



### Topic: 1.2 – EIGEN VALUES AND EIGEN VECTORS

Eigen-values (or) Proper values (or) Latent roots (or) Characteristic roots:

Let  $A = [a_{ij}]$  be square matrix. The characteristic eqn. of  $A$  is  $|A - \lambda I| = 0$ . The roots of the characteristic eqn. are called Eigen values of  $A$ .

Eigen Vector (or) Latent Vector:

Corresponding to each characteristic root  $\lambda$ , there corresponds non-zero vector  $X$  which satisfies the eqn.  $(A - \lambda I)X = 0$ . The non-zero vector  $X$  are called Characteristic vectors (or) Eigen vectors.

Working Rule to find Eigenvalues & Eigenvectors:

Step 1: To find the characteristic eqn.  $|A - \lambda I| = 0$ .

Step 2: To solve the characteristic eqn. we get characteristic roots. They are called Eigenvalues.

Step 3: To find Eigenvectors, solve  $(A - \lambda I)X = 0$  for the different values of  $\lambda$ .



AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Find the Eigenvalues and Eigen vectors of the matrix  $\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

Solu. Let  $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$

Step 1: To find the characteristic eqn.

The char. eqn. of  $A$  is  $\lambda^2 - s_1\lambda + s_2 = 0$ ,  
where  $s_1 =$  Sum of main diagonal elements  
 $= 1 + (-1) = 0$ .

$$s_2 = |A| = \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -1 - 3 = -4$$

$\therefore$  The char. eqn. is  $\lambda^2 - 0\lambda - 4 = 0$ .  
(or)  $\lambda^2 - 4 = 0$

Step 2: To solve the char. eqn.

$$\lambda^2 = 4$$

$$\lambda = \pm 2$$

Hence the Eigenvalues are  $-2, 2$





Step: 3: To find the Eigen vectors solve  $(A - \lambda I)x = 0$

$$\left[ \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\left[ \begin{array}{cc|c} 1-\lambda & 1 & 0 \\ 3 & -1-\lambda & 0 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \text{①}$$

Case (i):

If  $\lambda = -2$  then ① becomes

$$\begin{pmatrix} 3 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$3x_1 + x_2 = 0$$

$$3x_1 + x_2 = 0$$

We get only one eqn.  $3x_1 + x_2 = 0$

$$3x_1 = -x_2$$
$$\frac{x_1}{1} = \frac{-x_2}{-3}$$

$\therefore$  The corresponding Eigenvector  $\alpha_1$

$$\alpha_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Case (ii)

If  $\lambda = 2$  then eqn. ① becomes

$$\left[ \begin{array}{cc|c} -1 & 1 & 0 \\ 3 & -3 & 0 \end{array} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x_1 + x_2 = 0$$

$$3x_1 - 3x_2 = 0$$



(ie) we get only one eqn.  $x_1 - x_2 = 0$

$$x_1 = x_2$$

$$(ie) \frac{x_1}{1} = \frac{x_2}{1}$$

Hence the corresponding Eigen vector is  $x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Result:

1. Eigen values of A are  $(-2, 2)$

2. Eigen vectors:  $\lambda = -2 \Rightarrow x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

:  $\lambda = 2 \Rightarrow x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Find the Eigenvalues & Eigenvectors of

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

Solu:

Step 1: To find the Charac. eqn.

The Charac. eqn. of A is

$$\lambda^3 - s_1 \lambda^2 + s_2 \lambda - s_3 = 0$$

$s_1 =$  Sum of its leading diagonal elements

$$= 7 + 6 + 5 = 18$$





AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

$S_2$  = Sum of the minors of its leading diagonal.

$$= \begin{vmatrix} 6 & -2 \\ -2 & 5 \end{vmatrix} + \begin{vmatrix} 7 & 0 \\ 0 & 5 \end{vmatrix} + \begin{vmatrix} 7 & -2 \\ -2 & 6 \end{vmatrix}$$

$$= (30-4) + (35-0) + (42-4)$$

$$= 26 + 35 + 38 = 99$$

$$S_3 = |A| = \begin{vmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{vmatrix}$$

$$= 7[30-4] + 2[-10-0] + 0[1]$$

$$= 7(26) - 20$$

$$= 182 - 20 = 162$$

∴ The charac. eqn. is

$$\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$$

Step: 2 To solve the charac. eqn.

$$\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0 \quad \text{--- (1)}$$

If  $\lambda = 1$  then (1)  $\Rightarrow 1 - 18 + 99 - 162 \neq 0$ .

If  $\lambda = -1$  then (1)  $\Rightarrow -1 - 18 - 99 - 162 \neq 0$ .

If  $\lambda = 2$  then (1)  $\Rightarrow 8 - 72 + 198 - 162 \neq 0$

If  $\lambda = -2$  then (1)  $\Rightarrow -8 - 72 - 198 - 162 \neq 0$ .

If  $\lambda = 3$  then  $27 - 162 + 297 - 162 = 0$ .

∴  $\lambda = 3$  is a root



AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

By synthetic division

$$\begin{array}{r|rrrr} 3 & 1 & -18 & 99 & -162 \\ & & 3 & -45 & 162 \\ \hline & 1 & -15 & 54 & 0 \end{array}$$
$$(\lambda - 3)(\lambda^2 - 15\lambda + 54) = 0$$
$$(\lambda - 3)(\lambda - 6)(\lambda - 9) = 0$$
$$\Rightarrow \lambda = 3, \lambda = 6 \text{ \& } \lambda = 9$$

Step: 3: Solve  $(A - \lambda I)x = 0$ .

$$\begin{bmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{--- (A)}$$

Case (i)

If  $\lambda = 3$ ; (A)  $\Rightarrow 4x_1 - 2x_2 + 0x_3 = 0$  --- (i)

$$-2x_1 + 3x_2 - 2x_3 = 0 \quad \text{--- (ii)}$$
$$0x_1 - 2x_2 + 2x_3 = 0 \quad \text{--- (iii)}$$

Solving (i) & (ii)

$$\frac{x_1}{6-4} = \frac{x_2}{0+4} = \frac{x_3}{\lambda-0}$$
$$\frac{x_1}{2} = \frac{x_2}{4} = \frac{x_3}{4}$$
$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}$$





AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Hence the corresponding Eigen Vector is  $x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

Case (ii) : when  $\lambda = 6$ , (A) become .

$$x_1 - 2x_2 + 0x_3 = 0 \quad \text{--- (5)}$$

$$-2x_1 + 0x_2 - 2x_3 = 0 \quad \text{--- (6)}$$

$$0x_1 - 2x_2 - x_3 = 0 \quad \text{--- (7)}$$

Solving (6) & (7)

$$\frac{x_1}{0-4} = \frac{x_2}{0-2} = \frac{x_3}{4-0}$$

$$\frac{x_1}{-4} = \frac{x_2}{-2} = \frac{x_3}{4}$$

$$\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ -2 & 0 & -2 \\ 0 & -2 & -2 \end{array}$$

Hence the Eigen vector is  $x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

Case (iii) when  $\lambda = 9$ , (A) become .

$$-2x_1 - 2x_2 + 0x_3 = 0 \quad \text{--- (8)}$$

$$-2x_1 - 3x_2 - 2x_3 = 0 \quad \text{--- (9)}$$

$$0x_1 - 2x_2 - 2x_3 = 0 \quad \text{--- (10)}$$

Solving (9) & (10) we get

$$\frac{x_1}{12-4} = \frac{x_2}{0-8} = \frac{x_3}{4-0}$$

$$\frac{x_1}{8} = \frac{x_2}{-8} = \frac{x_3}{4}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ -2 & -3 & -2 \\ 0 & -2 & -2 \end{array}$$



SNS COLLEGE OF ENGINEERING  
Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade  
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai