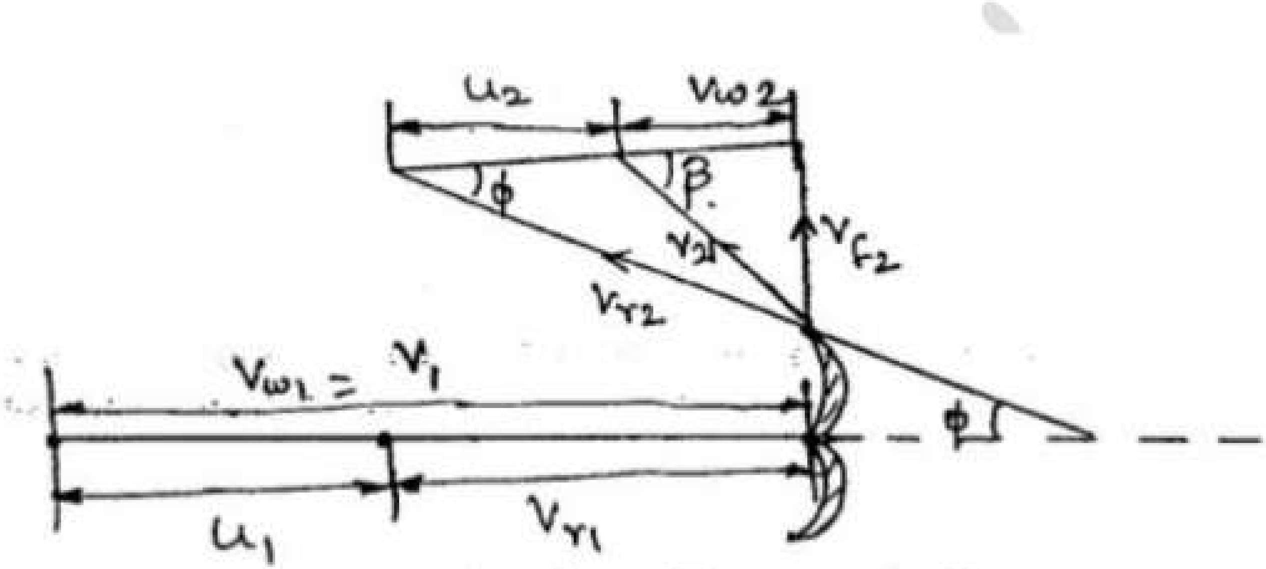
UNIT – V – TURBINES

PELTON WHEEL:



Symbol		Description	Unit
$u_1 \& u_2$	→	Tangential Velocity of Runner at Inlet & Outlet	$^m/_s$
$v_{r1} \& v_{r2}$	→	Relative Velocity at Inlet & Outlet	$^m/_s$
$v_{w1} \& v_{w2}$	→	Whirl Velocity at Inlet & Outlet	$^m/_s$
$V_1 \& V_2$	→	Absolute Velocity at Inlet & Outlet	$^m/_s$
$v_{f1} \& v_{f2}$	→	Flow Velocity at Inlet & Outlet	$^m/_s$
β	→	Angle made by Absolute Velocity at Outlet with the Direction of Motion of Vane	Degree
ϕ	→	Angle made by Relative Velocity at Outlet with the Direction of Motion of Vane	Degree

TANGENTIAL VELOCITY AT INLET AND OUTLET *(OR)* VELOCITY OF WHEEL:

$$u = \frac{\pi D N}{60}$$

Symbol		Description	Unit
D	⟶	Diameter of Runner	m
N	⟶	Speed of Impeller	rpm

VELOCITY OF JET:

$$V_1 = C_v\sqrt{2gH}$$

$$C_v = 0.97 - 0.99$$

Symbol		Description	Unit
C_v	⟶	Coefficient of Velocity	
g	⟶	Acceleration due to Gravity	m/s^2
H	⟶	Head	m

VELOCITY OF WHEEL:

$$u = k_u\sqrt{2gH}$$

$$k_u = 0.43 - 0.45$$

Symbol		Description	Unit
k_u	⟶	Speed Ratio	
g	⟶	Acceleration due to Gravity	m/s^2
H	⟶	Head	m

FROM INLET VELOCITY TRIANGLE DIAGRAM:

$$V_{w1} = V_1$$

$$V_{w1} = u_1 + V_{r1}$$

Symbol		Description	Unit
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	m/s
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
V_1	$\longrightarrow$	Absolute Velocity at Inlet	m/s

FROM OUTLET VELOCITY TRIANGLE DIAGRAM:

$$\cos\phi = \frac{u_2 + v_{w2}}{v_{r2}}$$

$$\tan\phi = \frac{v_{f2}}{u_2 + v_{w2}}$$

$$\sin\phi = \frac{v_{f2}}{v_{r2}}$$

$$\tan\beta = \frac{v_{f2}}{v_{w2}}$$

Symbol		Description	Unit
u_2	$\longrightarrow$	Tangential Velocity of Runner at Outlet	m/s
v_{r2}	$\longrightarrow$	Relative Velocity at Outlet	m/s
v_{w2}	$\longrightarrow$	Whirl Velocity at Outlet	m/s
v_{f2}	$\longrightarrow$	Flow Velocity at Outlet	m/s

WORK DONE BY JET PER SECOND:

$$W = \rho Q\,[v_{w1} + v_{w2}]u$$

Symbol		Description	Unit
u	$\longrightarrow$	Tangential Velocity of Runner	m/s
$v_{w1} \& v_{w2}$	$\longrightarrow$	Whirl Velocity at Inlet & Outlet	m/s
ρ	$\longrightarrow$	Density	kg/m^3
Q	$\longrightarrow$	Discharge	m^3/s

HYDRAULIC EFFICIENCY:

$$\eta_{hyd} = \frac{2[v_{w1} + v_{w2}]u}{{V_1}^2}$$

Symbol		Description	Unit
u	$\longrightarrow$	Tangential Velocity of Runner	m/s
$v_{w1} \& v_{w2}$	$\longrightarrow$	Whirl Velocity at Inlet & Outlet	m/s
V_1	$\longrightarrow$	Absolute Velocity at Inlet	m/s

OVERALL EFFICIENCY:

$$\eta_o = \frac{Shaft\ Power}{Input\ Power}$$

$$\eta_o = \frac{S.P}{\rho g Q H}$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	kg/m^3
Q	$\longrightarrow$	Discharge	m^3/s
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
H	$\longrightarrow$	Head	m
$S.P$	$\longrightarrow$	Shaft Power	W

DISCHARGE OF SINGLE JET:

$$q = \frac{\pi}{4} * d^2 * V_1$$

Symbol		Description	Unit
d	$\longrightarrow$	Diameter of Jet	m
V_1	$\longrightarrow$	Absolute Velocity at Inlet	m/s
q	$\longrightarrow$	Discharge of Single Jet	m^3/s

NUMBER OF JET:

$$n = \frac{Q}{q}$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	$m^3/_s$
q	$\longrightarrow$	Discharge of Single Jet	$m^3/_s$

NUMBER OF BUCKET:

$$Z = 15 + \frac{D}{2d}$$

Symbol		Description	Unit
d	$\longrightarrow$	Diameter of Jet	m
D	$\longrightarrow$	Diameter of Runner	m

DIMENSIONS OF BUCKET:

$$Axial\ Width \qquad B = 4.5d$$

$$Radial\ Length \quad L = 2.5d$$

$$Depth\ of\ Bucket\ T = d$$

Symbol		Description	Unit
d	$\longrightarrow$	Diameter of Jet	m

KINETIC ENERGY OF JET:

$$K.E\ of\ Jet = \frac{1}{2}\, m\, {V_1}^2$$

$$Since\ \ m = \rho A V$$

$$Therefore\ K.E\ of\ Jet = \frac{1}{2}\, \rho * A * V_1 * {V_1}^2$$

$$Since\ Q = AV$$

$$Therefore\ K.E\ of\ Jet = \frac{1}{2}\rho * Q * {V_1}^2$$

POWER LOST IN NOZZLE:

$$Input\ Power = Kinetic\ Energy + Power\ Lost\ in\ Nozzle$$

POWER LOST IN RUNNER:

$$\begin{aligned} Input\ Power &= Power\ of\ Shaft + Power\ Lost\ in\ Nozzle \\ &+ Power\ Lost\ in\ Runner \\ &+ Power\ Lost\ Due\ to\ Mechanical\ Resistance \end{aligned}$$

RESULTANT FORCE ON BUCKET:

$$F = \rho Q\,[v_{w1} + v_{w2}]$$

Symbol		Description	Unit
F	$\longrightarrow$	Resultant Force on Bucket	N
$v_{w1} \& v_{w2}$	$\longrightarrow$	Whirl Velocity at Inlet & Outlet	m/s
ρ	$\longrightarrow$	Density	kg/m^3
Q	$\longrightarrow$	Discharge	m^3/s

TORQUE:

$$T = F * \frac{D}{2}$$

Symbol		Description	Unit
F	$\longrightarrow$	Resultant Force on Bucket	N
D	$\longrightarrow$	Diameter of Runner	m
T	$\longrightarrow$	Torque	$N-m$

POWER:

$$P = \frac{2\pi NT}{60}$$

Symbol		Description	Unit
P	$\longrightarrow$	Power	W
T	$\longrightarrow$	Torque	$N-m$
N	$\longrightarrow$	Speed of Shaft	Rpm

SPECIFIC SPEED:

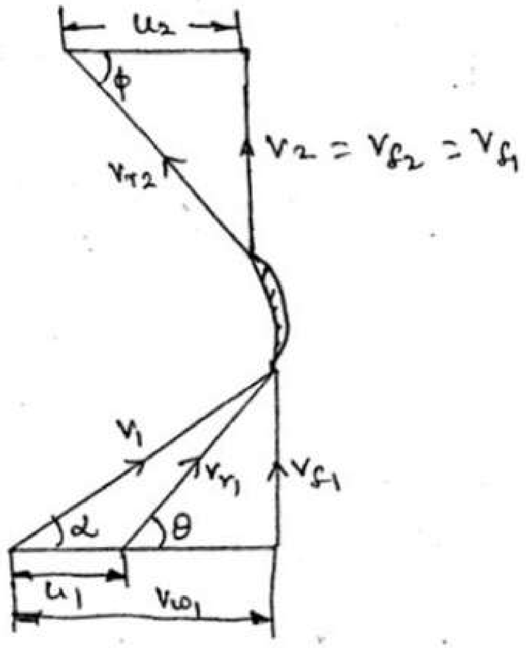
$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	m^3/s
H	$\longrightarrow$	Head	m
P	$\longrightarrow$	Power	kW
N	$\longrightarrow$	Speed	rpm
N_s	$\longrightarrow$	Specific Speed	

REACTION TURBINE:

INWARD FLOW REACTION TURBINE:



Symbol		Description	Unit
$u_1 \& u_2$	→	Tangential Velocity of Runner at Inlet & Outlet	$m/_s$
$v_{r1} \& v_{r2}$	→	Relative Velocity at Inlet & Outlet	$m/_s$
$v_{w1} \& v_{w2}$	→	Whirl Velocity at Inlet & Outlet	$m/_s$
$V_1 \& V_2$	→	Absolute Velocity at Inlet & Outlet	$m/_s$
$v_{f1} \& v_{f2}$	→	Flow Velocity at Inlet & Outlet	$m/_s$
α	→	Angle made by Absolute Velocity at Inlet with the Direction of Motion of Vane	Degree
θ	→	Angle made by Relative Velocity at Inlet with the Direction of Motion of Vane	Degree
ϕ	→	Angle made by Relative Velocity at Outlet with the	Degree

Direction of Motion of Vane

TANGENTIAL VELOCITY AT INLET:

$$u_1 = \frac{\pi d_1 N}{60}$$

Symbol		Description	Unit
d_1	⟶	Inlet (***or***) External Diameter	m
N	⟶	Speed of Turbine	rpm

TANGENTIAL VELOCITY AT OUTLET:

$$u_2 = \frac{\pi d_2 N}{60}$$

Symbol		Description	Unit
d_2	⟶	Outlet (***or***) Internal Diameter	m
N	⟶	Speed of Turbine	rpm

FROM INLET VELOCITY TRIANGLE DIAGRAM:

$$\sin \alpha = \frac{v_{f1}}{V_1}$$

$$\cos \alpha = \frac{v_{w1}}{V_1}$$

$$\tan \alpha = \frac{v_{f1}}{v_{w1}}$$

$$\sin \theta = \frac{v_{f1}}{v_{r1}}$$

$$\cos \theta = \frac{v_{w1} - u_1}{v_{r1}}$$

$$\tan\theta = \frac{v_{f1}}{v_{w1} - u_1}$$

Symbol		Description	Unit
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
V_1	$\longrightarrow$	Absolute Velocity at Inlet	m/s
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	m/s
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	m/s
α	$\longrightarrow$	Angle made by Absolute Velocity at Inlet with the Direction of Motion of Vane	Degree
θ	$\longrightarrow$	Angle made by Relative Velocity at Inlet with the Direction of Motion of Vane	Degree

RELATIVE VELOCITY AT INLET:

$$v_{r1} = \sqrt{{v_{f1}}^2 + (v_{w1} - u_1)^2}$$

Symbol		Description	Unit
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	m/s
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	m/s
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s

DISCHARGE:

$$Q = \pi d_1 b_1 v_{f1} = \pi d_2 b_2 v_{f2}$$

$$Q = A v_{f1} = A v_{f2} = A_{f1} v_{f1} = A_{f2} v_{f2}$$

Symbol	Description	Unit

$v_{f1} \& v_{f2}$	$\longrightarrow$	Flow Velocity at Inlet & Outlet	$m/_s$
$d_1 \& d_2$	$\longrightarrow$	Diameter of Impeller at Inlet & Outlet	m
$b_1 \& b_2$	$\longrightarrow$	Width of Impeller at Inlet & Outlet	m
Q	$\longrightarrow$	Discharge	$m^3/_s$
A	$\longrightarrow$	Area of Runner	m^2
$A_{f1} \& A_{f2}$	$\longrightarrow$	Area of Flow at Inlet & Outlet	$m/_s$

MASS OF WATER FLOWING THROUGH THE RUNNER:

$$m = \rho Q$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	$m^3/_s$
ρ	$\longrightarrow$	Density	$kg/_{m^3}$

HEAD AT INLET OF TURBINE:

$$H = \frac{1}{g} * v_{w1} * u_1 + \frac{{v_{f1}}^2}{2g}$$

Symbol		Description	Unit
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	$m/_s$
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	$m/_s$
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	$m/_s$
g	$\longrightarrow$	Acceleration due to gravity	$m/_{s^2}$

INPUT POWER TO TURBINE *(OR)* POWER GIVEN TO TURBINE:

$$P = \rho g Q H$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	$kg/_{m^3}$
Q	$\longrightarrow$	Discharge	$m^3/_s$

g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
H	$\longrightarrow$	Head	m

POWER DEVELOPED BY TURBINE:

$$P = \rho * Q * v_{w1} * u_1$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	kg/m^3
Q	$\longrightarrow$	Discharge	m^3/s
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s

HYDRAULIC EFFICIENCY:

$$\eta_{hyd} = \frac{v_{w1}u_1}{gH}$$

$$\eta_{hyd} = \frac{Head\ Inlet - Head\ Loss}{Head\ Inlet}$$

Symbol		Description	Unit
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
H	$\longrightarrow$	Head	m

OVERALL EFFICIENCY:

$$\eta_o = \frac{Shaft\ Power}{Input\ Power}$$

$$\eta_o = \frac{S.P}{\rho g Q H}$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	kg/m^3
Q	$\longrightarrow$	Discharge	m^3/s
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
H	$\longrightarrow$	Head	m
$S.P$	$\longrightarrow$	Shaft Power	W

SPEED RATIO:

$$K_u = \frac{u}{\sqrt{2gH}}$$

$$K_u = 0.6 - 0.9$$

Symbol		Description	Unit
u	$\longrightarrow$	Tangential Velocity	m/s
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
K_u	$\longrightarrow$	Speed Ratio	

FLOW RATIO:

$$K_f = \frac{v_{f1}}{\sqrt{2gH}}$$

$$K_f = 0.15 - 0.3$$

Symbol		Description	Unit
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	m/s
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
K_f	$\longrightarrow$	Flow Ratio	

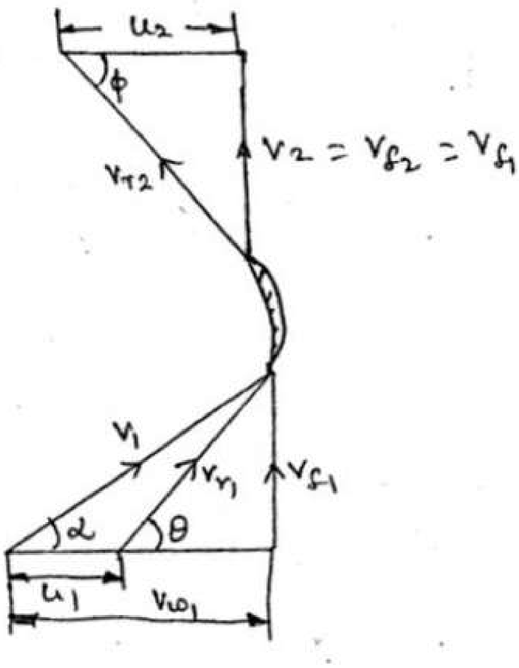
SPECIFIC SPEED:

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Symbol		Description	Unit
Q	→	Discharge	m^3/s
H	→	Head	m
P	→	Power	kW
N	→	Speed	rpm
N_s	→	Specific Speed	

OUTWARD FLOW REACTION TURBINE:



Symbol		Description	Unit
$u_1 \& u_2$	→	Tangential Velocity of Runner at Inlet & Outlet	m/s

$v_{r1} \& v_{r2}$	$\longrightarrow$	Relative Velocity at Inlet & Outlet	$m/_s$
$v_{w1} \& v_{w2}$	$\longrightarrow$	Whirl Velocity at Inlet & Outlet	$m/_s$
$V_1 \& V_2$	$\longrightarrow$	Absolute Velocity at Inlet & Outlet	$m/_s$
$v_{f1} \& v_{f2}$	$\longrightarrow$	Flow Velocity at Inlet & Outlet	$m/_s$
α	$\longrightarrow$	Angle made by Absolute Velocity at Inlet with the Direction of Motion of Vane	Degree
θ	$\longrightarrow$	Angle made by Relative Velocity at Inlet with the Direction of Motion of Vane	Degree
ϕ	$\longrightarrow$	Angle made by Relative Velocity at Outlet with the Direction of Motion of Vane	Degree

TANGENTIAL VELOCITY AT INLET:

$$u_1 = \frac{\pi d_1 N}{60}$$

Symbol		Description	Unit
d_1	$\longrightarrow$	Inlet (***or***) Internal Diameter	m
N	$\longrightarrow$	Speed of Turbine	rpm

TANGENTIAL VELOCITY AT OUTLET:

$$u_2 = \frac{\pi d_2 N}{60}$$

Symbol		Description	Unit
d_2	$\longrightarrow$	Outlet (***or***) External Diameter	m
N	$\longrightarrow$	Speed of Turbine	rpm

FROM INLET VELOCITY TRIANGLE DIAGRAM:

$$\sin\alpha = \frac{v_{f1}}{V_1}$$

$$\cos\alpha = \frac{v_{w1}}{V_1}$$

$$\tan\alpha = \frac{v_{f1}}{v_{w1}}$$

$$\sin\theta = \frac{v_{f1}}{v_{r1}}$$

$$\cos\theta = \frac{v_{w1} - u_1}{v_{r1}}$$

$$\tan\theta = \frac{v_{f1}}{v_{w1} - u_1}$$

Symbol		Description	Unit
v_{w1}	⟶	Whirl Velocity at Inlet	m/s
V_1	⟶	Absolute Velocity at Inlet	m/s
v_{f1}	⟶	Flow Velocity at Inlet	m/s
u_1	⟶	Tangential Velocity of Runner at Inlet	m/s
v_{r1}	⟶	Relative Velocity at Inlet	m/s
α	⟶	Angle made by Absolute Velocity at Inlet with the Direction of Motion of Vane	Degree
θ	⟶	Angle made by Relative Velocity at Inlet with the Direction of Motion of Vane	Degree

RELATIVE VELOCITY AT INLET:

$$v_{r1} = \sqrt{{v_{f1}}^2 + (v_{w1} - u_1)^2}$$

Symbol		Description	Unit
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	m/s
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	m/s
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s

DISCHARGE:

$$Q = \pi d_1 b_1 v_{f1} = \pi d_2 b_2 v_{f2}$$

$$Q = A v_{f1} = A v_{f2} = A_{f1} v_{f1} = A_{f2} v_{f2}$$

Symbol		Description	Unit
$v_{f1} \& v_{f2}$	$\longrightarrow$	Flow Velocity at Inlet & Outlet	m/s
$d_1 \& d_2$	$\longrightarrow$	Diameter of Impeller at Inlet & Outlet	m
$b_1 \& b_2$	$\longrightarrow$	Width of Impeller at Inlet & Outlet	m
Q	$\longrightarrow$	Discharge	m^3/s
A	$\longrightarrow$	Area of Runner	m^2
$A_{f1} \& A_{f2}$	$\longrightarrow$	Area of Flow at Inlet & Outlet	m/s

MASS OF WATER FLOWING THROUGH THE RUNNER:

$$m = \rho Q$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	m^3/s
ρ	$\longrightarrow$	Density	kg/m^3

INPUT POWER TO TURBINE *(OR)* POWER GIVEN TO TURBINE:

$$P = \rho g Q H$$

Symbol	Description	Unit
ρ	Density	kg/m^3
Q	Discharge	m^3/s
g	Acceleration due to Gravity	m/s^2
H	Head	m

POWER DEVELOPED BY TURBINE:

$$P = \rho * Q * v_{w1} * u_1$$

Symbol	Description	Unit
ρ	Density	kg/m^3
Q	Discharge	m^3/s
v_{w1}	Whirl Velocity at Inlet	m/s
u_1	Tangential Velocity of Runner at Inlet	m/s

HYDRAULIC EFFICIENCY:

$$\eta_{hyd} = \frac{v_{w1} u_1}{gH}$$

$$\eta_{hyd} = \frac{Head\ Inlet - Head\ Loss}{Head\ Inlet}$$

Symbol	Description	Unit
u_1	Tangential Velocity of Runner at Inlet	m/s
v_{w1}	Whirl Velocity at Inlet	m/s
g	Acceleration due to Gravity	m/s^2
H	Head	m

OVERALL EFFICIENCY:

$$\eta_o = \frac{Shaft\ Power}{Input\ Power}$$

$$\eta_o = \frac{S.P}{\rho g Q H}$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	kg/m^3
Q	$\longrightarrow$	Discharge	m^3/s
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H	$\longrightarrow$	Head	m
$S.P$	$\longrightarrow$	Shaft Power	W

SPEED RATIO:

$$K_u = \frac{u}{\sqrt{2gH}}$$

$$K_u = 0.6 - 0.9$$

Symbol		Description	Unit
u	$\longrightarrow$	Tangential Velocity	m/s
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
K_u	$\longrightarrow$	Speed Ratio	

FLOW RATIO:

$$K_f = \frac{v_{f1}}{\sqrt{2gH}}$$

$$K_f = 0.15 - 0.3$$

Symbol		Description	Unit
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	$m/_s$
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	$m/_{s^2}$
K_f	$\longrightarrow$	Flow Ratio	

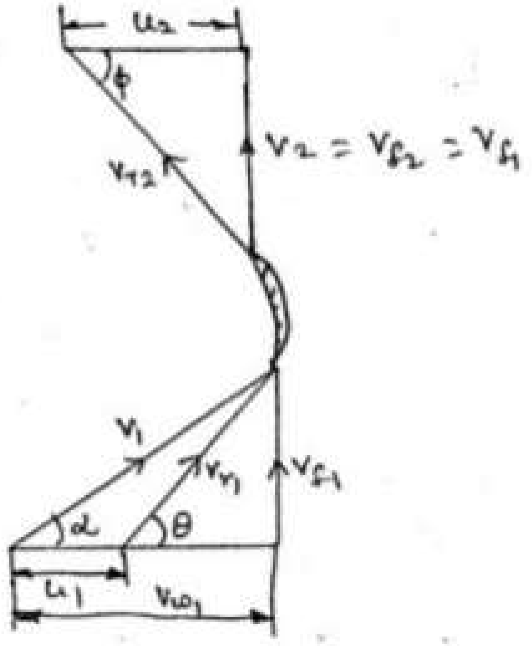
SPECIFIC SPEED:

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	$m^3/_s$
H	$\longrightarrow$	Head	m
P	$\longrightarrow$	Power	kW
N	$\longrightarrow$	Speed	rpm
N_s	$\longrightarrow$	Specific Speed	

FRANCIS TURBINE:



Symbol	Description	Unit
$u_1 \& u_2$	Tangential Velocity of Runner at Inlet & Outlet	$^m/_s$
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$v_{w1} \& v_{w2}$	Whirl Velocity at Inlet & Outlet	$^m/_s$
$V_1 \& V_2$	Absolute Velocity at Inlet & Outlet	$^m/_s$
$v_{f1} \& v_{f2}$	Flow Velocity at Inlet & Outlet	$^m/_s$
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TANGENTIAL VELOCITY AT INLET:

$$u_1 = \frac{\pi d_1 N}{60}$$

Symbol		Description	Unit
d_1	⟶	Inlet (*or*) External Diameter	m
N	⟶	Speed of Turbine	rpm

TANGENTIAL VELOCITY AT OUTLET:

$$u_2 = \frac{\pi d_2 N}{60}$$

Symbol		Description	Unit
d_2	⟶	Outlet (*or*) Internal Diameter	m
N	⟶	Speed of Turbine	rpm

FROM INLET VELOCITY TRIANGLE DIAGRAM:

$$\sin\alpha = \frac{v_{f1}}{V_1}$$

$$\cos\alpha = \frac{v_{w1}}{V_1}$$

$$\tan\alpha = \frac{v_{f1}}{v_{w1}}$$

$$\sin\theta = \frac{v_{f1}}{v_{r1}}$$

$$\cos\theta = \frac{v_{w1} - u_1}{v_{r1}}$$

$$\tan\theta = \frac{v_{f1}}{v_{w1} - u_1}$$

Symbol		Description	Unit
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
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u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	m/s
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RELATIVE VELOCITY AT INLET:

$$v_{r1} = \sqrt{{v_{f1}}^2 + (v_{w1} - u_1)^2}$$

Symbol		Description	Unit
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	m/s
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	m/s
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s

DISCHARGE:

$$Q = \pi d_1 b_1 v_{f1} = \pi d_2 b_2 v_{f2}$$

$$Q = A v_{f1} = A v_{f2} = A_{f1} v_{f1} = A_{f2} v_{f2}$$

Symbol		Description	Unit
$v_{f1} \& v_{f2}$	$\longrightarrow$	Flow Velocity at Inlet & Outlet	m/s
$d_1 \& d_2$	$\longrightarrow$	Diameter of Impeller at Inlet & Outlet	m

$b_1 \& b_2$	$\longrightarrow$	Width of Impeller at Inlet & Outlet	m
Q	$\longrightarrow$	Discharge	$m^3/_s$
A	$\longrightarrow$	Area of Runner	m^2
$A_{f1} \& A_{f2}$	$\longrightarrow$	Area of Flow at Inlet & Outlet	$m/_s$

CIRCUMFERENTIAL AREA OF RUNNER:

$$A = \pi d_1 b_1 = \pi d_2 b_2$$

Symbol		Description	Unit
$d_1 \& d_2$	$\longrightarrow$	Diameter of Impeller at Inlet & Outlet	m
$b_1 \& b_2$	$\longrightarrow$	Width of Impeller at Inlet & Outlet	m
A	$\longrightarrow$	Circumferential Area of Runner	m^2

MASS OF WATER FLOWING THROUGH THE RUNNER:

$$m = \rho Q$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	$m^3/_s$
ρ	$\longrightarrow$	Density	$kg/_{m^3}$

INPUT POWER TO TURBINE *(OR)* POWER GIVEN TO TURBINE:

$$P = \rho g Q H$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	$kg/_{m^3}$
Q	$\longrightarrow$	Discharge	$m^3/_s$
g	$\longrightarrow$	Acceleration due to Gravity	$m/_{s^2}$
H	$\longrightarrow$	Head	m

POWER DEVELOPED BY TURBINE:

$$P = \rho * Q * v_{w1} * u_1$$

Symbol		Description	Unit
ρ	⟶	Density	kg/m^3
Q	⟶	Discharge	m^3/s
v_{w1}	⟶	Whirl Velocity at Inlet	m/s
u_1	⟶	Tangential Velocity of Runner at Inlet	m/s

HYDRAULIC EFFICIENCY:

$$\eta_{hyd} = \frac{v_{w1}u_1}{gH}$$

$$\eta_{hyd} = \frac{Head\ Inlet - Head\ Loss}{Head\ Inlet}$$

Symbol		Description	Unit
u_1	⟶	Tangential Velocity of Runner at Inlet	m/s
v_{w1}	⟶	Whirl Velocity at Inlet	m/s
g	⟶	Acceleration due to Gravity	m/s^2
H	⟶	Head	m

OVERALL EFFICIENCY:

$$\eta_o = \frac{Shaft\ Power}{Input\ Power}$$

$$\eta_o = \frac{S.P}{\rho g Q H}$$

Symbol		Description	Unit
ρ	⟶	Density	kg/m^3
Q	⟶	Discharge	m^3/s

g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
H	$\longrightarrow$	Head	m
$S.P$	$\longrightarrow$	Shaft Power	W

SPEED RATIO:

$$K_u = \frac{u}{\sqrt{2gH}}$$

$$K_u = 0.6 - 0.9$$

Symbol		Description	Unit
u	$\longrightarrow$	Tangential Velocity	m/s
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
K_u	$\longrightarrow$	Speed Ratio	

FLOW RATIO:

$$K_f = \frac{v_{f1}}{\sqrt{2gH}}$$

$$K_f = 0.15 - 0.3$$

Symbol		Description	Unit
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	m/s
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
K_f	$\longrightarrow$	Flow Ratio	

BREADTH RATIO:

$$n = \frac{b_1}{d_1}$$

$$n = 0.1 - 0.4$$

Symbol		Description	Unit
b_1	$\longrightarrow$	Width of Runner at Inlet	m
d_1	$\longrightarrow$	Diameter of Runner at Inlet	m
n	$\longrightarrow$	Breadth Ratio	

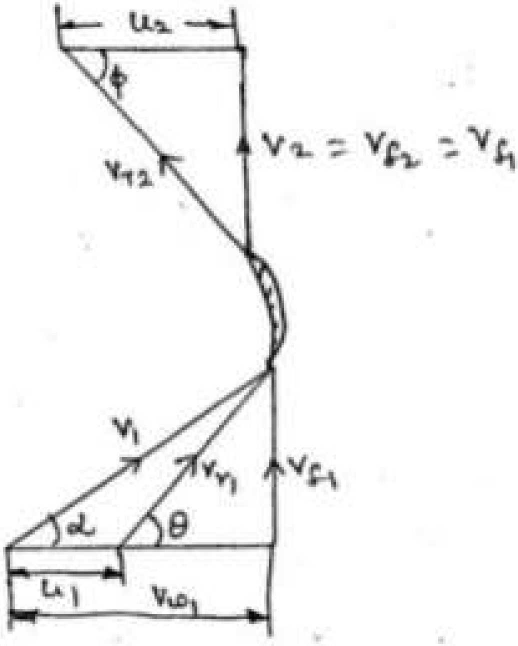
SPECIFIC SPEED:

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	m^3/s
H	$\longrightarrow$	Head	m
P	$\longrightarrow$	Power	kW
N	$\longrightarrow$	Speed	rpm
N_s	$\longrightarrow$	Specific Speed	

KAPLAN TURBINE:



Symbol	Description	Unit
$u_1 \& u_2$	Tangential Velocity of Runner at Inlet & Outlet	m/s
$v_{r1} \& v_{r2}$	Relative Velocity at Inlet & Outlet	m/s
$v_{w1} \& v_{w2}$	Whirl Velocity at Inlet & Outlet	m/s
$V_1 \& V_2$	Absolute Velocity at Inlet & Outlet	m/s
$v_{f1} \& v_{f2}$	Flow Velocity at Inlet & Outlet	m/s
α	Angle made by Absolute Velocity at Inlet with the Direction of Motion of Vane	Degree
θ	Angle made by Relative Velocity at Inlet with the Direction of Motion of Vane	Degree
ϕ	Angle made by Relative Velocity at Outlet with the Direction of Motion of Vane	Degree

TANGENTIAL VELOCITY AT INLET:

$$u_1 = \frac{\pi D_o N}{60}$$

Symbol		Description	Unit
D_o	$\longrightarrow$	Inlet (*or*) External Diameter	m
N	$\longrightarrow$	Speed of Turbine	rpm

TANGENTIAL VELOCITY AT OUTLET:

$$u_2 = \frac{\pi D_b N}{60} = \frac{\pi D_h N}{60}$$

Symbol		Description	Unit
D_b or D_h	$\longrightarrow$	Outlet (*or*) Boss (*or*) Hub Diameter	m
N	$\longrightarrow$	Speed of Turbine	rpm

FROM INLET VELOCITY TRIANGLE DIAGRAM:

$$\sin\alpha = \frac{v_{f1}}{V_1}$$

$$\cos\alpha = \frac{v_{w1}}{V_1}$$

$$\tan\alpha = \frac{v_{f1}}{v_{w1}}$$

$$\sin\theta = \frac{v_{f1}}{v_{r1}}$$

$$\cos\theta = \frac{v_{w1} - u_1}{v_{r1}}$$

$$\tan\theta = \frac{v_{f1}}{v_{w1} - u_1}$$

Symbol		Description	Unit
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	$^m/_s$
V_1	$\longrightarrow$	Absolute Velocity at Inlet	$^m/_s$
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	$^m/_s$
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	$^m/_s$
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	$^m/_s$
α	$\longrightarrow$	Angle made by Absolute Velocity at Inlet with the Direction of Motion of Vane	Degree
θ	$\longrightarrow$	Angle made by Relative Velocity at Inlet with the Direction of Motion of Vane	Degree

RELATIVE VELOCITY AT INLET:

$$v_{r1} = \sqrt{{v_{f1}}^2 + (v_{w1} - u_1)^2}$$

Symbol		Description	Unit
v_{r1}	$\longrightarrow$	Relative Velocity at Inlet	$^m/_s$
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	$^m/_s$
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	$^m/_s$
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	$^m/_s$

DISCHARGE:

$$Q = \frac{\pi}{4}\left[{D_0}^2 - {D_b}^2\right]v_{f1}$$

Symbol		Description	Unit
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	$^m/_s$
D_0	$\longrightarrow$	Inlet (***or***) External Diameter	m
D_b or D_h	$\longrightarrow$	Outlet (***or***) Boss (***or***) Hub Diameter	m

Q	$\longrightarrow$	Discharge	$m^3/_s$

CIRCUMFERENTIAL AREA OF RUNNER:

$$A = \frac{\pi}{4}\left[{D_0}^2 - {D_b}^2\right]$$

Symbol		Description	Unit
D_0	$\longrightarrow$	Inlet **(*or*)** External Diameter	m
D_b or D_h	$\longrightarrow$	Outlet **(*or*)** Boss **(*or*)** Hub Diameter	m
A	$\longrightarrow$	Circumferential Area of Runner	m^2

MASS OF WATER FLOWING THROUGH THE RUNNER:

$$m = \rho Q$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	$m^3/_s$
ρ	$\longrightarrow$	Density	$kg/_{m^3}$

INPUT POWER TO TURBINE *(OR)* POWER GIVEN TO TURBINE:

$$P = \rho g Q H$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	$kg/_{m^3}$
Q	$\longrightarrow$	Discharge	$m^3/_s$
g	$\longrightarrow$	Acceleration due to Gravity	$m/_{s^2}$
H	$\longrightarrow$	Head	m

POWER DEVELOPED BY TURBINE:

$$P = \rho * Q * v_{w1} * u_1$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	$kg/_{m^3}$
Q	$\longrightarrow$	Discharge	$m^3/_s$

v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s

HYDRAULIC EFFICIENCY:

$$\eta_{hyd} = \frac{v_{w1} u_1}{gH}$$

$$\eta_{hyd} = \frac{Head\ Inlet - Head\ Loss}{Head\ Inlet}$$

Symbol		Description	Unit
u_1	$\longrightarrow$	Tangential Velocity of Runner at Inlet	m/s
v_{w1}	$\longrightarrow$	Whirl Velocity at Inlet	m/s
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
H	$\longrightarrow$	Head	m

OVERALL EFFICIENCY:

$$\eta_o = \frac{Shaft\ Power}{Input\ Power}$$

$$\eta_o = \frac{S.P}{\rho g Q H}$$

Symbol		Description	Unit
ρ	$\longrightarrow$	Density	kg/m^3
Q	$\longrightarrow$	Discharge	m^3/s
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
H	$\longrightarrow$	Head	m
$S.P$	$\longrightarrow$	Shaft Power	W

SPEED RATIO:

$$K_u = \frac{u}{\sqrt{2gH}}$$

$$K_u = 0.6 - 0.9$$

Symbol		Description	Unit
u	$\longrightarrow$	Tangential Velocity	m/s
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
K_u	$\longrightarrow$	Speed Ratio	

FLOW RATIO:

$$K_f = \frac{v_{f1}}{\sqrt{2gH}}$$

$$K_f = 0.15 - 0.3$$

Symbol		Description	Unit
v_{f1}	$\longrightarrow$	Flow Velocity at Inlet	m/s
H	$\longrightarrow$	Head	m
g	$\longrightarrow$	Acceleration due to Gravity	m/s^2
K_f	$\longrightarrow$	Flow Ratio	

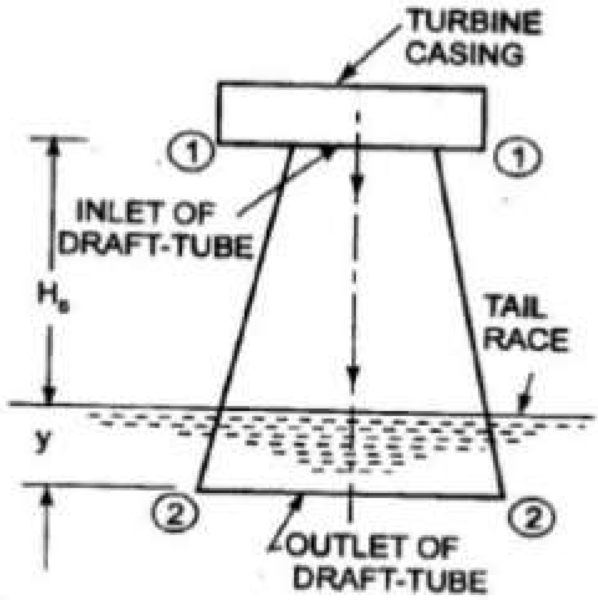
SPECIFIC SPEED:

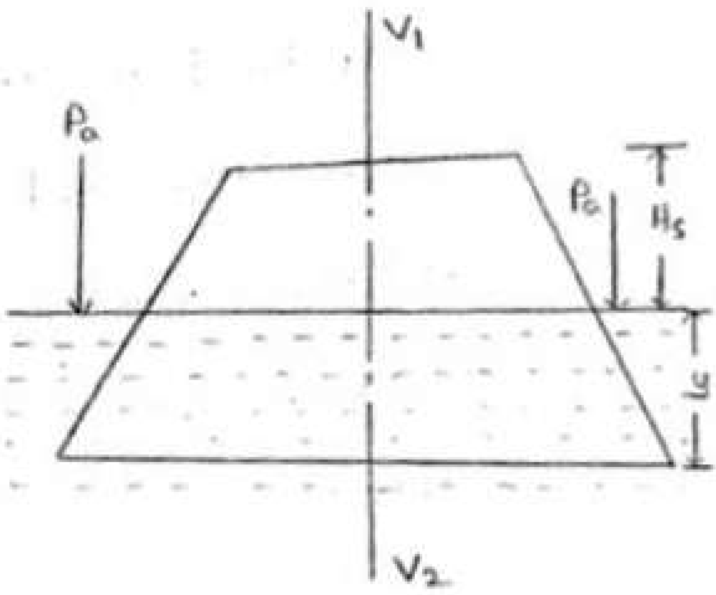
$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Symbol		Description	Unit
Q	$\longrightarrow$	Discharge	$m^3/_s$
H	$\longrightarrow$	Head	m
P	$\longrightarrow$	Power	kW
N	$\longrightarrow$	Speed	rpm
N_S	$\longrightarrow$	Specific Speed	

DRAFT TUBE:





Symbol		Description	Unit
$V_1 \& V_2$	$\longrightarrow$	Velocity at Inlet & Outlet	m/s
H_s	$\longrightarrow$	Vertical Height of Draft Tube Above Tail Race	m
y	$\longrightarrow$	Distance of Bottom of Draft Tube from Tail Race	m

FROM BERNOULLI'S EQUATION:

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} + z_2 + h_f$$

Symbol		Description	Unit
$P_1 \& P_2$	$\longrightarrow$	Pressure at Inlet & Outlet of Draft Tube	N/m^2
$V_1 \& V_2$	$\longrightarrow$	Velocity at Inlet & Outlet of Draft Tube	m/s
$z_1 \& z_2$	$\longrightarrow$	Datum Head Inlet & Outlet of Draft Tube	m
h_f	$\longrightarrow$	Head Loss	m
ρ	$\longrightarrow$	Density of Liquid	kg/m^3
g	$\longrightarrow$	Acceleration due to gravity	m/s^2

LENGTH OF DRAFT TUBE:

$$L = H_s + y$$

Symbol		Description	Unit
L	$\longrightarrow$	Length of Draft Tube	m
H_s	$\longrightarrow$	Vertical Height of Draft Tube Above Tail Race	m
y	$\longrightarrow$	Distance of Bottom of Draft Tube from Tail Race	m

EFFICIENCY OF DRAFT TUBE:

$$\eta_d = \frac{\left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right) - h_f}{\frac{{V_1}^2}{2g}}$$

Symbol		Description	Unit
V_1 & V_2	$\longrightarrow$	Velocity at Inlet & Outlet of Draft Tube	m/s
h_f	$\longrightarrow$	Head Loss	m
g	$\longrightarrow$	Acceleration due to gravity	m/s^2

HYDRAULIC EFFICIENCY OF DRAFT TUBE:

$$\eta_{hyd} = \frac{Head\ Utilized\ by\ Turbine}{Head\ Inlet\ of\ Turbine}$$

$$\eta_{hyd} = \frac{H - h_{ft} - h_{fd} - \frac{V_2^2}{2g}}{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1}$$

Symbol		Description	Unit
P_1	$\longrightarrow$	Pressure at Inlet of Draft Tube	N/m^2
V_1 & V_2	$\longrightarrow$	Velocity at Inlet & Outlet of Draft Tube	m/s
z_1	$\longrightarrow$	Datum Head Inlet of Draft Tube	m
h_f	$\longrightarrow$	Head Loss	m
ρ	$\longrightarrow$	Density of Liquid	kg/m^3
g	$\longrightarrow$	Acceleration due to gravity	m/s^2