TOPIC:4.6- MAXIMA AND MINIMA OF FUNCTIONS OF TWO VARIABLES

Maxima and Minima

State the condition for $f(x,y)$ be a maximum or minimum.

Maximum	Minimum
(i) $f_x = 0$	(i) $f_x = 0$
(ii) $f_y = 0$	(ii) $f_y = 0$
(iii) $f_{xx} f_{yy} - (f_{xy})^2 > 0$	(iii) $f_{xx} f_{yy} - (f_{xy})^2 > 0$
(iv) $f_{xx} < 0$ (or) $f_{yy} < 0$	(iv) $f_{xx} > 0$ (or) $f_{yy} > 0$

1) A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires least materials for its construction.

Let x, y, z be the dimensions of the rectangular box.

Surface area $f(x,y,z) = xy + 2xz + 2yz$

volume $g(x,y,z) = xyz = 32$

$g(x,y,z) = xyz - 32$

Hence $F(x,y,z) = f(x,y,z) + \lambda\, g(x,y,z)$

$= xy + 2xz + 2yz + \lambda(xyz - 32) \rightarrow (1)$

Diff. (1) partially w.r.t 'x', 'y', 'z'.

$$\frac{\partial F}{\partial x} = y + 2z + yz\lambda = 0$$

$$yz\lambda = -y - 2z$$

$$\lambda = -\left[\frac{y+2z}{yz}\right]$$

$$\lambda = -\left[\frac{1}{z} + \frac{2}{y}\right] \rightarrow (2)$$

$$\frac{\partial F}{\partial y} = x + 2z + xz\lambda = 0$$

$$xz\lambda = -x - 2z$$

$$\lambda = -\left[\frac{x+2z}{xz}\right]$$

$$\lambda = -\left[\frac{1}{z} + \frac{2}{x}\right] \rightarrow (3)$$

$$\frac{\partial F}{\partial z} = 2x + 2y + xy\lambda = 0$$

$$xy\lambda = -2x - 2y$$

$$\lambda = -\left[\frac{2x+2y}{xy}\right]$$

$$\lambda = -\left[\frac{2}{y} + \frac{2}{x}\right] \rightarrow (4)$$

From (2) & (3)

$$-\left[\frac{1}{z} + \frac{2}{y}\right] = -\left[\frac{1}{z} + \frac{2}{x}\right]$$

$$\frac{2}{y} = \frac{2}{x} \Rightarrow \boxed{x = y}$$

From (3) & (4).

$$-\left[\frac{1}{z} + \frac{2}{x}\right] = -\left[\frac{2}{y} + \frac{2}{x}\right]$$

$$\frac{1}{z} = \frac{2}{y} \Rightarrow \boxed{y = 2z}$$

$$\therefore \quad x = y = 2z$$

Sub. in $g(x, y, z)$.

$$xyz = 32 \Rightarrow 2z \cdot 2z \cdot z = 32$$

$$\Rightarrow 4z^3 = 32 \Rightarrow z^3 = 8 \Rightarrow \boxed{z = 2}$$

$\boxed{x = 4}$ and $\boxed{y = 4}$