UNIT I

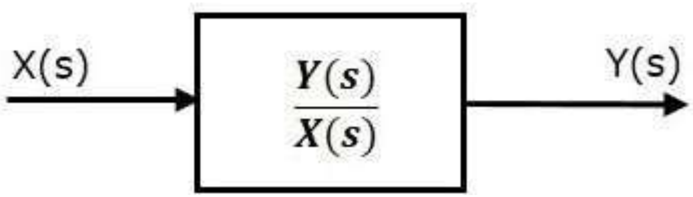
MATHEMATICAL MODELLING OF SYSTEMS

INTRODUCTION

- The control systems can be represented with a set of mathematical equations known as mathematical model. These models are useful for analysis and design of control systems.
- The following mathematical models are mostly used.
  - Differential equation model
  - Transfer function model
  - State space model

TRANSFER FUNCTION

- Transfer function model is an s-domain mathematical model of control systems.
- The Transfer function of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.

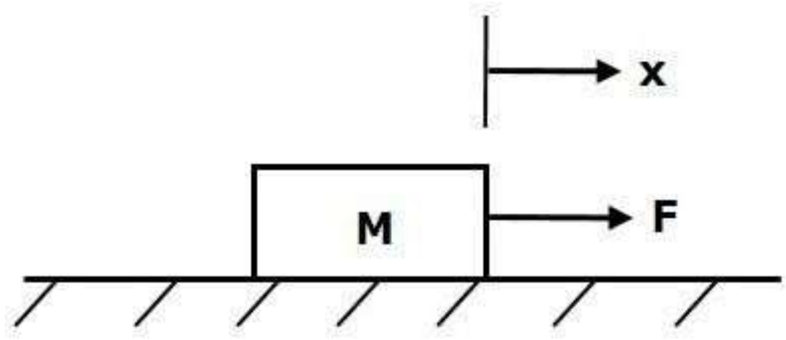


MECHNICAL SYSTEMS

- Mechanical systems mainly consists of three main elements namely **mass, dashpot and spring.**
- If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system.
- Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero

MECHNICAL SYSTEM

- **Mass:**



$$F_m \propto a$$

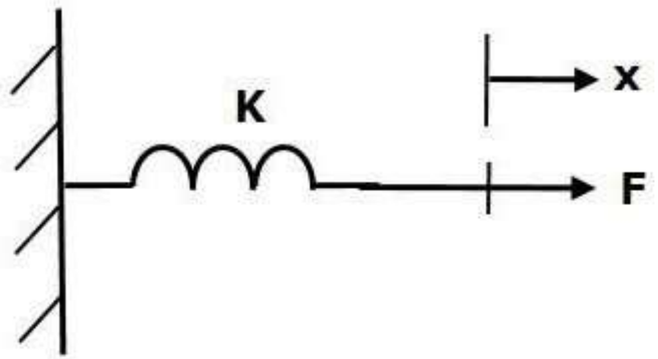
$$F_m = M_a = M\frac{d^2x}{dt^2}$$

$$F = F_m = M\frac{d^2x}{dt^2}$$

Where,
F is the applied force
F$_m$ is the opposing force due to mass
M is mass
a is acceleration
x is displacement

MECHNICAL SYSTEM

- **Spring:**



$$F \propto x$$

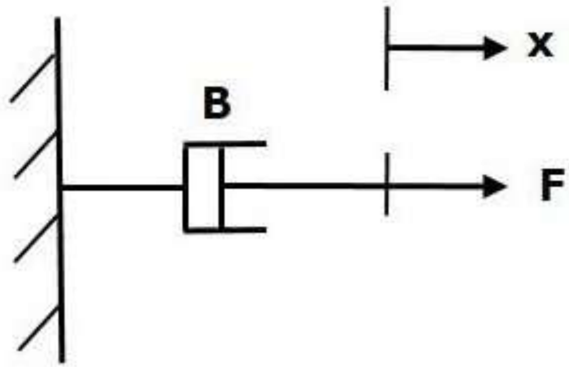
$$F_k = Kx$$

$$F = F_k = Kx$$

Where,
F is the applied force
$\mathbf{F_k}$ is the opposing force due to elasticity of spring
K is spring constant
x is displacement

MECHNICAL SYSTEM

- **Dashpot:**



$$F_b \propto v$$

$$F_b = Bv = B\frac{dx}{dt}$$

$$F = F_b = B\frac{dx}{dt}$$

Where,
F_b is the opposing force due to friction of dashpot
B is the frictional coefficient
v is velocity
x is displacement

ELECTRICAL SYSTEM

- The basic elements of electrical system are **resistor, inductor and capacitor.**