



The differential equation of physical phenomenon is given by $\frac{d^2y}{dx^2} + 500x^2 = 0$, $0 \leq x \leq 1$,

Trial function, $y = a_1(x - x^4)$, Boundary condition are, $y(0)=0$, $y(1)=0$ calculate the value of the parameter a_1 by the following methods. (i) Point collocation method (ii) Sub-domain collocation method (iii) least Square Method and (iv) Galerkin's method.

Given: Differential equation $\frac{d^2y}{dx^2} + 500x^2 = 0$, $0 \leq x \leq 1$
Trial function, $y = a_1(x - x^4)$
Boundary condition are, $y(0)=0$, $y(1)=0$

To find: The value of parameter a_1 by,
i. point collocation method, ii. Subdomain method, iii. Least Squares method, iv. Galerkin

Solution: First we have to verify, whether the trial function satisfies the boundary condition or not.

Trial function is, $y = a_1(x - x^4)$

When $x = 0$, $y = a_1(0 - 0) = 0$

$x = 1$, $y = a_1(1 - 1^4) = 0$

Hence it satisfies the boundary conditions,

(i) point collocation method:

$$y = a_1(x - x^4)$$

$$\frac{dy}{dx} = a_1(1 - 4x^3)$$

$$\frac{d^2y}{dx^2} = a_1(0 - 12x^2)$$

$$\frac{d^2y}{dx^2} = -12a_1x^2$$



Substituting $\frac{d^2y}{dx^2}$ value in given differential equation (1), $\rightarrow \frac{d^2y}{dx^2} + 500x^2 = 0$

$$\Rightarrow \text{Residual, } R = -12a_1x^2 + 500x^2 \rightarrow \textcircled{2}$$

In point collocation method, residuals are set to zero.

$$R = -12a_1x^2 + 500x^2 = 0 \rightarrow \textcircled{3}$$

In this problem, we have to find only one parameter, a_1 . So, only one collocation point is needed.

The point may be chosen between 0 and 1. Let us take $\frac{1}{2}$,

Substituting $x = \frac{1}{2}$ in equation $\textcircled{3}$

$$R = -12a_1\left[\frac{1}{2}\right]^2 + 500\left[\frac{1}{2}\right]^2 = 0$$

$$\Rightarrow -\frac{3}{1}a_1\left[\frac{1}{4}\right] + \frac{125}{1}a_1\left[\frac{1}{4}\right] = 0$$

$$-3a_1 + 125 = 0$$

$$a_1 = 41.66 \rightarrow \textcircled{4}$$

Hence the trial function is $y = 41.66(x - x^4)$



(ii) Subdomain collocation method:

This method requires $\int_0^1 R dx = 0$
Substitute R value

$$\rightarrow \int_0^1 [-12a_1 x^2 + 500x^2] dx = 0$$

$$= -12a_1 \left[\frac{x^3}{3} \right]_0^1 + 500 \left[\frac{x^3}{3} \right]_0^1 = 0$$

$$= \frac{-12a_1}{3} [1 - 0] + \frac{500}{3} [1 - 0] = 0$$

$$= \frac{-12a_1}{3} + \frac{500}{3} = 0$$

$$= -12a_1 + 500 = 0$$

$$= -12a_1 = -500$$

$$a_1 = \frac{500}{12} = 41.66 \rightarrow 5$$

Trial function is $y = 41.66(x - x^4)$

(iii) Least Squares method:

This method requires, $I = \int_0^1 R^2 dx$

It can also be written

$$\text{as } \frac{\partial I}{\partial a_1} = \int_0^1 R \frac{\partial R}{\partial a_1} dx \rightarrow 6$$



We know that, $R = -12a_1x^2 + 500x^2$

$$\frac{\partial R}{\partial a_1} = -12x^2$$

Substitute R and $\frac{\partial R}{\partial a_1}$ values in eq (6)

$$\Rightarrow \frac{\partial I}{\partial a_1} = \int_0^1 [-12a_1x^2 + 500x^2] (-12x^2) dx$$

The requirement is, $\frac{\partial I}{\partial a_1} = 0$

$$\Rightarrow \int_0^1 [-12a_1x^2 + 500x^2] (-12x^2) dx = 0$$

$$\int_0^1 [144a_1x^4 - 6000x^4] dx = 0$$

$$144a_1 \left[\frac{x^5}{5} \right]_0^1 - 6000 \left[\frac{x^5}{5} \right]_0^1 = 0$$

$$\frac{144a_1}{5} [1-0] - \frac{6000}{5} [1-0] = 0$$

$$\frac{144a_1}{5} - \frac{6000}{5} = 0 \Rightarrow a_1 = \frac{6000}{144} = 41.66$$

$$a_1 = 41.66$$



(iv) Galerkin's method: In this method, the trial function itself is considered as the weighting function, $w_i \Rightarrow \int w_i R dx = 0$

Here, the trial function is $y = w_i = a_1(x - x^4)$

Substitute w_i and R values in equation ⑧

$$\int_0^1 (x - x^4) (-12a_1 x^2 + 500x^2) dx = 0$$

$$a_1 \int_0^1 (x - x^4) (-12a_1 x^2 + 500x^2) dx = 0$$

$$a_1 \int_0^1 [-12a_1 x^3 + 500x^3 + 12a_1 x^6 - 500x^6] dx = 0$$

$$a_1 \left[-12a_1 \left[\frac{x^4}{4} \right]_0^1 + 500 \left[\frac{x^4}{4} \right]_0^1 + 12a_1 \left[\frac{x^7}{7} \right]_0^1 - 500 \left[\frac{x^7}{7} \right]_0^1 \right] = 0$$

$$-\frac{12a_1}{4} [1-0] + \frac{500}{4} [1-0] + \frac{12a_1}{7} (1-0) - \frac{500}{7} (1-0) = 0$$

$$-3a_1 + 125 + 1.714a_1 - 71.428 = 0$$

$$-1.286a_1 = -53.572$$

$$a_1 = 41.66$$

∴ ⑨

Trial function is $y = 41.66(x - x^4)$

From equation 4, 5, 7, and 9, we know that the value of parameter a_1 is same for all the four methods.

Prepared by Dr. M. SUBRAMANIAN/Professor/Mechanical/16ME402/ Finite Element Analysis

Result: parameter, a_1 [For all the four methods] = 41.66

