



Inference theory of statement calculus -

Def: Premises (or) Hypotheses:

Premises is a statement which is assumed to be true.

Inference theory

The main function of logic is to provide rules of inference, (or) principle of reasoning.

The theory associated with such rules is known as inference theory because it is concerned with the inferring of a conclusion from certain premises.

Formal Proof.

Process of determining a conclusion from a set of premises by using the accepted rules of reasoning is called formal proof and the argument (or) conclusion is called a valid argument (or) valid conclusion.

Def: Let A and B be two statement formulas we say that " B logically follows from A " (or) " B is a valid conclusion of the premise A " if $A \rightarrow B$ is a tautology, that is $A \rightarrow B$.



Implications

1.
$$\left. \begin{array}{l} p \wedge q \Rightarrow p \\ p \wedge q \Rightarrow q \end{array} \right\} \text{Simplification}$$
2.
$$\left. \begin{array}{l} p \Rightarrow p \vee q \\ q \Rightarrow p \vee q \end{array} \right\} \text{Addition}$$
3.
$$\neg p \Rightarrow p \rightarrow q$$
4.
$$q \Rightarrow p \rightarrow q$$
5.
$$\neg(p \rightarrow q) \Rightarrow p$$
6.
$$\neg(p \rightarrow q) \Rightarrow \neg q$$
7.
$$p, q \Rightarrow p \wedge q$$
8.
$$\neg p, p \vee q \Rightarrow q \quad \text{disjunctive syllogism}$$
9.
$$p, p \rightarrow q \Rightarrow q \quad \text{modus ponens}$$
10.
$$\neg q, p \rightarrow q \Rightarrow \neg p \quad \text{modus tollens}$$
11.
$$p \rightarrow q \rightarrow r \Rightarrow p \rightarrow r \quad \text{hypothetical syllogism}$$
12.
$$p \vee q, p \rightarrow r, q \rightarrow r \Rightarrow r \quad \text{dilemma}$$



Rules of inference:

A set of premises H_1, H_2, \dots, H_n and a conclusion C are given. We assume that H_1, H_2, \dots, H_n are all true. We want to conclude the conclusion C . (eg) we want to prove the conclusion C is true. The following rules are used.

Rule P: A premise may be introduced at any point in the derivation.

Rule T: A formula S may be introduced in a derivation if S is tautologically implied by any one (or) more of the preceding formulas in the derivation.

Rule CP: If we can derive S from R and a set of premises, then we can derive $R \rightarrow S$ from the set of premises alone.

2) Show that R is valid inference from the premises $P \rightarrow Q$, $Q \rightarrow R$ and P .

Soln:

Steps	Derivation	Rule	Reason
1	$P \rightarrow Q$	P	Given Premise
2	$Q \rightarrow R$	P	Given premise
3	$P \rightarrow R$	T	[(1), (2) Hypothetical Syllogism $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow (P \rightarrow R)$]
4	P	P	Given Premise
5	R	T	[(3), (4) modus ponens $P, P \rightarrow R \Rightarrow R$]

Here R is concluded from the given premises.