

3. Find the Fourier Series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$

Consider, $\phi_1(x) = 1 + \frac{2x}{\pi}$, $\phi_2(x) = 1 - \frac{2x}{\pi}$

$$\phi_1(-x) = 1 + 2\frac{(-x)}{\pi} = 1 - \frac{2x}{\pi} = \phi_2(x)$$

$$\phi_2(-x) = 1 - 2\frac{(-x)}{\pi} = 1 + \frac{2x}{\pi} = \phi_1(x)$$

$\therefore f(x)$ is an even function. $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} f(x) dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \left(1 - \frac{2x}{\pi}\right) dx$$

$$= \frac{2}{\pi} \left[x - \frac{x^2}{\pi} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\pi - \frac{\pi^2}{\pi} - 0 \right]$$

$$= \frac{2}{\pi} [\pi - \pi] = 0.$$

$$\therefore a_0 = 0.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx = \frac{2}{\pi} \int_0^{\pi} \left(1 - \frac{2x}{\pi}\right) \cos nx \, dx$$

$$\begin{array}{l} u = 1 - \frac{2x}{\pi} \\ u' = -\frac{2}{\pi} \\ u'' = 0 \end{array} \quad \left\{ \begin{array}{l} V = \cos nx \\ V_1 = \frac{\sin nx}{n} \\ V_2 = -\frac{\cos nx}{n^2} \end{array} \right.$$

$$a_n = \frac{2}{\pi} \left[\left(1 - \frac{2x}{\pi}\right) \frac{\sin nx}{n} - \frac{2}{\pi} \frac{\cos nx}{n^2} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\left(1 - \frac{2\pi}{\pi}\right) \frac{\sin n\pi}{n} - \frac{2}{\pi} \frac{\cos n\pi}{n^2} - \frac{\sin 0}{n} + \frac{2}{\pi} \frac{\cos 0}{n^2} \right]$$

$$= \frac{2}{\pi} \left[0 - \frac{2}{\pi} \frac{(-1)^n}{n^2} - 0 + \frac{2}{\pi} \frac{1}{n^2} \right]$$

$$= \frac{2}{\pi} \frac{2}{\pi n^2} [1 - (-1)^n]$$

$$a_n = \frac{4}{\pi^2 n^2} [1 - (-1)^n]$$

\therefore The Fourier series is

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2} [1 - (-1)^n] \cos nx.$$

$$f(x) = \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} [1 - (-1)^n] \cos nx$$

4. Find the Fourier Series for $f(x) = \begin{cases} L+x, & -L < x < 0 \\ L-x, & 0 < x < L \end{cases}$

$$\phi_1(x) = L+x \quad \& \quad \phi_2(x) = L-x.$$

$$\phi_1(-x) = L-x = \phi_2(x) \quad \& \quad \phi_2(-x) = L-(-x) = L+x = \phi_1(x)$$

$\therefore f(x)$ is an even function

$$\therefore b_n = 0.$$

The Fourier Series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L} \int_{-L}^L f(x) dx = \frac{2}{L} \int_0^L f(x) dx$$

$$= \frac{2}{L} \int_0^L (L-x) dx = \frac{2}{L} \left[Lx - \frac{x^2}{2} \right]_0^L$$

$$= \frac{2}{L} \left[L^2 - \frac{L^2}{2} \right] = \frac{2}{L} \frac{L^2}{2}$$

$$a_0 = L$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_0^L (L-x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$u = L-x$$

$$u' = -1$$

$$u'' = 0$$

$$V = \cos\left(\frac{n\pi x}{L}\right)$$

$$V_1 = \frac{\sin\left(\frac{n\pi x}{L}\right)}{\frac{n\pi}{L}}$$

$$V_2 = -\frac{\cos\left(\frac{n\pi x}{L}\right)}{\frac{n^2 \pi^2}{L^2}}$$

$$a_n = \frac{2}{L} \left[(L-x) \frac{\sin\left(\frac{n\pi x}{L}\right)}{\left(\frac{n\pi}{L}\right)} - (1) \frac{\cos\left(\frac{n\pi x}{L}\right)}{\frac{n^2 \pi^2}{L^2}} \right]_0^L$$

$$= \frac{2}{L} \left[0 - \frac{\cos n\pi}{\frac{n^2\pi^2}{L^2}} - 0 + \frac{\cos 0}{\frac{n^2\pi^2}{L^2}} \right]$$

$$= \frac{2}{L} \left[-\frac{(-1)^n L^2}{n^2\pi^2} + \frac{L^2}{n^2\pi^2} \right]$$

$$= \frac{2L^2}{Ln^2\pi^2} [1 - (-1)^n]$$

$$a_n = \frac{2L}{n^2\pi^2} [1 - (-1)^n]$$

$$x+x^2$$

$$a_0 = \frac{2\pi^2}{3}$$

$$a_n = 4 \frac{(-1)^n}{n^2}$$

$$b_n = \frac{-2(-1)^n}{n}$$

$$\therefore f(x) = \frac{L}{2} + \sum_{n=1}^{\infty} \frac{2L}{n^2\pi^2} [1 - (-1)^n] \cos\left(\frac{n\pi x}{L}\right)$$

$$= \frac{L}{2} + \frac{2L}{\pi^2} \sum_{n=1}^{\infty} [1 - (-1)^n] \cos\left(\frac{n\pi x}{L}\right)$$