Unit-I

Fourier Series

Definition:

Periodic function:

A function $f(x)$ is said to have a period T for all x, $f(x+T) = f(x)$, where T is a +ve constant. The least value of $T > 0$ is called the period of $f(x)$.

e.g: $\sin x$, $\cos x$ are periodic function with period 2π.

Fourier series:

If $f(x)$ is a periodic function and satisfies Dirichlet conditions, then it can be represented by an infinite series called Fourier series.

Uses of Fourier series:

Fourier series are particularly suitable for expansion of periodic functions. we come across many periodic functions in voltage, current flex density, applied force, potential and electromagnetic force in electricity, hence

fourier series are very useful in electrical engineering problems.

Deduction: Sum of the Fourier series

Continuous   Discontinuous

                  End point   Middle point

Substitute the value   Average values $\dfrac{L.H.S + R.H.S}{2}$

directly   at end points

Problems:

① Sum of the Fourier series for

$$f(x) = \begin{cases} x^2 & -\pi \le x \le 0 \\ 0 & 0 \le x \le \pi \end{cases} \quad \text{at } x = \frac{\pi}{2}, \frac{-\pi}{2}.$$

sol:

$x = \frac{\pi}{2}$ is a continuous point at $(0, \pi)$

$\therefore f(x) = 0.$

$x = -\frac{\pi}{2}$ is a continuous point at $(-\pi, 0)$

$f(x) = x^2$

$\Rightarrow f\left(\dfrac{-\pi}{2}\right) = \left(\dfrac{-\pi}{2}\right)^2 = \dfrac{\pi^2}{4}.$

$f(x) = \dfrac{\pi^2}{4}.$

② Sum the Fourier series for

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2, & 1 < x < 2 \end{cases} \quad at \ x=0.$$

Sol:

$x=0$ is a discontinuous and end point.

Sum of the fourier series $= \dfrac{f(0) + f(2)}{2}$

$$= \dfrac{0 + 2}{2} = \dfrac{2}{2}$$

Sum of the fourier series $\Big\} = 1.$

③ Sum of the fourier series for

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ x^2, & \pi < x < 2\pi \end{cases} \quad at \ x=\pi.$$

Sol:

$x=\pi$ is a discontinuous and middle point.

Sum of the fourier series $= \dfrac{f(\pi-) + f(\pi+)}{2}$

$$= \dfrac{\pi + \pi^2}{2}.$$

Dirichlet condition:

i) $f(x)$ is periodic, single valued and finite

ii) $f(x)$ has a finite no. of finite discontinuous.

iii) $f(x)$ has no infinite discontinuous

iv) $f(x)$ has a finite no. of maxima and minima.

Formula for fourier series in $(0, 2\pi)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where

$$a_0 = \frac{2}{b-a} \int_{a}^{b} f(x)\, dx$$

$$a_n = \frac{2}{b-a} \int_{a}^{b} f(x) \cos nx\, dx$$

$$b_n = \frac{2}{b-a} \int_{a}^{b} f(x) \sin nx\, dx$$

Problems:

① Expand $f(x) = x^2$ in $(0, 2\pi)$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.