Fourier Series

Periodic function:

A fn $f(x)$ is said to be periodic if $f(x+t) = f(x)$ $\forall$ real x and for some +ve no. t.

't' is known as period of $f(x)$.

Eg: $\cos x$, $\sin x$ are all periodic functions with period 2π

Dirichlet's Condition:

Any fn $f(x)$ can be developed as a Fourier series provided,

i) $f(x)$ is periodic, single valued and finite.

ii) $f(x)$ has a finite number of discontinuity in any one period.

iii) $f(x)$ has a finite number of maxima and minima.

General Fourier Series

If $f(x)$ is a periodic fn and satisfies Dirichlet's condition defined for the interval $[c, c+2l]$, then it can be represented by an infinite series is called Fourier Series as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

where a_0 , a_n & b_n are called Fourier Co-efficients

$$a_0 = \frac{1}{l} \int_{c}^{c+2l} f(x)\, dx$$

$$a_n = \frac{1}{l} \int_{c}^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l} \int_{c}^{c+2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$