



Topic: 1.8 – Problems On Reduction of Quadratic Form

$$2x_1x_2 + 2x_1x_3 - 2x_2x_3$$
$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$
$$S_1 = 0$$
$$S_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= (-1) + (-1) + (-1)$$
$$= -3$$
$$S_3 = 0 - 1(0 + 1) + (-1)$$
$$= -1 - 1$$
$$S_3 = -2$$
$$\lambda^3 - 3\lambda + 2 = 0$$

$$\begin{array}{c|ccc} 1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$
$$\lambda^3 + \lambda^2 - 2 = 0$$
$$(\lambda - 1)(\lambda + 2)(\lambda + 1) = 0$$
$$\lambda = 1, 1, -2$$
$$(A - \lambda I)x = 0$$



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$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$
$$\begin{bmatrix} -\lambda + 1 & -1 & 1 \\ 1 & -\lambda & -1 \\ 1 & -1 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\lambda = 1$

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{aligned} -x_1 + x_2 + x_3 &= 0 \\ x_1 - x_2 - x_3 &= 0 \\ x_1 - x_2 - x_3 &= 0 \\ x_1 &= 0 \\ x_2 &= -x_3 \\ \frac{x_2}{-1} &= \frac{x_3}{1} \\ x_2 &= \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \end{aligned}$$

$\lambda = -2$

$$x_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Let $x_3 = \begin{bmatrix} 1 \\ m \\ n \end{bmatrix}$

$$(2 \ m \ n) \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 0$$



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$$\begin{aligned} -l + m + n &= 0 \\ X_3 X_2^T &= 0 \\ (l \ m \ n) \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} &= 0 \end{aligned}$$
$$\frac{l}{1+1} = \frac{m}{0+1} = \frac{n}{1}$$
$$\frac{1}{2} = \frac{m}{1} = \frac{n}{1}$$
$$X_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$
$$N = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} -\frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$
$$N^T = \begin{bmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$



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$$AN = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$
$$= \begin{bmatrix} 2/\sqrt{3} & 0 & 2/\sqrt{6} \\ -2/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ -2/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$
$$AN N^T = \begin{bmatrix} -1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 2/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 2/\sqrt{3} & 0 & 2/\sqrt{3} \\ -2/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ -2/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

C.F = $-2y_1^2 + y_2^2 + y_3^2 = 0$

② $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

Charact. equ is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$$S_1 = 4$$
$$S_2 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}$$
$$= (1-4) + (2-1) + 2(-1)$$
$$= -3 + 1 + 1 = -1$$



$$\begin{aligned} S_3 &= 2(1-4) - 1(1-2) - 1(-2+1) \\ &= -6 + 1 + 1 \\ &= -4 \\ \lambda^3 - 4\lambda^2 - \lambda + 4 &= 0 \\ \boxed{\lambda=1} \quad 1-4-1+4 &= 0 \\ \begin{array}{r|rrrr} 1 & 1 & -4 & -1 & 4 \\ & 0 & 1 & -3 & -4 \\ & 1 & -3 & -4 & 0 \end{array} \\ (\lambda-1) (\lambda^2 - 3\lambda - 4) & \\ (\lambda-1) (\lambda-4) (\lambda+1) & \\ \lambda = 1, -1, 4 & \end{aligned}$$

$$\begin{aligned} \boxed{\lambda=1} \quad \begin{pmatrix} 2-\lambda & 1 & -1 \\ 1 & 1-\lambda & -2 \\ -1 & -2 & 1-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ 2-\lambda x_1 + x_2 - x_3 &= 0 \\ x_1 + (1-\lambda)x_2 - 2x_3 &= 0 \\ -x_1 - 2x_2 + x_3(1-\lambda) &= 0 \\ x_1 + x_2 - x_3 &= 0 \\ x_1 + 0x_2 - 2x_3 &= 0 \\ -x_1 - 0x_2 + 0x_3 &= 0 \end{aligned}$$



$$\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 1 \\ 1 & 0 & -2 & 1 & 0 \end{array}$$
$$\frac{x_1}{-2} = \frac{x_2}{-1+2} = \frac{x_3}{-1}$$
$$x_1 = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$$

$\lambda = -1$

$$\begin{aligned} 3x_1 + x_2 - x_3 &= 0 \\ x_1 + 2x_2 - 2x_3 &= 0 \\ -x_1 - 2x_2 + 2x_3 &= 0 \end{aligned}$$
$$\begin{array}{cccc|c} 3 & 1 & -1 & 3 & 1 \\ 1 & 2 & -2 & 1 & 2 \end{array}$$
$$\frac{x_1}{-2+2} = \frac{x_2}{-1+6} = \frac{x_3}{6-1}$$
$$x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$\lambda = 4$

$$\begin{aligned} -2x_1 + x_2 - x_3 &= 0 \\ x_1 - 3x_2 - 2x_3 &= 0 \\ -x_1 - 2x_2 - 3x_3 &= 0 \end{aligned}$$



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$$\begin{array}{cccc|c} -2 & 1 & -1 & -2 & 1 \\ 1 & -3 & -2 & 1 & -3 \end{array}$$
$$\frac{x_1}{-2-3} = \frac{x_2}{-1-2} = \frac{x_3}{6-1}$$
$$\frac{x_1}{-5} = \frac{x_2}{-3} = \frac{x_3}{5}$$
$$\begin{array}{cccc|c} -3 & -2 & 1 & -3 \\ -1 & -2 & -3 & -1 & -2 \end{array}$$
$$\frac{x_1}{9-4} = \frac{x_2}{2+3} = \frac{x_3}{-2-3}$$
$$x_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$N = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$
$$N = \begin{bmatrix} 0/\sqrt{2} & 2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \end{bmatrix}$$



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$$N^T = \begin{bmatrix} 0/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix}$$
$$AN = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0/\sqrt{2} & 2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{0+1-1}{\sqrt{2}} & \frac{4-1-1}{\sqrt{6}} & \frac{2+1+1}{\sqrt{3}} \\ \frac{0+1-2}{\sqrt{2}} & \frac{2-1-2}{\sqrt{6}} & \frac{1+1+2}{\sqrt{3}} \\ \frac{0-2+1}{\sqrt{2}} & \frac{-2+2+1}{\sqrt{6}} & \frac{-1-2-1}{\sqrt{3}} \end{bmatrix}$$

$$AN = \begin{bmatrix} 0 & 2/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & -4/\sqrt{3} \end{bmatrix}$$
$$N^T AN = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 0 & 2/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & -4/\sqrt{3} \end{pmatrix}$$
$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$