$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

1. Find the Fourier Series for the function $f(x) = x^2$ in $(0, 2\pi)$

$f(x) = x^2$ in $(0, 2\pi)$ $\quad c = 0, \; l = \pi$

Fourier series for the fn $f(x)$ in $[0, 2\pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

To find a_0:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} x^2\,dx = \frac{1}{\pi}\left[\frac{x^3}{3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\,\frac{8\pi^3}{3} = \frac{8\pi^2}{3}$$

To find a_n:

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x^2 \cos nx\,dx$$

$u = x^2 \qquad V = \cos nx$

$u' = 2x \qquad V_1 = \frac{\sin nx}{n}$

$u'' = 2 \qquad V_2 = -\frac{\cos nx}{n^2}$

$u''' = 0 \qquad V_3 = -\frac{\sin nx}{n^3}$

$$= \frac{1}{\pi}\left[x^2\frac{\sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{(2\pi)^2 \sin n(2\pi)}{n} + \frac{2(2\pi)\cos n(2\pi)}{n^2} - \frac{2\sin n(2\pi)}{n^3}\right] - \frac{1}{\pi}\left[0 + 0 - \frac{2\sin 0}{n^3}\right]$$

$$= \frac{1}{\pi}\left[0 + \frac{4\pi}{n^2} - 0 - 0 - 0 + 0\right]$$

$$= \frac{4}{n^2}$$

To find b_n:

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx\,dx$$

$u = x^2$ | $v = \sin nx$

$u' = 2x$ | $v_1 = -\frac{\cos nx}{n}$

$u'' = 2$ | $v_2 = -\frac{\sin nx}{n^2}$

$u''' = 0$ | $v_3 = \frac{\cos nx}{n^2}$

$$= \frac{1}{\pi}\left[-x^2\frac{\cos nx}{n} + \frac{2x\sin nx}{n^2} + 2\frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-(2\pi)^2\frac{\cos n(2\pi)}{n} + \frac{2(2\pi)\sin n(2\pi)}{n^2} + \frac{2\cos n(2\pi)}{n^2}\right]$$

$$-\frac{1}{\pi}\left[0 + 0 + 2\frac{\cos 0}{n^2}\right]$$

$$= \frac{1}{\pi}\left[-\frac{4\pi^2}{n} + 0 + \frac{2}{n^2} - \frac{2}{n^2}\right] = -\frac{4\pi}{n}$$

$$\therefore f(x) = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2}\cos nx - \sum_{n=1}^{\infty}\frac{4\pi}{n}\sin nx .$$

2. Find the Fourier Series for the function $f(x) = \frac{(\pi - x)^2}{2}$ in $0 \leq x \leq 2\pi$

$$f(x) = \frac{(\pi - x)^2}{2} .$$

Fourier Series for the function $f(x)$ in the interval $[0, 2\pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

To find a_0:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi}\frac{(\pi - x)^2}{2}\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi}(\pi - x)^2\,dx = \frac{1}{2\pi}\left[\frac{(\pi - x)^3}{-3}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{-\pi^3}{-3}\right] - \left[\frac{\pi^3}{-3}\right]$$

$$= \frac{1}{2\pi}\,\frac{2\pi^3}{3} = \frac{\pi^2}{3}$$

To find a_n:

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{(\pi-x)^2}{2}\cos nx\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (\pi-x)^2\cos nx\,dx$$

$$= \frac{1}{2\pi}\left[\frac{4\pi}{n^2}\right] = \frac{2}{n^2}$$

To find b_n:

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{(\pi-x)^2}{2}\sin nx\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (\pi-x)^2\sin nx\,dx$$

$u = (\pi-x)^2$	$v = \sin nx$
$u' = 2(\pi-x)(-1)$	$v_1 = -\frac{\cos nx}{n}$
$= -2(\pi-x)$	$v_2 = -\frac{\sin nx}{n^2}$
$u'' = 2$	$v_3 = \frac{\cos nx}{n^3}$
$u''' = 0$	

$$b_n = \frac{1}{2\pi}\left[-(\pi-x)^2\frac{\cos nx}{n} - 2(\pi-x)\frac{\sin nx}{n^2} + 2\frac{\cos nx}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[-\pi^2\frac{\cos n(2\pi)}{n} + 2\pi\frac{\sin n(2\pi)}{n^2} + \frac{2\cos n(2\pi)}{n^3}\right] - \frac{1}{2\pi}\left[-\pi^2\frac{\cos 0}{n} - 2\pi\frac{\sin 0}{n^2} + \frac{2\cos 0}{n^3}\right]$$

$$= \frac{1}{2\pi}\left[-\frac{\pi^2}{n} + 0 + \frac{2}{n^3} + \frac{\pi^2}{n} + 0 - \frac{2}{n^3}\right]$$

$$= 0.$$

The Fourier Series is $f(x) = \frac{\frac{\pi^2}{3}}{2} + \sum_{n=1}^{\infty}\frac{2}{n^2}\cos nx + 0$

$$= \frac{\pi^2}{6} + \sum_{n=1}^{\infty}\frac{2}{n^2}\cos nx$$