SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
(An Autonomous Institution)
Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A+’ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

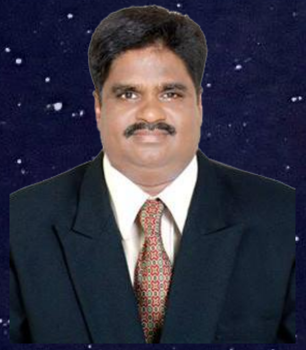
DEPARTMENT OF MECHANICAL ENGINEERING

16ME401 Finite Element Analysis

IV Year VII Sem

Unit I Introduction

Topic – Mathematical Modeling of field problems in Engineering-Governing Equations



SNS *Design Thinkers*

Dr.M.SUBRAMANIAN, Professor& Mech

Mathematical Modeling of field problems in Engineering- Governing Equations

Physical phenomenon → Model → Governing equations & BCs → Approximation → Finite element equations

- **Solid Mechanics**
  e.g. Axially loaded elastic bar
  $$\frac{d}{dx}\left(AE\frac{du}{dx}\right)+b=0$$
- **Fluid Mechanics**
  e.g. Poiseuille flow in pipe
  $$\frac{d}{dx}\left(A\frac{D^2}{32\mu}\frac{dp}{dx}\right)+Q=0$$
- **Thermal Conduction**
  e.g. 1-D heat flow
  $$\frac{d}{dx}\left(Ak\frac{dT}{dx}\right)+Q=0$$
- **Diffusion**
  e.g. 1-D diffusion
  $$\frac{d}{dx}\left(AD\frac{dC}{dx}\right)+Q=0$$
- **Electrical Conduction**
  e.g. 1-D electric current flow
  $$\frac{d}{dx}\left(A\sigma\frac{dV}{dx}\right)+Q=0$$

+ BCs (Boundary Conditions) → *System of equations:*

$$[K]\{u\}=\{F\}$$

Stiffness matrix, Nodal vector, Force vector

Figure Governing equations for various physical phenomena

One-dimensional Heat Transfer

The governing differential equation for the steady state one-dimensional conduction heat transfer with convective heat loss from lateral surfaces is given by

$$k\frac{d^2T}{dx^2} + q = \left(\frac{P}{A_c}\right)h(T - T_\infty)$$

where

k = coefficient of thermal conductivity of the material,
T = temperature,
q = internal heat source per unit volume,
P = perimeter,
A_c = the cross-sectional area,
h = convective heat transfer coefficient, and
T_∞ = ambient temperature.

Governing Equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$M = \frac{EI}{R}$$

$$M = EI\left(\frac{d^2y}{dx^2}\right)$$

Shear force [SF]= $\frac{dM}{dx}$ = $= EI\frac{d^3y}{dx^3}$

Load distribution q= $\frac{dF}{dx} = EI\frac{d^4y}{dx^4}$

Differential equation

$$EI\frac{d^4y}{dx^4} - q = 0$$

For small curve

ds=dx=di.R

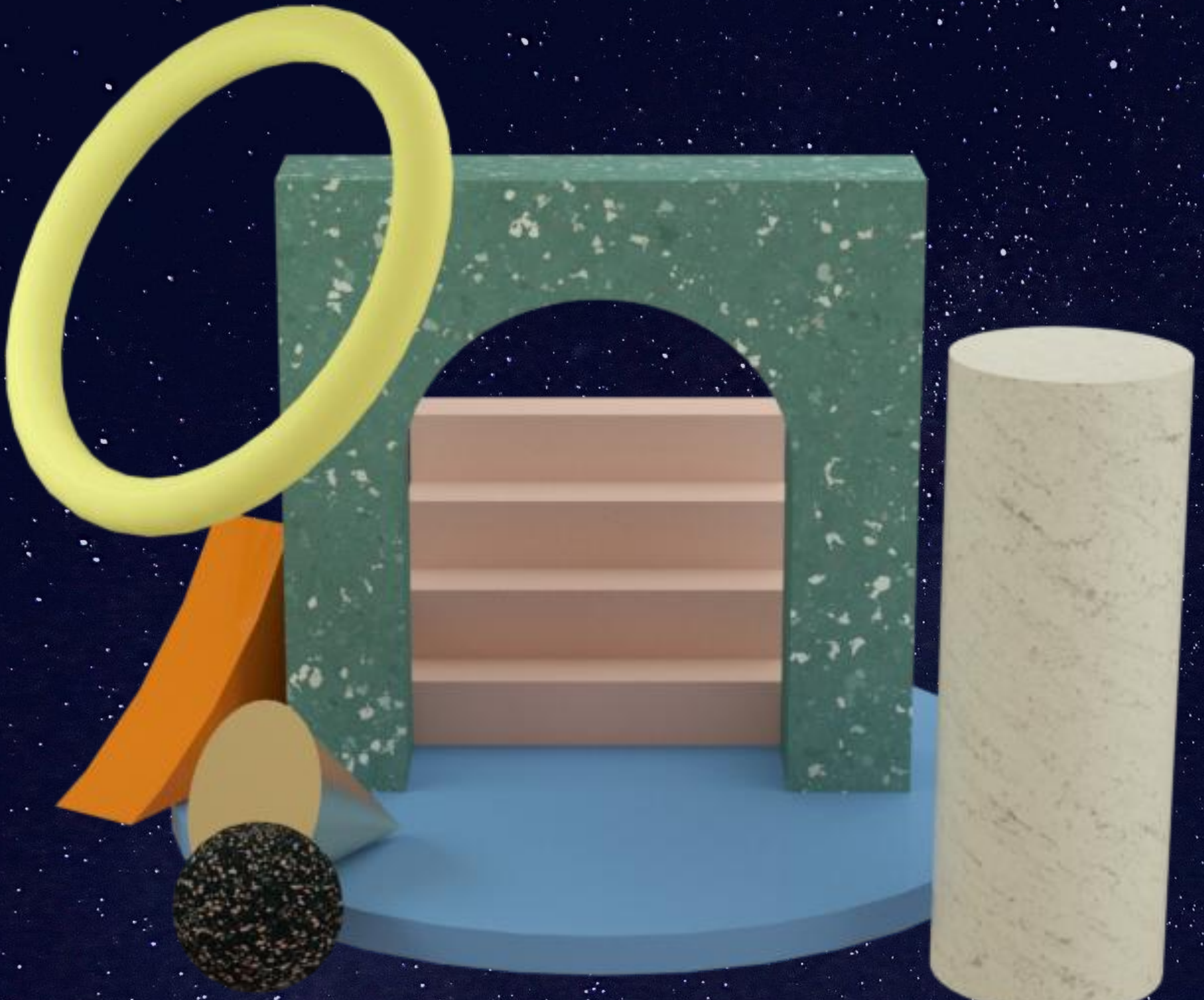
$$i = \frac{dy}{dx} = slope$$

$$\frac{1}{R} = \frac{di}{dx}$$

$$\frac{1}{R} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Check Google classroom for a Lecture material



Thank you