2. Express $f(x) = x(\pi - x)$, $0 < x < \pi$ as a Fourier series of periodicity 2π containing i) sine terms only

ii) cosine terms only

Half range sine series:

$$f(x) = x(\pi - x), \quad 0 < x < \pi$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x(\pi - x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \pi x \sin nx\,dx - \frac{2}{\pi}\int_0^{\pi} x^2 \sin nx\,dx$$

$$= \frac{2}{\cancel{\pi}}\Big[$$

$u = \pi x$	$v = \sin nx$	$u = x^2$	$v = \sin nx$
$u' = \pi$	$v_1 = -\frac{\cos nx}{n}$	$u' = 2x$	$v_1 = -\frac{\cos nx}{n}$
$u'' = 0$	$v_2 = -\frac{\sin nx}{n^2}$	$u'' = 2$	$v_2 = -\frac{\sin nx}{n^2}$
		$u''' = 0$	$v_3 = \frac{\cos nx}{n^3}$

$$b_n = \frac{2}{\pi}\left[-\frac{\pi x\cos nx}{n} + \frac{\pi \sin nx}{n^2}\right]_0^{\pi} - \frac{2}{\pi}\left[-\frac{x^2\cos nx}{n} + \frac{2x\sin nx}{n^2} + \frac{2\cos nx}{n^3}\right]_0^{\pi}$$

$$= 2\left[\frac{-\pi(-1)^n}{n}\right] - \frac{2}{\pi}\left[-\frac{\pi^2(-1)^n}{n} + 0 + \frac{2(-1)^n}{n^3} - \frac{2}{n^3}\right]$$

$$= -\frac{2\pi(-1)^n}{n} + \frac{2\pi(-1)^n}{n} - \frac{4(-1)^n}{\pi n^3} + \frac{4}{\pi n^3}$$

$$= \frac{4}{\pi n^3}\left[1 - (-1)^n\right]$$

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \dfrac{8}{\pi n^3} & \text{if } n \text{ is odd} \end{cases}$$

$$\therefore f(x) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$$

4. Find the half range sine series for $f(x) = x$ in $(0, l)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$b_n = \frac{2}{l}\int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{2}{l}\int_0^l x \sin\left(\frac{n\pi x}{l}\right) dx$$

$u = x$ | $v = \sin\left(\frac{n\pi x}{l}\right)$

$u' = 1$ | $v_1 = \dfrac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}$, $v_2 = \dfrac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}$

$u'' = 0$

$$= \frac{2}{l}\left[\frac{-x\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}} + \frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} \right]_0^l$$

$$= \frac{2}{l}\left[\frac{-l\cos\frac{n\pi l}{l}}{\frac{n\pi}{l}} + \frac{\sin\frac{n\pi l}{l}}{\left(\frac{n\pi}{l}\right)^2} + 0 - \frac{\sin\frac{n\pi(0)}{l}}{\left(\frac{n\pi}{l}\right)^2} \right]$$

$$= \frac{2}{l}\left[\frac{-l^2(-1)^n}{n\pi} + 0 - 0 \right]$$

$$b_n = -\frac{2l}{n\pi}(-1)^n$$

$$\therefore f(x) = -\frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi x}{l}\right)$$