EVEN FUNCTION

A real Function $f(x)$ is said to be even if $\boxed{f(x) = f(-x)}$.

If $f(x)$ is an even function then

$$\int_{-l}^{l} f(x)\,dx = 2\int_{0}^{l} f(x)\,dx.$$

ODD FUNCTION:

A real Function $f(x)$ is said to be odd if $\boxed{f(x) = -f(x)}$

If $f(x)$ is an odd Function then

$$\int_{-l}^{l} f(x)\,dx = 0.$$

Note:

If $f(x)$ does not satisfies even and odd Function then it is called neither even nor odd function.

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Example:

1. $f(x) = x^2$

$f(-x) = (-x)^2$

$= x^2$

$= f(x)$

It is Even Function

2. $f(x) = x \cos x.$

$f(-x) = (-x) \cos(-x)$

$= -x \cos x$

$= f(x)$

It is odd Function

3. $f(x) = x \sin x \rightarrow$ Even Function

4. $f(x) = |x| \rightarrow$ Even Function

5. $f(x) = x + x^2 \rightarrow$ Neither Even nor odd.

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Note:

1. Even Function $\times$ Even Function = Even Fn.
2. Odd Fn $\times$ odd Fn = Even Fn.
3. Even Fn $\times$ odd Fn = odd Fn.
4. odd Fn $\times$ Even Fn = odd Fn.

* For even Function, $\boxed{b_n = 0}$
* For odd Function,
  $\boxed{a_0 = 0}$ and $\boxed{a_n = 0}$.

1. Find the Fourier series for the Function $f(x) = |x|$, $-\pi \leq x \leq \pi$.

Soln:

$$f(x) = |x| = x$$

$$f(-x) = |-x| = x$$

$$f(x) = f(-x)$$

$\therefore$ $f(x)$ is Even Function.

$\therefore$ $b_n = 0$

The fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

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To Find a_0:

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx.$$

$$= \frac{2}{\pi} \int_{0}^{\pi} f(x)\, dx.$$

Since for even Function

$$\int_{-l}^{l} f(x)\, dx = 2 \int_{0}^{l} f(x)\, dx$$

$$= \frac{2}{\pi} \int_{0}^{\pi} x\, dx$$

$$= \frac{2}{\pi} \left(\frac{x^2}{2} \right)_0^{\pi}$$

$$= \frac{\cancel{2}}{\cancel{\pi}} \left(\frac{\pi^{\cancel{2}}}{\cancel{2}} \right)$$

$$\boxed{a_0 = \pi}$$

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To find a_n:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \cos nx \, dx.$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x \cos nx \, dx.$$

$u = x$ $\qquad v = \cos nx.$

$u' = 1$ $\qquad v_1 = \frac{\sin nx}{n}$

$u'' = 0$ $\qquad v_2 = -\frac{\cos nx}{n^2}$

$$= \frac{2}{\pi}\left[x\frac{\sin nx}{n} - (1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[0 + \frac{\cos n\pi}{n^2} - 0 - \frac{\cos 0}{n^2}\right]$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$a_n = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

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$\therefore$ The fourier series is

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left[(-1)^n - 1\right]\cos nx.$$

2. Find the fourier series $f(x) = x$ in $(-\pi, \pi)$.

Soln:

$f(x) = x$

$f(-x) = -x = -f(x)$

$\therefore f(-x) = -f(x)$

$\therefore f(x)$ is odd Function.

$\therefore a_0 = 0$ and $a_n = 0$.

The Fourier Series is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx.$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx \, dx .$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x \sin nx \, dx \quad [\because x \sin nx \text{ is even Fn}]$$

$u = x$ $\qquad$ $v = \sin nx$

$u' = 1$ $\qquad$ $v_1 = -\frac{\cos nx}{n}$

$u'' = 0$ $\qquad$ $v_2 = -\frac{\sin nx}{n^2}$

$$= \frac{2}{\pi}\left[-x \frac{\cos nx}{n} - (1)\left(-\frac{\sin nx}{n^2}\right) \right]_0^{\pi}$$

$$= \frac{2}{\cancel{\pi}}\left[-\cancel{\pi}\frac{\cos n\pi}{n} + 0 + 0 - 0 \right]$$

$$\boxed{b_n = -\frac{2(-1)^n}{n}}$$

$$\therefore \quad f(x) = \sum_{n=1}^{\infty} \frac{(-2)(-1)^n}{n} \sin nx .$$

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