

SNS COLLEGE OF ENGINEERING Coimbatore - 641 107

TOPIC : 1 - INTRODUCTION AND APPLICATIONS TO PARTIAL DIFFERENTIAL EQUATIONS



limited II - find bittery constants Partial Differential Equations Formation of Partial Differential equations by Elimination of arbitrary constants Consider an equation \$(x, y, z, a, b)=0-20 where a Bb denote asbitrary constants. A pdie is formed by eliminating the arbitrary constants that occur in the functional relation between the variables. Using $\frac{\partial z}{\partial x} = \beta$, $\frac{\partial z}{\partial y} = q$. 1. Form the p.d.e by eliminating the arbitrary constants a & b form z=axtby Diff p. w. r. to x we get a fla $\frac{\partial z}{\partial x} = a \Rightarrow p = a$ siff p.w.r. to y are get $\frac{\partial z}{\partial y} = b \Rightarrow q = b$ Eqn D becomes, z= px+qy.

SNS COLLEGE OF ENGINEERING Coimbatore - 641 107 2. Eliminate the arbitrary constants at b form $z = (x^2 + a) (y^2 + b)$. Sel: $Z = (a^2 + a) (y^2 + b)$ Diff P.w.n. to x. Diff P.W. $(y^2+b) \Rightarrow \frac{b}{2\pi} = \frac{y^2+b}{2\pi}$ piff provento y. $q = 2y(x^2+a) = \frac{q}{2y} = x^2+a$ i Eqn () becomes along positive all $z = \frac{p}{2a} \cdot \frac{q}{2y}$ with a land positive () Harryz = bq. z = a(x+y)+bDiff winto x. piff w.r. to y. q= a - 90 From O to we get pq=a, p=q.



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4. Find the pastial differential equation of all planes having equal intercepts on the xkyaxis Sol: Intercept form of the plane equation is $\frac{x}{6} + \frac{y}{6} + \frac{z}{c} = 1$. Given a= b (Equal Intercepts on the Qiven a= b (Equal Intercepts on the QAY axis) x+y+z=1. Diffj.w.r.to x, we get $\frac{1}{2} + \frac{1}{2} = 0$ $=) \frac{1}{a} = \frac{-1}{c} \frac{\partial z}{\partial x} = \frac{-1}{c} \frac{p}{p} \rightarrow 0$ Diff puir to y, we get $\frac{1}{a} + \frac{1}{c} \frac{\partial z}{\partial y} = 0$ $=) \frac{1}{a} = -\frac{1}{c} \frac{\partial z}{\partial q} = -\frac{1}{c} \frac{q}{2} \rightarrow 0$ From O & D. $-\frac{1}{c}P = -\frac{1}{c}q$ p = q



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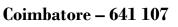
5. $(\chi_{-}a)^2 + (\gamma_{-}b)^2 + 2^2 = 1.$ Bol: piff w.r. to a, we get 2 (2-a) + 22 22 =0 2(n-a) = - p Z px-a=-zp -20 piff P.w.r.to y.we get $2(y-b) + 22 \frac{\partial 2}{\partial y} = 0$ 2(y-b) = -229

y-b= -29-) 2

Using O DO , we get $(-2p)^2 + (-2q)^2 + 2^2 = 1$ 2 p2 + 22 q2 + 22 = 1 $z^{2}(p^{2}+q^{2}+1)=1.$



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() Z=(x+a)²+ (y-b)² > 5 5 + Sol: Diff p.w.r. to x. we get $\frac{\partial z}{\partial x} = 3(x+a)^2 \cdot 1$ $=) \frac{p}{3} = (x+a)^2$ Diff p.w.r. to yive get $\frac{\partial z}{\partial y} = 2(y-b)$ $\frac{q}{2} = y-b.$ $Z = \left(\frac{p}{3}\right)^{3} + \left(\frac{q}{2}\right)^{2} Z = \left(\frac{p}{3}\sqrt{\frac{p}{3}}\right)^{3} + \left(\frac{q}{2}\right)^{2}$ $Z = \left(\frac{p^{3/2}}{3^{3/2}}\right)^{3} + \left(\frac{q}{2}\right)^{2}.$ $(z-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$ (7) $(x-a)^2 + (y-b)^2 = z^2 \cot^2 d$ Sol: Diff w.r. to x we get $2(x-a) = \beta z \beta cot \alpha$ x-a = zp cot2 x piff w.r. to y, we get 2(y-b) = 229 cot2 y-b = 29 cot 2 $z^2 p^2 \cot^4 \alpha + z^2 q^2 \cot^4 \alpha = z^2 \cot^2 \alpha$ $\pm z^2 \cot^2 \alpha$, $p^2 \cot^2 \alpha \pm q^2 \cot^2 \alpha = 1$.