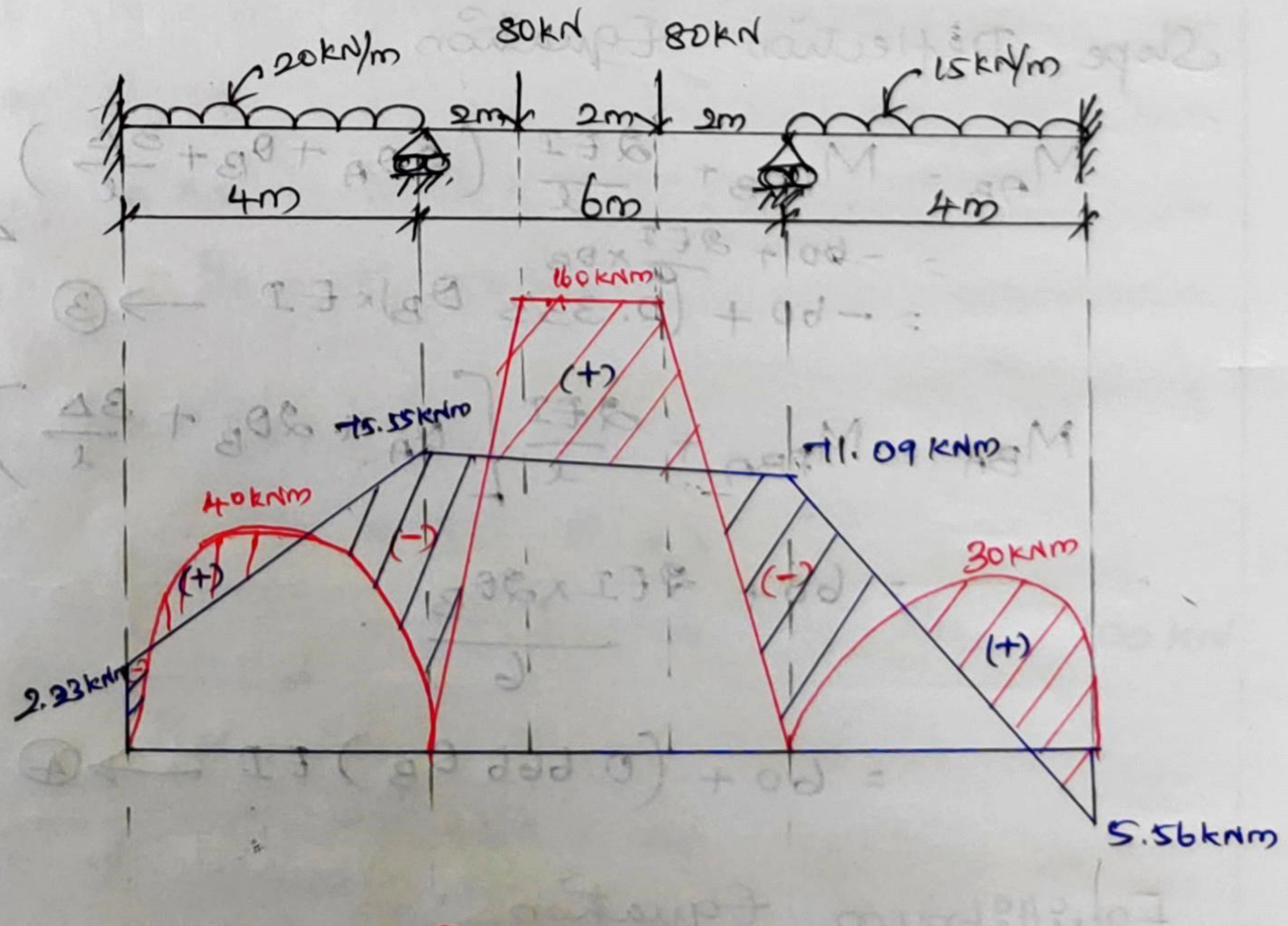
$$M_{CB} = 106.67 + EI\left(\left(0.666 \times \frac{48.88}{EI}\right) + \left(1.333 \times \frac{-51.11}{EI}\right)\right)$$

$$= 71.09\,kNm$$

$$M_{CD} = -20 + EI\left(\frac{-51.11}{EI}\right) = -71.11\,kNm$$

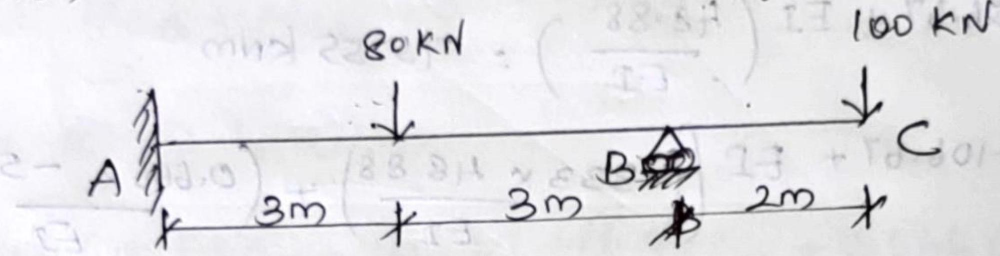
$$M_{DC} = 20 + \left(0.5EI \times \frac{-51.11}{EI}\right) = -5.56\,kNm$$



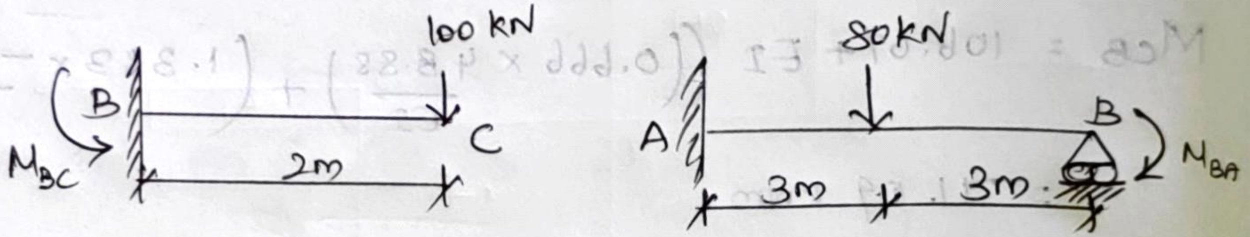


BMD

2. Analyse the beam loaded as shown in fig by the Slope deflection method. Draw bending moment diagram and shear force diagram. EI is constant.



Soln :



$M_{BC} = -200 \text{ kNm} \rightarrow ①$ $\quad M_{BA} = 200 \text{ kNm} \rightarrow ②$

Fixed End Moment :

$$M_{FAB} = -\frac{Wl}{8} = -\frac{80 \times 6}{8} = -60 \text{ kNm}$$

$$M_{FBA} = \frac{Wl}{8} = \frac{80 \times 6}{8} = 60 \text{ kNm}$$

Slope Deflection Equation :

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\Delta}{l}\right)$$

$\Delta = 0$.

$$= -60 + \frac{2EI}{6} \times \theta_B$$

$$= -60 + (0.333\ \theta_B) \times EI \rightarrow ③$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B + \frac{3\Delta}{l}\right)$$

$$= 60 + \frac{2EI \times 2\theta_B}{6}$$

$$= 60 + (0.666\ \theta_B)\ EI \rightarrow ④$$

Equillibrium Equation :

$M_{BA} = 200 \text{ kNm}$

④ $\Rightarrow$ $200 = 60 + 0.666\ \theta_B\ EI$

$$\theta_B = \frac{210.21}{EI}$$

(or) sub ① & ④ in ⑤

$$M_{BA} + M_{BC} = 0 \rightarrow ⑤$$

$$60 + 0.666\,\theta_B EI - 200 = 0$$

$$0.666\,\theta_B EI = 140$$

$$\theta_B = \frac{210.21}{EI}$$

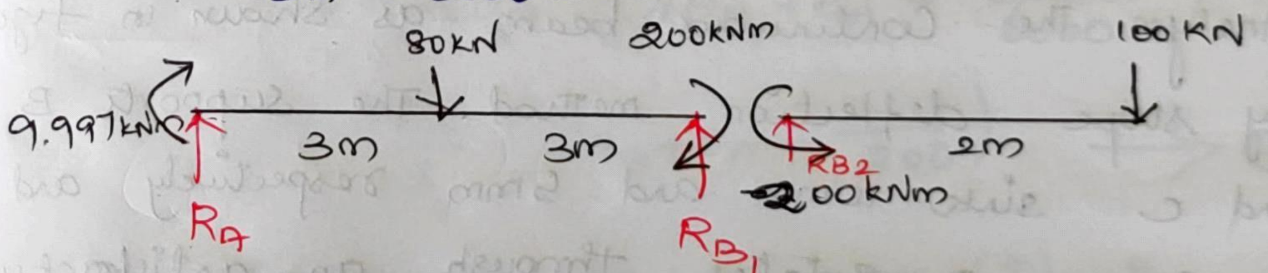
Final Moments :

$$M_{AB} = -60 + EI \times 0.333\,\theta_B$$

$$= -60 + EI \times 0.333 \times \frac{210.21}{EI}$$

$$= 9.997 \text{ kNm}$$

$$M_{BA} = 200 \text{ kNm}$$



Shear Force :

$\Sigma M_B = 0$, $R_A \times 6 - 80 \times 3 + 9.997 + 200 = 0$

$$R_A = 8.33 \text{ kN}$$

$\Sigma V = 0$, $R_A + R_{B_1} = 80$

$$R_{B_1} = 80 - 8.33$$

$$= 71.667 \text{kN}, \quad R_{B_2} = 100 \text{kN}$$

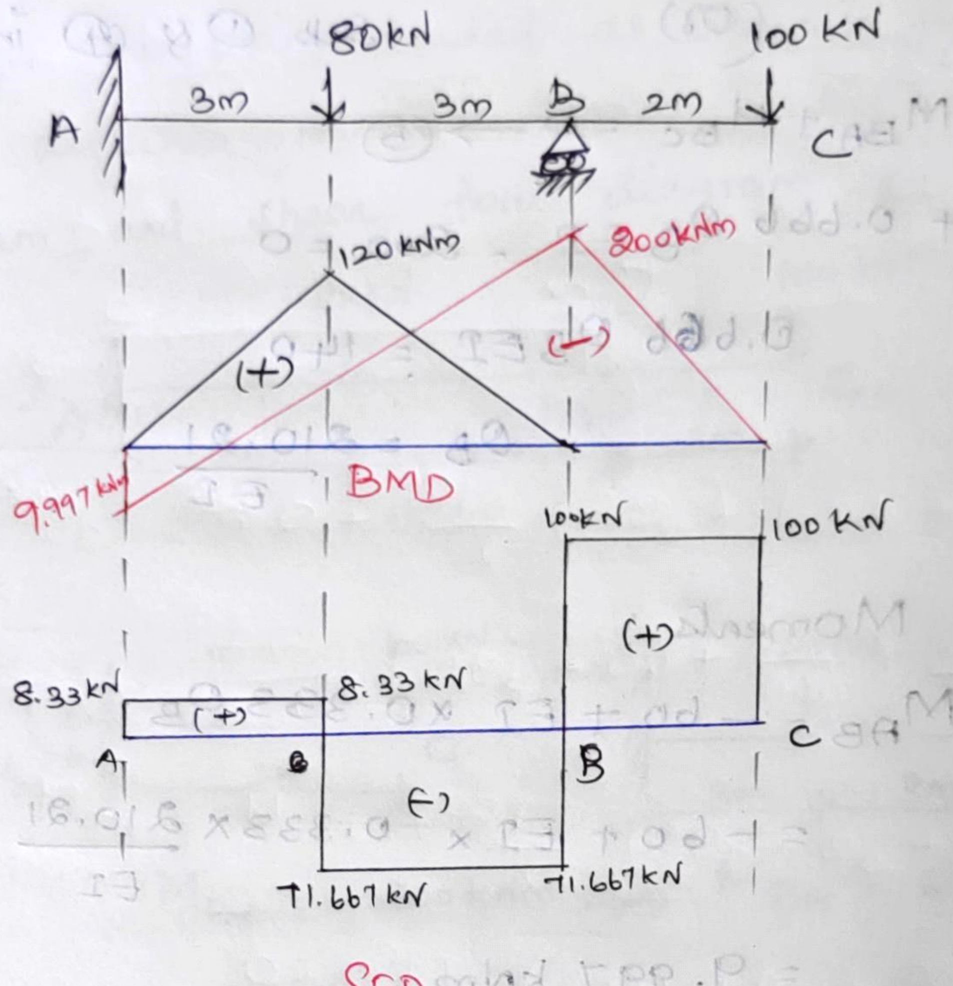
$\Sigma V = 0$, $R_B = R_{B_1} + R_{B_2}$

$$= 71.667 + 100$$

$$= 171.667 \text{ kN}$$

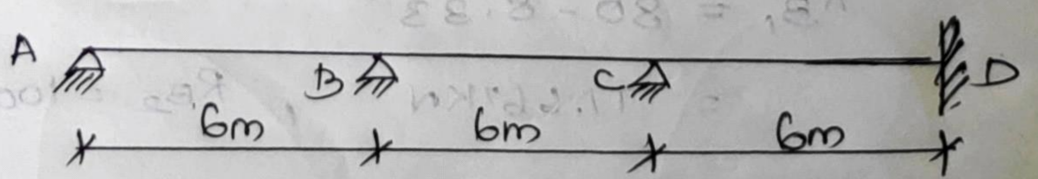
Maximum Bending Moment :

$$M_{AB} = \frac{Wl}{4} = \frac{80 \times 6}{4} = 120 \text{ kNm}$$



SFD.

3. Analyse the continuous beam as shown in figure by slope deflection method. The supports B and C sink 10mm and 5mm respectively and the support D rotates through an anticlockwise angle of 0.1 radian. There are no loads on the beam. Values of E and I are constant throughout the length of the beam. Take, $E = 2\times10^5$ Mpa, $I = 4\times10^7\ mm^4$. Sketch the BMD.



Soln :

$$E = 2\times10^5\times10^6\ N/m^2 = 2\times10^5\times10^6\ N/10^6\times mm^2 = 2\times10^5\ N/mm^2$$

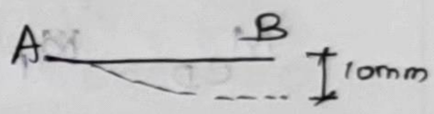
All the spans are not loaded, the fixed end moments in all the span is zero due to loading. The fixed end moments due to settlement of supports B and C and rotation of end D will be included in the slope deflection equation.

Slope Deflection Equation :

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\Delta}{l}\right)$$

$$\boxed{\frac{2EI}{l} = \frac{2\times 2\times 10^5 \times 4\times 10^7}{6000} = \frac{16}{6}\times 10^9 = \frac{8\times 10^9}{3}}$$

Support B sinks at 10mm,



$$M_{AB} = 0 + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\times 10}{6000}\right)$$

$$= \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{1}{200}\right) \longrightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\times 10}{6000}\right)$$

$$= \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{1}{200}\right) \longrightarrow (2)$$

Net Settlement at B = 5mm (↑)

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{3\times 5}{6000}\right)$$

$$= \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{1}{400}\right) \longrightarrow (3)$$

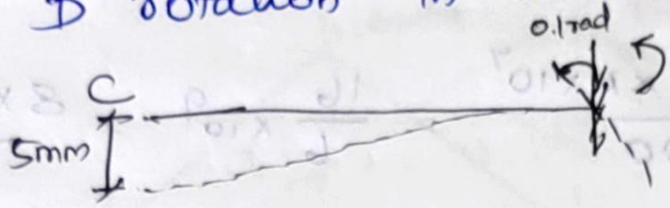
$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{3\times 5}{6000}\right)$$

$$= \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{1}{400}\right) \longrightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D + \frac{3\Delta}{l}\right.$$

At D rotation in anticlockwise 0.1 radians.



$$M_{CD} = M_F\, 0 + \frac{2EI}{l}\left(2\theta_C + (-0.1) + \frac{3\times5}{6000}\right)$$

$$= \frac{2EI}{l}\left(2\theta_C - 0.1 + \frac{1}{400}\right) \rightarrow (5)$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{l}\left(-2\times0.1 + \theta_C + \frac{3\times5}{6000}\right)$$

$$= \frac{2EI}{l}\left(-0.2 + \theta_C + \frac{1}{400}\right) \rightarrow (6)$$

Equillibrium Equations:

$M_{AB} = 0$ (At A) $\rightarrow (7)$

$M_{BA} + M_{BC} = 0$ (At B) $\rightarrow (8)$

$M_{CB} + M_{CD} = 0$ (At C) $\rightarrow (9)$

Sub (1) in (7) $\Rightarrow$

$$\frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{1}{200}\right) = 0$$

$$2\theta_A + \theta_B - \frac{1}{200} = 0$$

$$2\theta_A + \theta_B = 0.005 \rightarrow (10)$$

Sub (2) & (3) in (8) $\Rightarrow$

$$\frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{1}{200}\right) + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{1}{400}\right) = 0$$

$$\frac{2EI}{l}\left(4\theta_B + \theta_A + \theta_C - \frac{1}{400}\right) = 0$$

$\theta_A + 4\theta_B + \theta_C - 0.0025 = 0$

$\theta_A + 4\theta_B + \theta_C = 0.0025 \longrightarrow$ (10)

sub (4) & (5) in (9) ⇒

$$\frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{1}{400}\right) + \frac{2EI}{l}\left(2\theta_C - 0.1 + \frac{1}{400}\right) = 0$$

$$\frac{2EI}{l}\left(4\theta_C + \theta_B - 0.1 + 0.005\right) = 0$$

$$4\theta_C + \theta_B - 0.095 = 0$$

$$4\theta_C + \theta_B = 0.095 \longrightarrow (12)$$

(11) × 4 ⇒ $4\theta_A + 16\theta_B + 4\theta_C = 0.010 \longrightarrow$ (13)

(13) − (12) ⇒ $4\theta_A + 15\theta_B = -0.085 \longrightarrow$ (14)

(10) × 2 ⇒ $4\theta_A + 2\theta_B = 0.010 \longrightarrow$ (15)

(14) − (15) ⇒ $13\theta_B = -0.095$

$\boxed{\theta_B = -0.0073}$, $\boxed{\theta_A = 0.0062}$

(14) ⇒ $\theta_C = 0.0025 - \theta_A - 4\theta_B$

$= 0.0025 - 0.0062 - 4(-0.0073)$

$\boxed{\theta_C = 0.0255}$

Final Moments:

$$M_{AB} = \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{1}{200}\right)$$

$$= \frac{8\times10^9}{3}\left(2\times0.0062 - 0.0073 - \frac{1}{200}\right)$$

$$= 0.027\times10^7 \text{ Nmm}$$

$$M_{BA} = \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{1}{200}\right)$$

$$= \frac{8\times10^9}{3}\left(2\times-0.0073 + 0.0062 - \frac{1}{200}\right)$$

$$= -3.573\times10^7 \text{ Nmm}$$

$$M_{BC} = \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{1}{400}\right)$$

$$= \frac{8\times10^9}{3}\left(2\times -0.0073 + 0.0255 + \frac{1}{400}\right)$$

$$= 3.573\times10^7 \text{ Nmm}$$

$$M_{CB} = \frac{2EI}{l}\left(2\theta_C + \theta_B + \frac{1}{400}\right)$$

$$= \frac{8\times10^9}{3}\left(2\times0.0255 - 0.0073 + \frac{1}{400}\right)$$

$$= 12.32\times10^7 \text{ Nmm}$$

$$M_{CD} = \frac{2EI}{l}\left(2\theta_C - 0.1 + \frac{1}{400}\right)$$

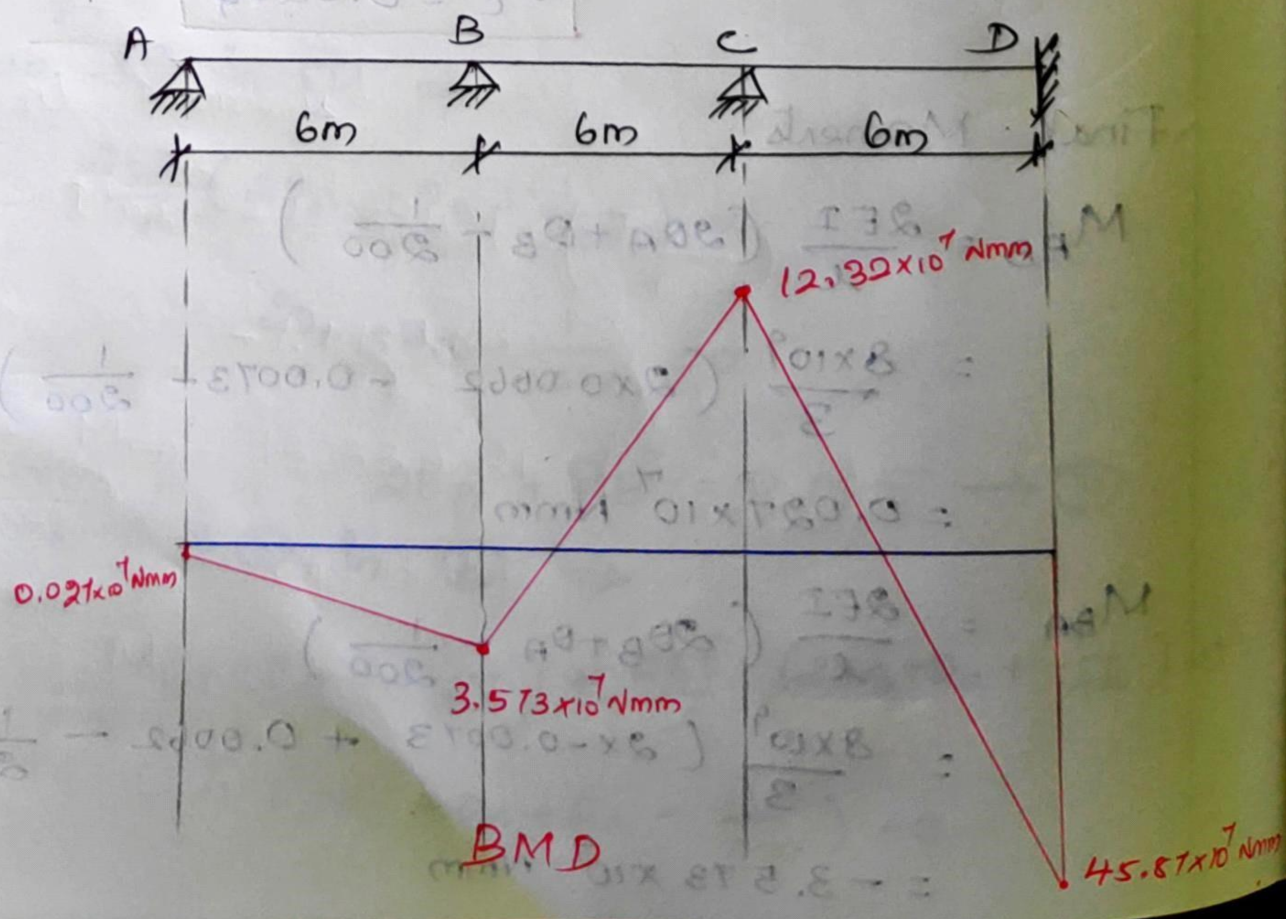
$$= \frac{8\times10^9}{3}\left(2\times0.0255 - 0.1 + \frac{1}{400}\right)$$

$$= -12.40\times10^7 \text{ Nmm}$$

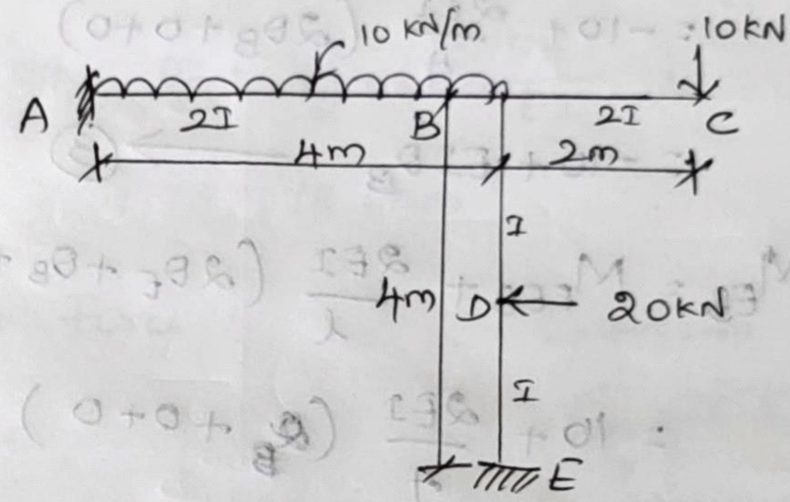
$$M_{DC} = \frac{2EI}{l}\left(-0.2 + \theta_C + \frac{1}{400}\right)$$

$$= \frac{8\times10^9}{3}\left(-0.2 + 0.0255 + \frac{1}{400}\right)$$

$$= -45.87 \text{ Nmm}$$



A) Analyze the structure as shown in figure by the slope deflection method. Sketch the BMD and SFD.



Soln :

Fixed End Moments :

Span AB, $M_{FAB} = -\frac{wl^2}{12} = -\frac{10 \times 16}{12} = -13.33$ kNm

$M_{FBA} = \frac{wl^2}{12} = \frac{10 \times 16}{12} = 13.33$ kNm

Span BC, $M_{FBC} = -10 \times 2 = -20$ kNm

Span BE, $M_{FBE} = -\frac{wl}{8} = -\frac{20 \times 4}{8} = -10$ kNm

$M_{FEB} = \frac{wl}{8} = \frac{20 \times 4}{8} = 10$ kNm

Slope Deflection Equation : $\theta_A = 0$ (fixed end)
$\delta = 0$ (No settlement)

Span AB, $M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\Delta}{l}\right)$

$= -13.33 + \frac{2E(2I)}{4}(0 + \theta_B + 0)$

$= -13.33 + EI\theta_B \longrightarrow (1)$

$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A + \frac{3\Delta}{l}\right)$

$= 13.33 + \frac{2E \times 2I}{4}(2\theta_B + 0 + 0)$

$= 13.33 + 2\theta_B EI \longrightarrow (2)$

Span BE, $M_{BE} = M_{FBE} + \frac{2EI}{l}\left(2\theta_B + \theta_E + \frac{3\Delta}{l}\right)$

$= -10 + \frac{2EI}{4}(2\theta_B + 0 + 0)$

$= -10 + EI\theta_B \longrightarrow (3)$

$M_{EB} = M_{FEB} + \frac{2EI}{l}\left(2\theta_E + \theta_B + \frac{3\Delta}{l}\right)$

$= 10 + \frac{2EI}{4}(\theta_B + 0 + 0)$

$= 10 + \frac{EI}{2}\theta_B \longrightarrow (4)$

Joint Equilibrium Equations:

$M_{BA} + M_{BC} + M_{BE} = 0$

Sub (2) & (4), M_{BE} in the above equation,

$13.33 + 2EI\theta_B - 10 + EI\theta_B - 20 = 0$

$3\theta_B EI = 16.67$

$\theta_B = \frac{16.67}{3EI}$

$\theta_B = \frac{5.557}{EI}$

Final Moments:

$M_{AB} = -13.33 + EI\left(\frac{5.557}{EI}\right)$

$= -7.773$ kNm

$M_{BA} = 13.33 + 2EI\left(\frac{5.557}{EI}\right)$

$= 24.447$ kNm

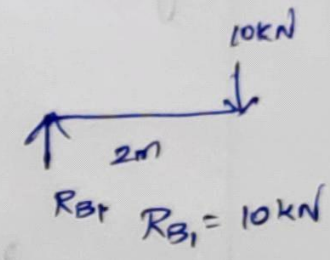
$M_{BC} = -20$ kNm

$$M_{BE} = -10 + EI\left(\frac{5.557}{EI}\right)$$

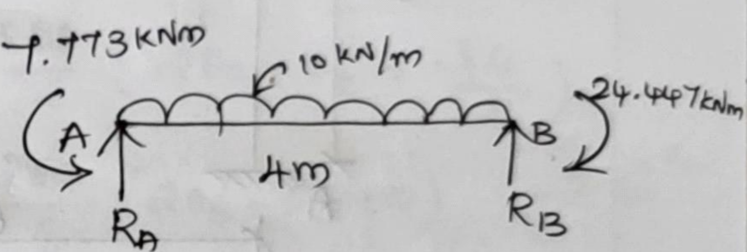
$$= -4.443 \text{ kNm}$$

$$M_{EB} = 10 + \frac{EI}{2}\left(\frac{5.557}{EI}\right)$$

$$= 12.778 \text{ kNm}$$



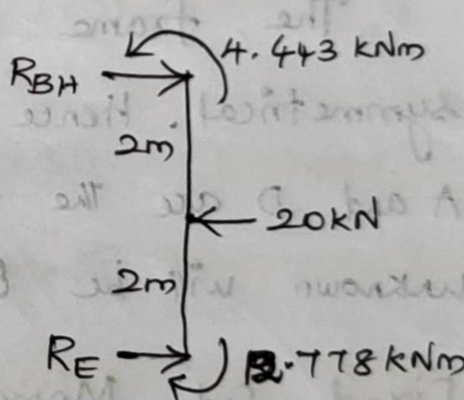
Shear force :



$\Sigma M_B = 0,$

$$R_A \times 4 - (10 \times 4 \times 4/2) - 7.773 + 24.447 = 0$$

$$R_A = 15.832 \text{ kN}$$



$\Sigma V = 0,$

$$R_{BV} = 10 \times 4 - R_A = 40 - 15.832$$

$$= 24.168 \text{ kN}$$

$$R_B = R_{BV} + R_{B1} = 24.168 + 10$$

$$= 34.168 \text{ kN}$$

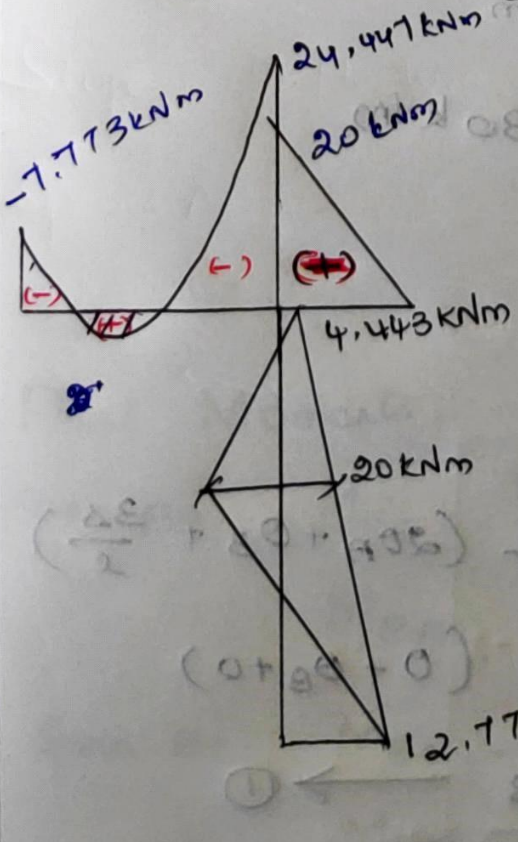
$\Sigma M_B = 0,$

$$-R_E \times 4 - 4.443 + 12.778 + (20 \times 2) = 0$$

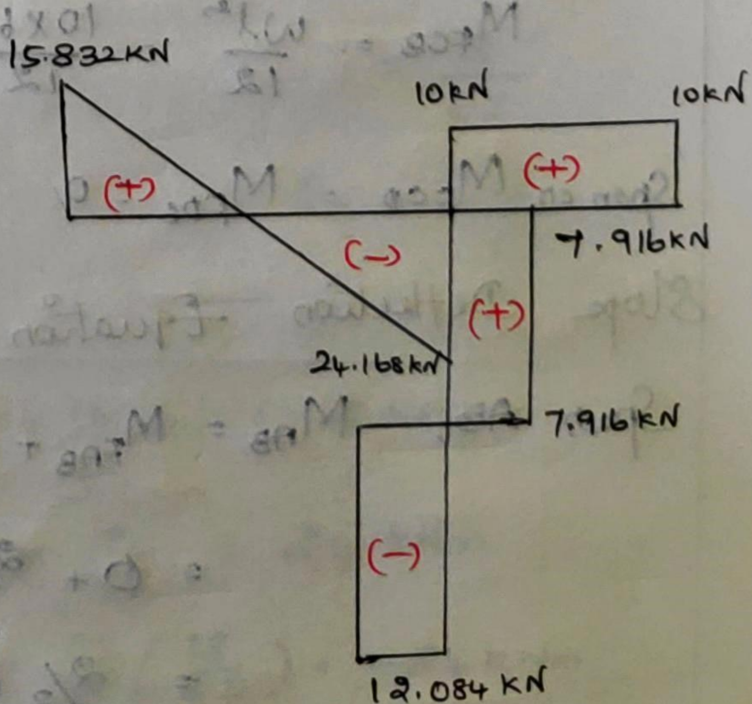
$$R_E = 12.084 \text{ kN}$$

$$R_{BH} = T.L - R_E = 20 - 12.084$$

$$= 7.916 \text{ kN}$$

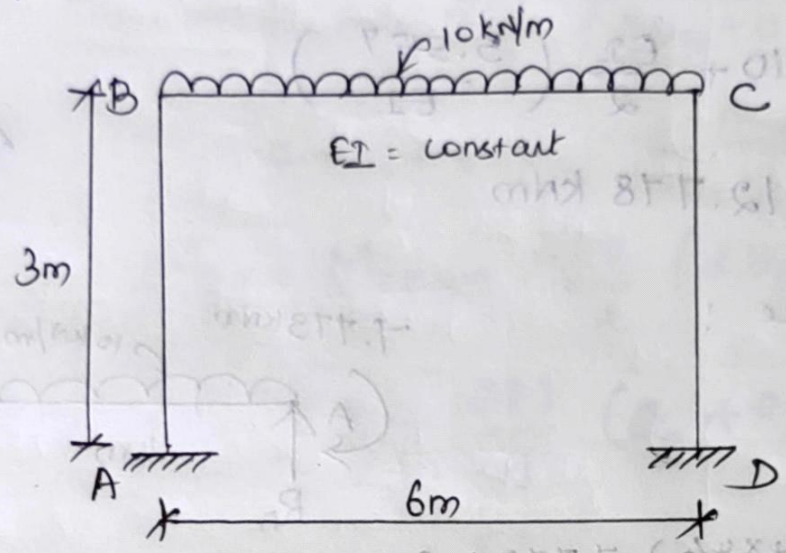


BMD



SFD.

5. Analyse the portal frame loaded as shown in figure by slope deflection method. Sketch the BMD and SFD.



Soln:

The frame end conditions and loadings are symmetrical. Hence there will be no side sway. A and D are the fixed ends ($\theta_A = \theta_D = 0$). The unknown will be θ_B and θ_C.

Fixed End Moments:

Span AB, $M_{FAB} = M_{FBA} = 0$

Span BC, $M_{FBC} = -\frac{wl^2}{12} = \frac{-10 \times 6^2}{12}$

$= -30$ kNm

$M_{FCB} = \frac{wl^2}{12} = \frac{10 \times 6^2}{12} = 30$ kNm

Span CD, $M_{FCD} = M_{FDC} = 0$

Slope Deflection Equation:

Span AB, $M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B + \frac{3\Delta}{l}\right)$

$= 0 + \frac{2EI}{3}(0 + \theta_B + 0)$

$= 2/3 \, EI\theta_B \longrightarrow (1)$

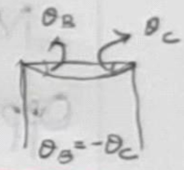
$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A + \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{3}(2\theta_B + 0 + 0)$$

$$= \frac{4}{3} EI\,\theta_B \longrightarrow (2)$$

Span BC, $M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C + \frac{3\Delta}{l}\right)$

$$= -30 + \frac{2EI}{6}(2\theta_B - \theta_B + 0)$$



$$= -30 + \frac{EI}{3}\theta_B \longrightarrow (3) \quad \boxed{(\theta_C = -\theta_B)}$$

Joint Equilibrium Equations:

$$M_{BA} + M_{BC} = 0 \longrightarrow (4)$$

$$M_{CB} + M_{CD} = 0 \longrightarrow (5)$$

Sub (2) & (3) in (4),

$$\frac{4EI}{3}\theta_B - 30 + \frac{EI}{3}\theta_B = 0$$

$$\frac{5EI}{3}\theta_B = 30$$

$$\theta_B = \frac{30 \times 3}{5EI}$$

$$= \frac{18}{EI}$$

$$\theta_C = -\frac{18}{EI}$$

Final Moments:

Span AB, $M_{AB} = \frac{2EI}{3} \times \frac{18}{EI} = 12$ kNm

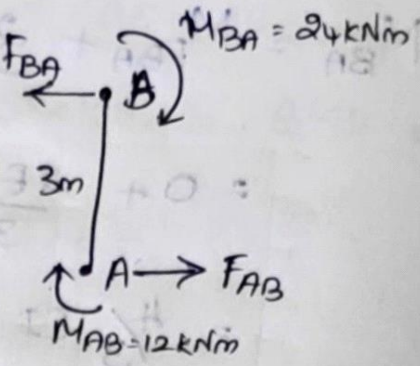
$M_{BA} = \frac{4EI}{3} \times \frac{18}{EI} = 24$ kNm

Span BC, $M_{BC} = -30 + \frac{EI}{3}\left(\frac{18}{EI}\right) = -24$ kNm

$M_{CB} = 24$ kNm

Span CD, $M_{CD} = -24$ kNm, $M_{DC} = -12$ kNm } Due to Symmetry

Shear force :

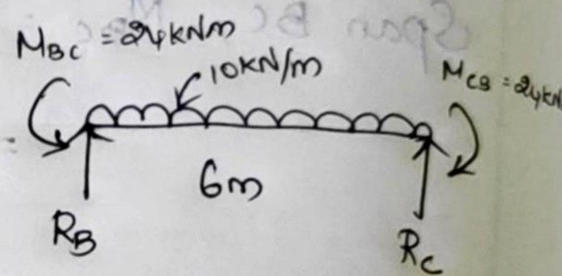


$\Sigma M_A = 0, \quad -F_{BA} \times 3 + M_{BA} + M_{AB} = 0$

$F_{BA} \times 3 = 24 + 12$

$F_{BA} = \frac{36}{3} = 12\ kN$

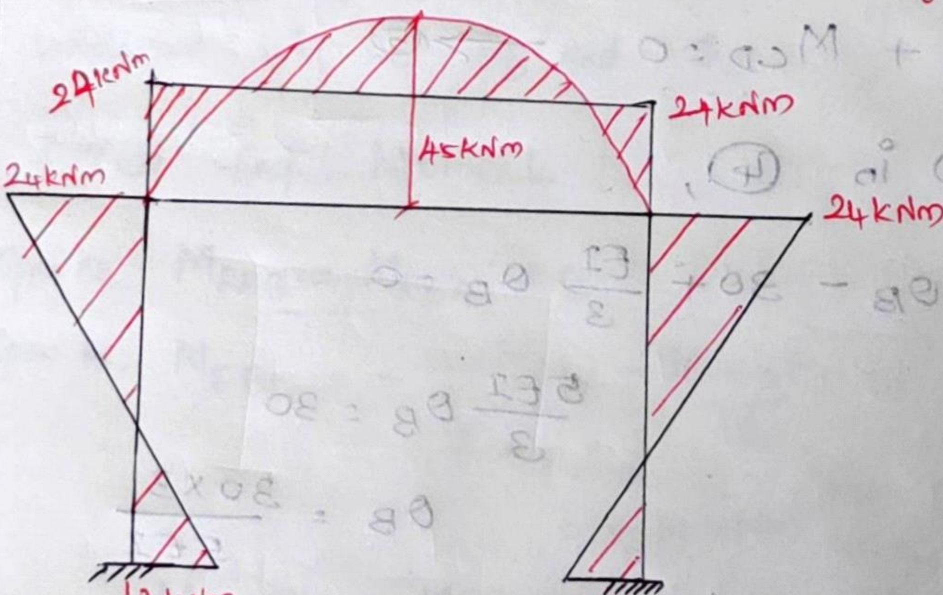
$F_{AB} = 12\ kN$



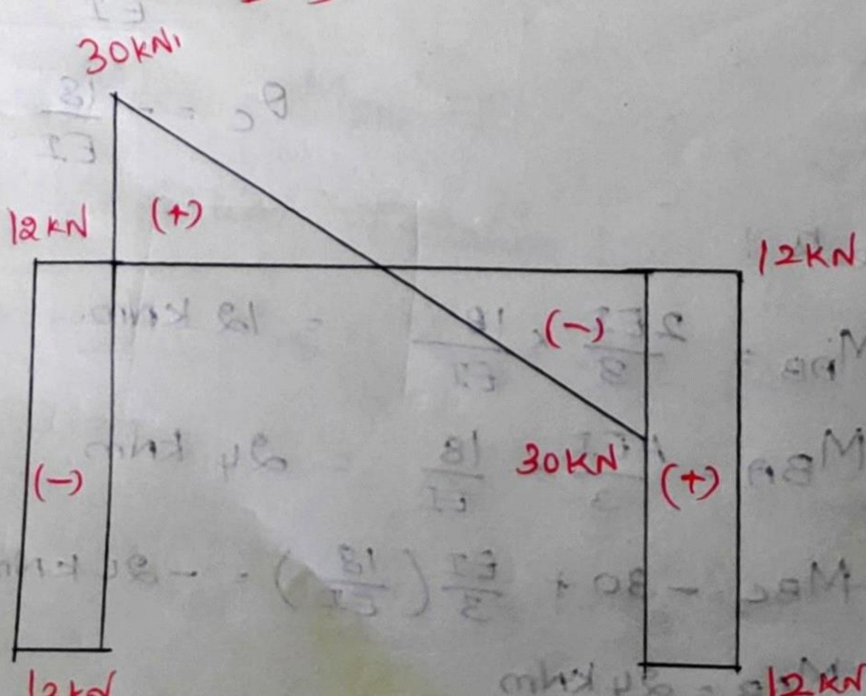
Moments are same and Opposite so there is no ~~moment~~ Shear due to Support moments.

$R_B = R_C = \frac{10 \times 6}{2} = 30\ kN$

$F_{CD} = F_{DC} = 12kN$ (Due to Symmetry)



BMD



SFD

Portal Frames with Side sway:

Portal frames may sway due to one of the following reasons.

(i). Eccentric or unsymmetrical loading on the portal frames.

(ii). Unsymmetrical shape of the frame.

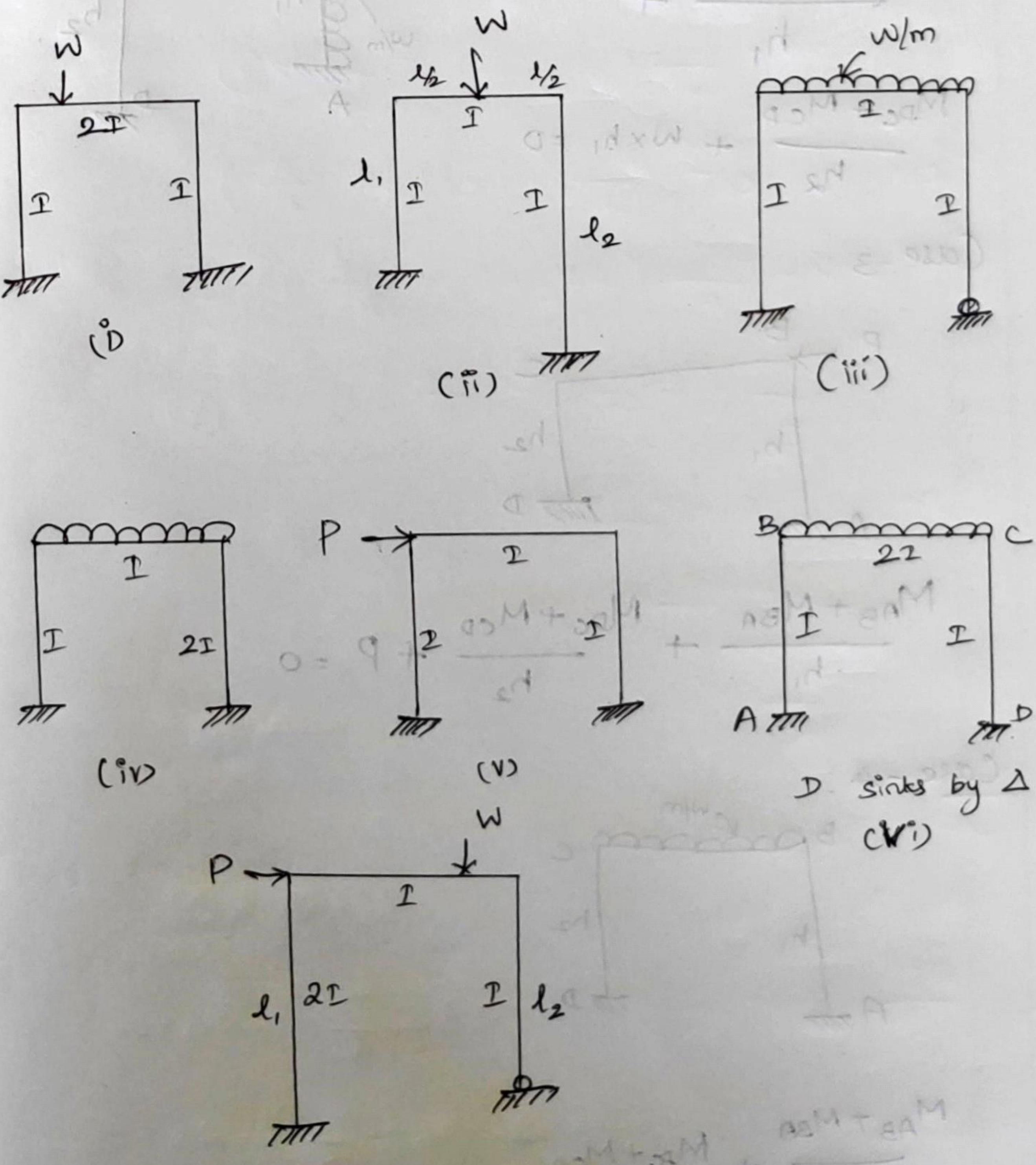
(iii) Different end conditions of the columns of the portal frame.

(iv) Non Uniform Section of the members of the frame.

(v) Horizontal loading on the columns of the frames.

(vi) Settlement of the supports of the frame.

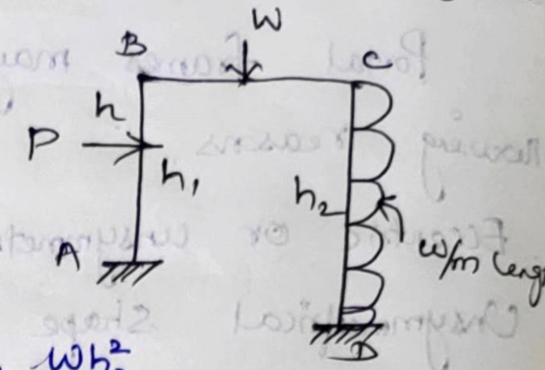
(vii) A combination of the above.



Frame Sway Conditions

Genaral Equations for different conditions :(Shear Equation)

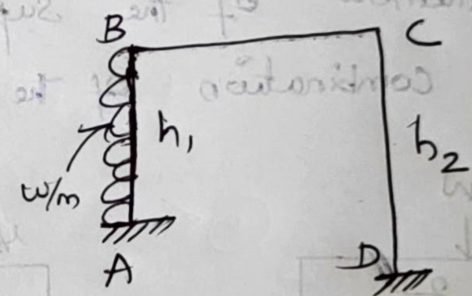
Case - 1 :



$$H_A = \frac{M_{BA} + M_{AB} - (P \times h)}{h_1}$$

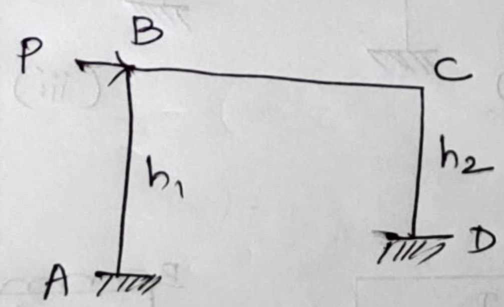
$$\frac{M_{AB} + M_{BA} - P \times h}{h_1} + \frac{M_{DC} + M_{CD} + \frac{wh_2^2}{2}}{h_2} + P - w \times h_2 = 0$$

Case - 2 :



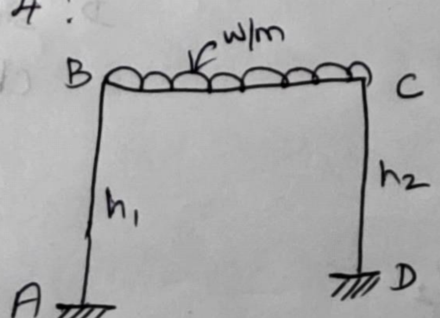
$$\frac{M_{AB} + M_{BA} - \frac{wh_1^2}{2}}{h_1} + \frac{M_{DC} + M_{CD}}{h_2} + w \times h_1 = 0$$

Case - 3 :



$$\frac{M_{AB} + M_{BA}}{h_1} + \frac{M_{DC} + M_{CD}}{h_2} + P = 0$$

Case - 4 :



$$\frac{M_{AB} + M_{BA}}{h_1} + \frac{M_{DC} + M_{CD}}{h_2} = 0$$