

## Harmonic Analysis.

The process of finding the Fourier Series for a function given by numerical values is known as harmonic analysis.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

$$a_0 = 2 \left( \frac{\sum y}{N} \right)$$

$$a_n = 2 \left( \frac{\sum y \cos nx}{N} \right) \quad \& \quad b_n = 2 \left( \frac{\sum y \sin nx}{N} \right)$$

Fundamental (or) First Harmonic function:

$$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x$$

Second Harmonic function:

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x.$$

Third Harmonic function:

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x$$

There are four types:

Type 1: The value of  $x$  given in terms of  $\pi$

Type 2: The value of  $x$  given in terms of degrees.

Type 3: The value of  $x$  given in terms of time function  $T$ .

Type 4: The value of  $x$  given in numerical values.

Type 1: The value of  $x$  given in terms of  $\pi$

1) Find the Fourier series expansion of period  $2\pi$  for  $y=f(x)$  defined in  $(0, 2\pi)$  by means of the values given below, [upto second harmonic]

$x$	0	$\pi/3$	$2\pi/3$	$\pi$	$4\pi/3$	$5\pi/3$	$2\pi$
$y$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

$$N=7$$

If the function value of 1st & last ordinates coincide we can omit any one of them.

$$\therefore N=6$$

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$$

$x$	$y=f(x)$	$y \cos x$	$y \cos 2x$	$y \sin x$	$y \sin 2x$
0	1.0	1.0	1.0	0	0
$\pi/3$	1.4	0.7	-0.7	1.212	1.212
$2\pi/3$	1.9	-0.95	-0.95	1.645	-1.645
$\pi$	1.7	-1.7	1.7	0	0
$4\pi/3$	1.5	-0.75	-0.75	-1.299	1.299
$5\pi/3$	1.2	0.6	-0.6	-1.039	-1.039
	$\Sigma y = 8.7$	$\Sigma y \cos x = -1.1$	$\Sigma y \cos 2x = -0.3$	$\Sigma y \sin x = 0.519$	$\Sigma y \sin 2x = -0.1732$

$$a_0 = 2 \left( \frac{\Sigma y}{N} \right) = 2 \frac{8.7}{6} = 2.9$$

$$a_1 = 2 \frac{\Sigma y \cos x}{N} = \frac{2(-1.1)}{6} = -0.367$$

$$a_2 = 2 \frac{\Sigma y \cos 2x}{N} = \frac{2(-0.3)}{6} = -0.1$$

$$b_1 = 2 \frac{\Sigma y \sin x}{N} = \frac{2(0.519)}{6} = 0.173$$

$$b_2 = 2 \frac{\Sigma y \sin 2x}{N} = \frac{2(-0.1732)}{6} = -0.0577$$

$$f(x) = 1.45 - 0.367 \cos x - 0.1 \cos 2x + 0.173 \sin x - 0.057 \sin 2x$$

2. Find the Fourier Series as far as the  $\Pi$  harmonic to represent the function given in the following data.

$x$	0	1	2	3	4	5
$y$	9	18	24	28	26	20

Here  $N=6$   
 $2l=6 \Rightarrow l=3$

$\therefore$  The Fourier Series is

$$f(x) = \frac{a_0}{2} + a_1 \cos \frac{\pi x}{l} + a_2 \cos \frac{2\pi x}{l} + b_1 \sin \frac{\pi x}{l} + b_2 \sin \frac{2\pi x}{l}$$

$$= \frac{a_0}{2} + a_1 \cos \frac{\pi x}{3} + a_2 \cos \frac{2\pi x}{3} + b_1 \sin \frac{\pi x}{3} + b_2 \sin \frac{2\pi x}{3}$$

$x$	$y=f(x)$	$y \cos \frac{\pi x}{3}$	$y \cos \frac{2\pi x}{3}$	$y \sin \frac{\pi x}{3}$	$y \sin \frac{2\pi x}{3}$
0	9	9	9	0	0
1	18	9	-9	15.7	15.6
2	24	-12	-24	20.9	0
3	28	-28	28	0	0
4	26	-13	-13	-22.6	22.6
5	20	10	10	-17.4	-17.4
	$\Sigma y = 125$	$\Sigma y \cos \frac{\pi x}{3} = -25$	$\Sigma y \cos \frac{2\pi x}{3} = -19$	$\Sigma y \sin \frac{\pi x}{3} = -3.4$	$\Sigma y \sin \frac{2\pi x}{3} = 20.8$

$$a_0 = 2 \frac{\Sigma y}{N} = 2 \left( \frac{125}{6} \right) = 41.66$$

$$a_1 = 2 \frac{\Sigma y \cos \frac{\pi x}{3}}{N} = 2 \left( \frac{-25}{6} \right) = -8.33$$

$$a_2 = 2 \frac{\Sigma y \cos \frac{2\pi x}{3}}{N} = 2 \left( \frac{-19}{6} \right) = -6.33$$

$$b_1 = 2 \frac{\sum y \sin \frac{\pi x}{3}}{N} = 2 \left( \frac{-3.4}{6} \right) = -1.13$$

$$b_2 = 2 \frac{\sum y \sin \frac{2\pi x}{3}}{N} = 2 \left( \frac{20.8}{6} \right) = 6.9$$

$$\therefore f(x) = 20.83 - 8.33 \cos \frac{\pi x}{3} - 6.33 \cos \frac{2\pi x}{3} - 1.13 \sin \frac{\pi x}{3} + 6.9 \sin \frac{2\pi x}{3}$$

3. Find the Fourier Series expansion defined in  $(0, \pi)$  by means of the table values given below. Find the series upto the 2nd harmonic

t sec	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	$T$
A temp	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

$$N = 6, T = 2\pi = 360^\circ$$

$$2l = 2\pi \Rightarrow l = \pi$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\therefore f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$$

Sub  $T = 2\pi$  in the table.

t sec	0	$60^\circ$	$120^\circ$	$180^\circ$	$240^\circ$	$300^\circ$
A temp	1.98	1.30	1.05	1.30	-0.88	-0.25

x	y = f(x)	y = cos x	y = cos 2x	y sin x	y sin 2x
0	1.98	1.98	1.98	0	0
$60^\circ$	1.30	0.65	-0.65	1.126	1.126
$120^\circ$	1.05	-0.525	-0.525	0.909	-0.909
$180^\circ$	1.30	-1.3	1.3	0	0
$240^\circ$	-0.88	0.44	0.44	0.762	-0.762
$300^\circ$	-0.25	-0.125	0.125	0.217	0.217
	$\sum y = 4.5$	$\sum y \cos x = 1.12$	$\sum y \cos 2x = 2.67$	$\sum y \sin x = 3.014$	$\sum y \sin 2x = -0.328$

$$a_0 = 2 \frac{\sum y}{N} = 2 \left( \frac{4.5}{6} \right) = 1.5$$

$$a_1 = 2 \frac{\sum y \cos x}{N} = 2 \left( \frac{1.12}{6} \right) = 0.373$$

$$a_2 = 2 \frac{\sum y \cos 2x}{N} = 2 \left( \frac{2.67}{6} \right) = 0.89$$

$$b_1 = 2 \left( \frac{\sum y \sin x}{N} \right) = 2 \left( \frac{3.014}{6} \right) = 1.005$$

$$b_2 = 2 \left( \frac{\sum y \sin 2x}{N} \right) = 2 \left( \frac{-0.328}{6} \right) = -0.109$$

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$$

$$= 0.75 + 0.373 \cos x + 0.89 \cos 2x + 1.005 \sin x - 0.109 \sin 2x$$