



DISCRETE MATHEMATICS

UNIT - I

LOGIC AND PROOFS

Proposition (or) Statement:

Proposition is a declarative statement that is either true (or) false, but not both. The truth value of proposition is true (or) false.

Eg: (i) Chennai is the capital of Tamil Nadu (True)
(ii) $2+7=10$ (False)

The following sentences are not propositions:

1. This statement is false [we cannot say True (or) False]
2. Do you speak English? [is a question, not a statement]
3. Close the door. [is a command, not a statement]
4. $2+4=2$ is neither true (or) false.

In the above sentences we cannot assign true (or) false.

Atomic Statement (or) primary (or) simple statement

Declarative sentences which cannot be further split into simple sentences are called atomic statements [also called primary statements (or) primitive statements].

Eg: Rama is a boy.

Compound Proposition:

It is a proposition consisting of two (or) more simple propositions using logical operators.

Eg: Raju is a boy and Sita is a girl.

Truth table:

It displays the relationship between the truth values of propositions.



Connectives:

Negation:

If p is a proposition (statement), then its negation is denoted by $\neg p$ (or) $\sim p$ read as "not p ".

Truth table for negation

p	$\neg p$
T	F
F	T

Eg: p - Ram is intelligent
 $\neg p$ - Ram is not intelligent.

Conjunction [AND]

Let p and q be two propositions. The proposition $p \wedge q$ is called the conjunction of p & q .

Truth table for Conjunction

p	q	$p \wedge q$
T	F	F
F	T	F
T	T	T
F	F	F

Note: $p \wedge q$ is True when both p and q are True. Otherwise $p \wedge q$ is False.

Eg: p - Rani wears a ring
 q - Rani wears a chain
 $p \wedge q$ - Rani wears a ring and chain

Disjunction [OR]

If p & q are two statements, then statement $p \vee q$ is (read as p (or) q) called a disjunction.

Truth table for Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note: $p \vee q$ is False when p & q are False. Otherwise $p \vee q$ is True.



Eg:

P: Raju appointed in wipro

Q: Raju appointed in HCL

$P \oplus Q$: Raju appointed in wipro or HCL

Exclusive OR:

If the p and q are two propositions, the exclusive OR of p and q is denoted by $P \oplus Q$ is the proposition that is true, when exactly one of p and q is true and false otherwise.

P	q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement [If... then] [\rightarrow]

If p and q are any two statements then the statement $p \rightarrow q$ which is read as "If p then q" is called a conditional statement.

Truth table

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Note:
 $P \rightarrow Q$ is false when P is true and q is false.
otherwise it is true.

Eg: P: Ram is a Computer Science Student [T]

q: Ram study DBMS [T]

$P \rightarrow q$: If Ram is a Computer Science Student, then he will study DBMS [T]



Biconditional Statement [\leftrightarrow] [if and only if]

If 'p' and 'q' are any two statements, then the statement $p \leftrightarrow q$ which is read as "p if and only if q" is called bi-conditional statement.

Truth table

P	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Note

$p \leftrightarrow q$ is true if both p and q have same truth values. Otherwise $p \leftrightarrow q$ is false.

Eg: P: You can take the flight (T)

q: You buy a ticket (T)

$p \leftrightarrow q$: You can take the flight if and only if you buy a ticket (T).

Eg: P: You cannot take the flight (F)

q: You do not buy a ticket (F)

$p \leftrightarrow q$: You cannot take the flight if and only if you do not buy a ticket (T)

Symbolize the statements using logical connectives.

1) Using the statements. R: Mark is rich
H: Mark is happy.

Write the following statements in a symbolic form:

(a) Mark is poor but happy.

(b) Mark is rich (or) unhappy.

(c) Mark is neither rich nor happy.

(d) Mark is poor (or) he is both rich and unhappy.



Solu: Symbolic form

(a) $\neg R \wedge H$.

(b) $R \vee \neg H$

(c) $\neg R \vee \neg H$

(d) $\neg R \vee (R \wedge H)$.

2) Let p, q, r represent the following Propositions

p : It is raining

q : The Sun is shining

r : There are clouds in the sky

Symbolize the following statements.

(i) If it is raining, then there are clouds in the sky.

(ii) If it is not raining, then the Sun is not shining and there are clouds in the sky.

(iii) The Sun is shining if and only if it is not raining.

Solu: Symbolic form

(i) $p \rightarrow r$

(ii) $\neg p \rightarrow (\neg q \wedge r)$

(iii) $q \leftrightarrow \neg p$.

Construction of Truth Table.

Construct the truth table for $p \wedge (p \vee q)$

Solu:

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F