SNS College of Technology

Finite Element Analysis

Weighted Residual Method-Example problem

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Finite Element Analysis

The differential equation of physical phenomenon is given by Trial function,

$$\frac{d^2y}{dx^2} + 500x^2 = 0,\ 0 \le x \le 1, Trial\ function, y = a(x^2 - x)$$

Boundary condition are, y(0)=0, y(1)=0 calculate the value of the parameter a1by the following weighted residual methods. (i) Point collocation method (ii) Sub-domain collocation method (iii) least Square Method and (iv) Galerkin's method.

$$\frac{d^2y}{dx^2} + 500x^2 = 0$$

Boundary condition $y(0) = 0,\ y(1) = 0$

$$0 \le x \le 1$$

solution:

$$y(x) = a_0 + a_1 x + a_2 x^2 \quad \Rightarrow \ x = 0, y = 0 \ \ a_0 = 0$$

$$\Rightarrow x = 1, y = 0 \quad a_1 = -a_2$$

$$y(x) = -a_2 + a_2 x^2 \quad \Rightarrow \ y(x) = a_2(x^2 - x) \Rightarrow y(x) = a(x^2 - x)$$

$$\Rightarrow x = 1, y = 0 \quad a_1 = -a_2$$

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$$\frac{d^2y}{dx^2} + 500x^2 = 0$$

$$y(x) = a(x^2 - x)$$

$$\frac{dy}{dx} = a(2x - 1)$$

$$\frac{d^2y}{dx^2} = 2a$$

$$2a + 500x^2 = 0$$

Residual $R = 2a + 500x^2$

Finite Element Analysis

Residual $R = 2a + 500x^2$

Point collocation Method

$$R = 0,$$

Residual=0

$$2a + 500x^2 = 0 \quad 0 \le x \le 1$$

$$a = -\frac{500}{2}x^2 = -250x^2$$

$$a = -250\left(\frac{1}{2}\right)^2 = -62.5$$

$$y(x) = a(x^2 - x) \longrightarrow y(x) = 62.5(x - x^2)$$

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Residual $R = 2a + 500x^2$

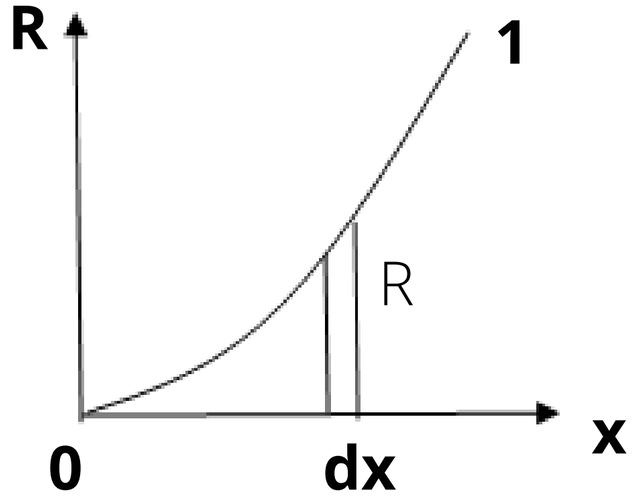
Sub domain Method $\int_0^1 R\,dx = 0$

$$\int_0^1 \left(2a + 500x^2\right)dx = 0$$

$$\left[2ax + 500\frac{x^3}{3}\right]_0^1 = 0 \quad \Bigg| \quad 2a + \frac{500}{3} = 0$$

$$2a(1) + 500\frac{1^3}{3} = 0 \quad \Bigg| \quad a = -\frac{500}{6} = -83.33$$

$$y(x) = a(x^2 - x) \longrightarrow y(x) = 83.33(x - x^2)$$



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Residual $R = 2a + 500x^2$

Least square Method $\int_0^1 R^2 dx = 0$

$$\int_0^1 R.R\,dx = 0,$$

$$\int_0^1 R.\frac{\partial R}{\partial a}dx = 0, \qquad \frac{\partial R}{\partial a} = 2$$

$$\int_0^1 2\times\left(2a + 500x^2\right)dx = 0$$

$$\left[4ax + 1000\frac{x^3}{3}\right]_0^1 = 0$$

$$4a(1) + 1000\frac{1^3}{3} = 0$$

$$4a + \frac{1000}{3} = 0$$

$$a = -\frac{1000}{12} = -83.33$$

$$y(x) = a(x^2 - x) \longrightarrow \qquad y(x) = 83.33(x - x^2)$$

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Residual $R = 2a + 500x^2$

Galerkin Method $\int_0^1 W_i R\,dx = 0$

$$\int_0^1 \left(x^2 - x\right)\left(2a + 500x^2\right)dx = 0,$$

$$\int_0^1 \left(2ax^2 + 500x^4 - 2ax - 500x^3\right)dx = 0,$$

$$\left[2a\frac{x^3}{3} + 500\frac{x^5}{5} - 2a\frac{x^2}{2} - 500\frac{x^4}{4}\right]_0^1 = 0$$

$$\left[2a\frac{1^3}{3} + 500\frac{1^5}{5} - 2a\frac{1^2}{2} - 500\frac{1^4}{4}\right] - [0] = 0$$

$$\left[2a\frac{1}{3} + 500\frac{1}{5} - 2a\frac{1}{2} - 500\frac{1}{4}\right] = 0$$

$$\left[\frac{2a}{3} + \frac{500}{5} - a - \frac{500}{4}\right] = 0$$

$$\left[-\frac{a}{3} + 100 - \frac{500}{4}\right] = 0$$

$$a = 75$$

$$y(x) = a(x^2 - x)$$

$$y(x) = 75(x - x^2)$$

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Thank you