q

Methods of proving theorem:

1) Direct proof:

It is a proof that the implication $p \to q$ is true that proceeds by showing that q must be true when p is true.

Eg: Give a direct proof of the theorem "If n is an odd integer then n^2 is an odd integer.

Soln:

Given: "If n is an odd integer, then n^2 is an odd integer"

Let p: n is an odd integer.

Q: n^2 is an odd integer.

Hypothesis: First assume that p is true.

(ie) n is an odd integer

Analysis: By def. of odd integer $n = 2k+1$, where k is some intege

$n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

Conclusion: $= 2m^2 + 1$ where $m = 2k^2 + 2k$

We observe that R.H.S value is not divisible by 2.

$\therefore$ n^2 is not divisible by 2

$\therefore$ n^2 is an odd integer.

(ie) $p \to Q$ is true.

1) Indirect proof (i) Contraposition) :

It is a proof that the implication $p \to q$ is true that proceeds by showing that p must be False when q is false.

(i.e) the implication $p \to q$ can be proved by showing that its contrapositive $\neg q \to \neg p$ is true.

Eg: Give an indirect proof of the theorem "If $3n+2$ is odd then n is odd"

Soln:

P : $3n+2$ is odd.

Q : n is odd.

Hypothesis: ~~Assume~~ Since $p \to Q$ its contrapositive $\neg Q \to \neg p$ are logically equivalent.

So assume that $\neg Q$ is true (ie) n is even

Analysis: If n is even, then $n = 2K$, for some integer.

$$\therefore 3n+2 = 3(2k)+2$$
$$= 6k+2$$
$$= 2(3k+1)$$
$$= \text{even integer.}$$

Conclusion: R.H.S of $3n+2$ is divisible by 2.

This means that $3n+2$ is an even integer.

(ie) $\neg p$ is true.

$\therefore$ $\neg Q \to \neg p$ is true.

(ie) If $3n+2$ is odd then n is odd.

Proof by contradiction.

A proof by contradiction establishes by assuming that the hypothesis P is true and that the conclusion Q is false and then, using P and ¬Q as well as other axioms and definitions derives a contradiction.

Eg: Give a proof by contradiction of the theorem "If 3n+2 is odd, then n is odd"

Soln:

Let P: 3n+2 is odd

Q: n is odd

Hypothesis: Assume that $P \rightarrow Q$ is false.

(ie) Assume that P is true and Q is false

(ie) n is not odd $\Rightarrow$ n is ~~ed~~ even.

Analysis: If n is even then $n=2k$ for some integer k

$$3n+2 = 3(2k)+2$$
$$= 6k+2$$
$$= 2(3k+1)$$

Conclusion.

We observe that R.H.S value of 3n+2 is divisible by 2. This means that 3n+2 is even. This contradicts the assumption P is true.

$\therefore P \rightarrow Q$ is true.

2. Prove that $\sqrt{2}$ is irrational by giving a Proof using Contradiction.

Soln:

Let p be the proposition "$\sqrt{2}$ is irrational"

Suppose that $\neg p$ is true. Then $\sqrt{2}$ is rational.

(ie) assume $\sqrt{2}$ is rational.

$\therefore \sqrt{2} = \frac{a}{b}$ for some integers a and b such that a and b have no common factors.

$\therefore \frac{a^2}{b^2} = 2$

$\Rightarrow 2b^2 = a^2$

This means that a^2 is even, implying that a is even, $a = 2c$ for some integer c.

Thus $2b^2 = 4c^2$

So $b^2 = 2c^2$

This means that b^2 is even. Hence b must even as well.

$\therefore b = 2k$ for some integer k.

Thus a and b are even. Hence they have a common factor 2. This contradicts the assumption a & b have no common factors. Thus our assumption $\sqrt{2}$ is rational is wrong.

Hence $\sqrt{2}$ is irrational