TOPIC : 11 - HARMONIC ANALYSIS

Harmonic Analysis:

The process of finding the fourier series for a function given by such values of the function and independent variable is known as Harmonic analysis.

$$y = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where $a_0 = 2\left[\dfrac{\Sigma y}{n}\right]$

$a_n = 2\left[\dfrac{\Sigma y \cos nx}{n}\right]$

① find the series upto the second harmon

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Sol:

$$y = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$$

$$a_0 = 2\left[\frac{\Sigma y}{n}\right] = 2\left(\frac{8.7}{6}\right) = 2(1.45) = 2.9$$

$$a_1 = 2\left[\frac{\Sigma y \cos x}{n}\right] = 2\left(\frac{-1.1}{6}\right) = -0.3667$$

$$a_2 = 2\left[\frac{\Sigma y \cos 2x}{n}\right] = 2\left(\frac{-0.3}{6}\right) = -0.1$$

$$b_1 = 2\left[\frac{\Sigma y \sin x}{n}\right] = 2\left(\frac{0.5196}{6}\right) = 0.1732$$

$$b_2 = 2\left[\frac{\Sigma y \sin 2x}{n}\right] = 2\left(\frac{-0.1732}{6}\right) = -0.0577$$

Sub these values in ①

$$y = \frac{2.9}{2} + (0.3667) \cos x + (-0.1) \cos 2x$$
$$+ (0.1732) \sin x - (0.0577) \sin 2$$

x	Cosx	Sinx	Cos2x	Sin2x	ycosx	ysinx ycos2α	ycos2x	ysinx
0	1	0	1	0	1	0	1	0
	0.5	0.866	-0.5	+0.866	0.7	1.2124	-0.7	1.2124
	-0.5	0.866	-0.5	-0.866	-0.95	1.6454	-0.95	1.6454
	-1	0	1	0	-1.7	0	1.7	0
	-0.5	-0.866	-0.5	0.866	-0.75	-1.299	-0.75	1.299
	0.5	-0.866	-0.5	-0.866	0.6	-1.0392	-0.6	-1.0392
	1	0	1	0	1	0	1	0
					-1.1	0.9646	-0.3	-0.1732

2. Compute first two harmonic of the fourier series of $f(x)$ from the table below.

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
y	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

Sol:

$$\theta = \frac{2\pi x}{T}.$$

$$a_0 = 2\left[\frac{\sum y}{n}\right] = 2\left[\frac{4.5}{6}\right] = 1.5$$

$$a_1 = 2\left[\frac{\sum y \cos\theta}{n}\right] = 2\left[\frac{1.12}{6}\right] = 0.3733$$

$$a_2 = 2\left[\frac{\sum y \cos 2\theta}{n}\right] = 2\left[\frac{2.67}{6}\right] = 0.8900$$

$$b_1 = 2\left[\frac{\sum y \sin\theta}{n}\right] = 2\left[\frac{3.0137}{6}\right] = 1.0046$$

$$b_2 = 2\left[\frac{\sum y \sin 2\theta}{n}\right] = 2\left[\frac{-0.3291}{6}\right]$$

$$= -0.1097.$$

Sub all these values in ①.

$$y = 1.5 + (0.3733)\cos\theta + (0.8900)\cos 2\theta$$
$$+ (1.0046)\sin\theta + (-0.1097)\sin 2\theta$$

$x = \theta$	y	$\cos\theta$	$\sin\theta$	$\cos 2\theta$	$\sin 2\theta$	$y\cos\theta$	$y\sin\theta$	$y\cos 2\theta$	$y\sin 2\theta$
0	1.98	1	0	1	0	1.98	0	1.98	0
$\dfrac{\Pi}{3}\,(60°)$	1.3	0.5	0.8660	-0.5	0.866	0.65	1.1258	-0.65	1.1258
$\dfrac{2\Pi}{3}\,(120°)$	1.05	-0.5	0.8660	-0.5	-0.866	-0.525	0.9093	-0.525	-0.9093
$\Pi\,(180°)$	1.3	-1	0	1	0	-1.3	0	1.3	0
$\dfrac{4\Pi}{3}\,(240°)$	-0.88	-0.5	-0.866	-0.5	0.866	$+0.44$	0.7621	0.44	-0.7621
$\dfrac{5\Pi}{3}\,(300°)$	-0.25	0.5	-0.866	-0.5	-0.866	-0.125	0.2165	0.125	0.216
	4.5					1.12	3.037	2.67	-0.324

③ Find the Fourier series as far as the second harmonic to represent the function given in the following data:

x	0	1	2	8	4	5
$f(x)$	9	18	24	28	26	20

Sol:

$$2l = 6$$
$$l = \frac{6}{2} = 3.$$
$$l = 3.$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{3} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3}$$

$$a_0 = 2\left[\frac{\Sigma y}{6}\right] = 2\left[\frac{125}{6}\right] = 41.6667$$

$$a_1 = 2\left[\frac{\Sigma y \cos\frac{\pi x}{3}}{6}\right] = 2\left[\frac{-25}{6}\right] = -8.333$$

$$a_2 = 2\left[\frac{\Sigma y \cos\frac{2\pi x}{3}}{6}\right] = 2\left[\frac{-7}{6}\right] = -2.333$$

$$b_1 = 2\left[\frac{\Sigma y \sin\frac{\pi x}{3}}{6}\right] = 2\left[\frac{-3.464}{6}\right] = -1.1547$$

$$b_2 = 2\left[\frac{\Sigma y \sin\frac{2\pi x}{3}}{6}\right] = 2\left[0\right] = 0.$$

$$f(x) = \frac{41.6667}{2} + (-8.333)\cos\frac{\pi x}{3} + (-2.333)\cos\frac{2\pi x}{3}$$
$$+ (-1.1547)\sin\pi x + 0.$$

y	$\cos\frac{\pi x}{3}$	$\sin\frac{\pi x}{3}$	$\cos\frac{2\pi x}{3}$	$\sin\frac{2\pi x}{3}$	$y\cos\frac{\pi x}{3}$	$y\sin\frac{\pi x}{3}$	$y\cos\frac{2\pi x}{3}$	$y\sin\frac{2\pi x}{3}$
9	1	0	1	0	9	0	9	0
18	0.5	0.866	-0.5	0.866	9	15.588	-9	15.588
24	-0.5	0.866	-0.5	-0.866	-12	20.7846	-12	-20.784
28	-1	0	1	0	-28	0	28	0
26	-0.5	-0.866	-0.5	0.866	-13	-22.516	-13	+22.516
20	0.5	-0.866	-0.5	-0.866	10	-17.2200	-10	-17.32
12.5	1				-25	-3.464	-7	0