Graph Representation

## Graph Representation

- Graph consists of a non empty set of points called vertices and a set of edges that link vertices.
Definition: A graph $G=(V, E)$ consists of
- a set $V=\left\{v_{1}, v_{2} \ldots . ., v_{n}\right\}$ of $n>1$ vertices and
- a set of $E=\left\{e_{1}, e_{2}, \ldots . ., e_{m}\right\}$ of $m>0$ edges
- such that each edge $e_{k}$ is corresponds to an un ordered pair of vertices ( $\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}$ )
- A road network is a simple example of a graph, in which vertices reprents cities and road connecting them are correspond to edges.
- Loop is an edge that connects a vertex to itself. Edge $\mathrm{e}_{6}$ in the figure below is a loop.
- Edges with same end vertices are called parallel edges . Edges $\mathrm{e}_{4}$ and $e_{5}$ are parallel edges in the below figure

- A Graph without loops and parallel edges is called a simple graph.
- A graphs with isolated vertices (no edges) is called null graph.
- Set of edges $\boldsymbol{E}$ can be empty for a graph but not set of vertices $\boldsymbol{V}$.

Incidence: if an vertex $v_{i}$ is an end vertex of an edge $e_{k}$, we say vertex $v_{i}$ is incident on $e_{k}$ and $e_{k}$ is incident on $v_{i}$.

- $e_{1}$ is incident on $v_{1}$ and $v_{3}$ in the below figure.
- $\mathrm{v}_{4}$ is incident on $\mathrm{e}_{3}, e_{4}$, and $\mathrm{e}_{5}$ in the figure below

Degree: Degree of an vertex is number of edges incident on it, with loops counted twice.


## Basic Operations

- Adjacent Edges: Two non parallel edges are adjacent if they have a vertex in common.
- $\quad e_{1}$ and $e_{2}, e_{2}$ and $e_{6}, e_{2}$ and $e_{3}, e_{1}$ and $e_{4}$ are adjacent edges in the above diagram.
- Adjacent vertices: Two vertices are adjacent if they are connected by an edge.
- $v_{1}$ and $v_{3}, v_{1}$ and $v_{2}, v_{2}$ and $v_{4}$ are adjacent vertices in the above diagram.


## Graph Representation

Graph Representation: There are several different ways to represent graphs in a computer. Two main representations are Adjacency Matrix and Adjacency list.

## Adjacency Matrix Representation:

- An adjacency matrix of a graph $G=(V, E)$ (let $\left.V=\left\{v_{1}, v_{2} \ldots . ., v_{n}\right\}\right)$ is a $n X$ $n$ matrix $A$, such that $A[i, j]=1$ if there is edge between $v_{i}$ and $v_{j}$.
- 0,other wise


|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 2 | 1 | 0 | 0 | 1 | 0 |
| 3 | 1 | 0 | 0 | 1 | 1 |
| 4 | 1 | 1 | 1 | 0 | 1 |
| 5 | 0 | 0 | 1 | 1 | 0 |

## Adjacency List Representation:

- It consists of a list of vertices, which can be represented either by linked list or array. For each vertex, adjacent vertices are represented in the form of a linked list.

$2 \square \rightarrow v_{1}\left|\rightarrow v_{4}\right|$
$3 \rightarrow\left|v_{1}\right| \rightarrow\left|v_{4}\right| \rightarrow>v_{5} \mid$

$5 \longrightarrow>v_{3}\left|>v_{4}\right|$

