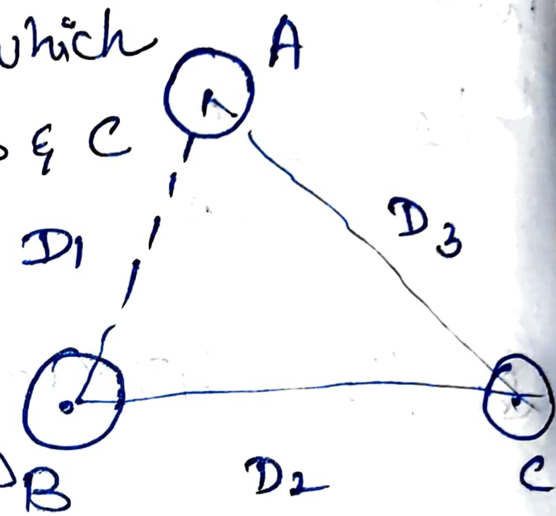
Inductance of a 3ϕ Tr. line

Consider a 3ϕ Tr. line which consists of 3 conductors A, B & C of radius r meters and D_1, D_2 and D_3 are the distances b/w the conductors.



The conductors carry currents of I_A, I_B & I_C respectively.

"Depending upon the distances b/w the conductors, we have two kinds of configurations

i) Symmetrical Spacing

ii) Unsymmetrical "

If the three conductors A, B & C are placed symmetrically at the corners of an equilateral triangle, then it is called symmetrical spacing.

If the three conductors A, B & C are not equally placed, then it is called as unsymmetrical spacing.

For a balanced circuit $I_A + I_B + I_C = 0$.

Flux linkage with conductor A due to its own current $= \mu_0 \frac{I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right]$

External flux produced due to I_B & I_C with conductor A

∴ flux linkage with A due to current $I_B = \mu_0 \frac{I_B}{2\pi} \int_{D_1}^{\infty} \frac{dx}{x}$.

$I_C = \mu_0 \frac{I_C}{2\pi} \int_{D_3}^{\infty} \frac{dx}{x}$

Total flux linkage with conductor A

$$\psi_A = \mu_0 \frac{I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right] + \mu_0 \frac{I_B}{2\pi} \int_{D_1}^{\infty} \frac{dx}{x} + \mu_0 \frac{I_C}{2\pi} \int_{D_3}^{\infty} \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A[\ln x]_r^{\infty} + I_B[\ln x]_{D_1}^{\infty} + I_C[\ln x]_{D_3}^{\infty}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \ln\infty - I_A \ln r + I_B \ln\infty - I_B \ln D_1 + I_C \ln\infty - I_C \ln D_3\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \ln\infty\,(I_A + I_B + I_C) - I_A \ln r - I_B \ln D_1 - I_C \ln D_3\right]$$

$\because$ $I_A + I_B + I_C = 0$ for balanced circuit

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \ln\infty\,(0) - I_A \ln r - I_B \ln D_1 - I_C \ln D_3\right]$$

$$\boxed{\psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r - I_B \ln D_1 - I_C \ln D_3\right]}$$

With Symmetrical Spacing.

$$D_1 = D_2 = D_3 = D$$

$$\psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r - I_B \ln D - I_C \ln D\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r - \ln D\,(I_B + I_C)\right]$$

$\because I_B + I_C = -I_A$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r + I_A \ln D\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \ln \frac{D}{r}\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \ln \frac{D}{r}\right] \text{ wb-turn}$$

Inductance of conductor A

$$L_A = \frac{\psi_A}{I_A} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln \frac{D}{r}\right] \; H/m$$

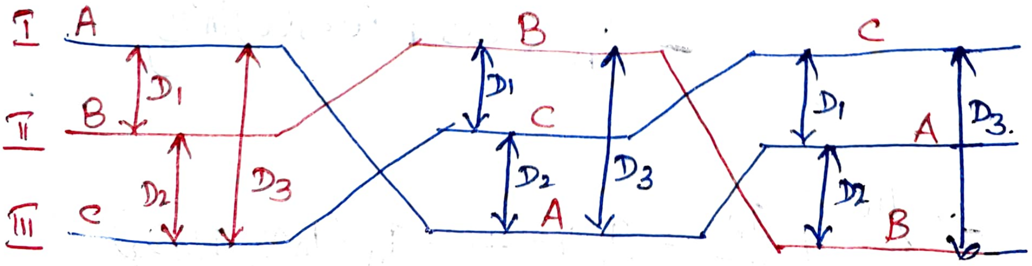
$$L_A = 2\times 10^{-7}\left(\ln \frac{D}{r'}\right) \quad \text{where } r' = 0.7788\,r$$

|||ly

$$L_B = \frac{\psi_B}{I_B} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln \frac{D}{r}\right] \; H/m$$

$$L_C = \frac{\psi_C}{I_C} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln \frac{D}{r}\right] \; H/m$$

Unsymmetrical Spacing.



* flux linkage & inductance of each phase are not equal, voltage drop is each phase is different & receiving end voltage is not same in all the three phases.

* In order to have equal voltage drop in all the three phases, positions of the conductor are interchanged at equal distance.

* Such an exchange of position is known as Transposition.

Total flux linkage with the conductor A in position I is.

$$\psi_{A1} = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r - I_B \ln D_1 - I_C \ln D_3\right]$$

$$I_A = I_A \angle 0^\circ = I_A (1+j0)$$

$$I_B = I_A \angle -120^\circ = I_A (-0.5 - j0.866)$$

$$I_C = I_A \angle +120^\circ = I_A (-0.5 - j0.866)$$

$$\psi_{A1} = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r - I_A(-0.5+j0.866)\ln D_1 - I_A(-0.5+j0.866)\ln D_3\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r + 0.5 I_A (\ln D_1 + \ln D_3) + j0.866 I_A (\ln D_1 - \ln D_3)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r + 0.5 I_A \ln D_1 D_3 + j0.866 I_A \ln \frac{D_1}{D_3} + \ln (D_1 D_3)^{0.5}\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} - \ln r + \ln \sqrt{D_1 D_3} + j0.866 \ln \frac{D_1}{D_3}\right]$$

Illy Total flux linkage at position II

$$\psi_{AII} = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r - I_B \ln D_3 - I_C \ln D_2\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} - \ln r + \ln \sqrt{D_3 D_2} + j0.866 \ln \frac{D_3}{D_2}\right]$$

Illy Total flux linkage at position III

$$\psi_{AIII} = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \ln r - I_B \ln D_2 - I_C \ln D_1\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} - \ln r + \ln \sqrt{D_2 D_1} + j0.866 \ln \frac{D_2}{D_1}\right]$$

Total flux linkages is average of flux linkages.

$$\psi_A = \frac{\psi_{AI} + \psi_{AII} + \psi_{AIII}}{3}$$

$$= \frac{\mu_0 I_A}{3 \times 2\pi}\left[\frac{3}{4} - 3\ln r + \ln \sqrt{D_1 D_3} \times \ln \sqrt{D_3 D_2} \times \ln \sqrt{D_2 D_1} + j0.866 \ln \frac{D_1}{D_3} \times \frac{D_3}{D_2} \times \frac{D_2}{D_1}\right]$$

$$= \frac{\mu_0 I_A}{3 \times 2\pi}\left[\frac{3}{4} - 3\ln r + \ln D_1 D_2 D_3 + j0.866 \ln 1\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} - \ln r + \frac{1}{3}\ln D_1 D_2 D_3\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \ln \frac{(D_1 D_2 D_3)^{1/3}}{r}\right]$$

Inductance of conductor A

$$L_A = \frac{\psi_A}{I_A} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln \frac{(D_1 D_2 D_3)^{1/3}}{r}\right] H/m$$

$$L_A = 2\times 10^{-7}\left[\ln \frac{\sqrt[3]{D_1 D_2 D_3}}{r'}\right]$$

$$\boxed{L_A = 2\times 10^{-7}\left(\ln \frac{D_m}{r'}\right)}$$

where $r' = 0.7788\,r$.

$D_m = \sqrt[3]{D_1 D_2 D_3}$ is D_{eq}.

GMD. equivalent spacing.