u1

✱ The differential equation of a physical phenomenon is given by

$$\frac{d^2y}{dx^2} + y = 4x, \quad 0 \le x \le 1$$

The boundary conditions are : $y(0)=0$ $y(1)=1$

obtain one term approximate solution by using galerkin's method of weighted residuals.

Sol

Here, the bc are not homogeneous, so, we assume a trail function as.

$$y = a_1 x(x-1) + x$$

1st we have to verify whether the trial function satisfies the boundary conditions or not.

$$y = a_1 x(x-1) + x$$

$\Rightarrow$ $x=0$ $y=0$

$x=1$ $y=1$

Hence it satisfies the boundary conditions.

Residual, R

$$y = a_1 x(x-1) + x$$

$$= a_1 (x^2 - x) + x$$

$$\frac{dy}{dx} = a_1(2x-1) + 1$$

$$\frac{d^2y}{dx^2} = a_1(2)$$

$$\frac{d^2y}{dx^2} = 2a_1$$

$\Rightarrow d^2y/dx^2 + y = 4x$

$2a_1 + y = 4x$

Substitute y value

$2a_1 + a_1 x(x-1) + x = 4x$

Residual, $R = 2a_1 + a_1 x(x-1) + x - 4x$

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Galerkin's method $\int_0^1 y \cdot R \cdot dx = 0$

$y = w_i = x(x-1)$

$$\int_0^1 x(x-1)\left[2a_1 + a_1 x(x-1) + x - 4x\right] dx = 0$$

$$\int_0^1 x(x-1)\left[2a_1 + a_1 x^2 - a_1 x + x - 4x\right] dx = 0$$

$$\int_0^1 (x^2 - x)\left[2a_1 + a_1 x^2 - a_1 x - 3x\right] dx = 0$$

$$\int_0^1 \left[2a_1 x^2 + a_1 x^4 - a_1 x^3 - 3x^3 - 2a_1 x - a_1 x^3 + a_1 x^2 + 3x^2\right] dx = 0$$

$$2a_1\left[\frac{x^3}{3}\right]_0^1 + a_1\left[\frac{x^5}{5}\right] - a_1\left[\frac{x^4}{4}\right]_0^1 - 3\left[\frac{x^4}{4}\right]_0^1 - 2a_1\left[\frac{x^2}{2}\right]_0^1 -$$

$$a_1\left[\frac{x^4}{4}\right]_0^1 + a_1\left[\frac{x^3}{3}\right]_0^1 + 3\left[\frac{x^3}{3}\right]_0^1 = 0$$

$$\frac{2a_1}{3} + \frac{a_1}{5} - \frac{a_1}{4} - 3/4 - \frac{2a_1}{2} - \frac{a_1}{4} + \frac{a_1}{3} + \frac{3}{3} = 0$$

$$0.666a_1 + 0.2a_1 - 0.25a_1 - 0.75 - a_1 - 0.25a_1$$
$$+ 0.333a_1 + 1 = 0$$
$$-0.301a_1 = -0.25$$
$$a_1 = 0.830$$

$y = 0.830x(x-1) + x$

$= 0.830x^2 - 0.830x + x$

$$\boxed{y = 0.830x^2 + 0.170x}$$